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TEMPERATURE OF A TEMPERATE GLACIER*

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ABSTRACT. The closure of water-filled glacier bore holes is considered and it is concluded that freezing due to conduction to the surrounding ice is usually the dominant process. On this basis englacial temperatures are obtained from a bore hole near the equilibrium line of Blue Glacier, U.S.A., where the ice thickness is about 125 m. Temperatures range from −0.03° C near the surface to −0.13° C at a depth of 105 m, with an estimated uncertainty of 0.02 or 0.03 deg. On the average the temperature is about 0.05 deg colder than the equilibrium temperature of ice and pure water. It is shown that at this temperature small amounts of water-soluble impurities play an important role in the thermal behavior of the ice. This leads to a new definition of temperate ice in terms of its effective heat capacity. The effective heat capacity of the Blue Glacier ice is apparently much larger than that of pure ice at the same temperature.

RESUMÉ. *Température d'un glacier tempéré.* On a observé les processus de fermeture de trous de forage remplis d'eau dans un glacier. On en conclut que le regel dû à la conduction vers la glace du pourtour est ordinairement le processus dominant. Sur cette base, les températures dans la glace ont été mesurées à partir d'un forage près de la ligne d'équilibre du Blue Glacier, U.S.A., où l'épaisseur de la glace est d'environ 125 m. Les températures s'étagent depuis −0,03°C près de la surface jusqu'à −0,14°C à une profondeur de 105 m avec une incertitude estimée à 0,02 ou 0,03 deg. En moyenne, la température est d'environ 0,05 deg plus froide que la température d'équilibre entre la glace et l'eau pure. On montre qu'à cette température des petites quantités d'impuretés solubles jouent un rôle important dans le comportement thermique de la glace. Ceci conduit à une nouvelle définition de la glace tempérée d'après la réserve effective de chaleur qu'elle contient. La réserve effective de chaleur de la glace du Blue Glacier est apparemment beaucoup plus grande que celle d'une glace pure à la même température.

ZUSAMMENFASSUNG. *Temperatur eines temperierten Gletschers.* Aus dem Schliessvorgang wassergefüllter Gletscherbohrlöcher wurde abgeleitet, dass gewöhnlich das Gefrieren durch Wärmeleitung in das umgebende Eis der bestimmende Prozess ist. Auf dieser Basis wurden Gletschertemperaturen aus einem Bohrloch nahe der Gleichgewichtslinie des Blue Glacier, USA, gewonnen, wo das Eis etwa 125 m mächtig ist. Die Temperaturen reichten von −0,03° C nahe der Oberfläche bis −0,13° C in einer Tiefe von 105 m mit einer geschätzten Unsicherheit von 0,02 oder 0,03 deg. Im Mittel ist die Temperatur ungefähr 0,05 deg kälter als die Gleichgewichtstemperatur von Eis und reinem Wasser. Es wird gezeigt, dass bei dieser Temperatur kleine Mengen von wasserlöslichen Verunreinigungen eine wichtige Rolle im thermischen Verhalten des Eises spielen. Dies führt zu einer neuen Definition für temperiertes Eis in Abhängigkeit von seiner effektiven Wärmekapazität. Die effektive Wärmekapazität des Blue Glacier-Eises ist sichtlich viel grösser als die von reinem Eis bei gleicher Temperatur.

1. INTRODUCTION

Glaciers have long been classified as "temperate" and "polar" (Ahlmann, 1935), a temperate glacier being one that is at the melting point throughout except for a thin surface layer subject to seasonal cooling. Although this concept has been an important guide to thinking about glacier behavior, it has been of little comfort to many drillers who have melted bore holes in "temperate" ice. The persistent tendency of their holes to refreeze, often at all depths, has raised the question of whether or not temperate glaciers really exist. Even on Blue Glacier, seemingly an ideal example of a temperate glacier (LaChapelle, 1959), the problem has been encountered (Shreve and Sharp, 1970).

The present work proposes an explanation for the observed rates of closure of bore holes, and uses these rates to determine glacier temperature. The basic concept is that the closure of a water-filled bore hole by freezing indicates a heat flux away from the hole wall which can be interpreted in terms of ice temperature at the time the hole was drilled. Although the measurement of temperature by this method is indirect, it is attractive because the measurement of closure rate is simple and results can be obtained soon after the hole is drilled. On the other hand, direct temperature measurement is considerably more difficult,

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mainly because of the problem of freezing temperature sensing elements into temperate or nearly temperate ice, and the necessity to wait (typically one season) for equilibrium to become re-established.

The experimental procedure used here is described in Section 2, and the evaluation of englacial temperature from the field data in Section 3. In Section 4 it is shown that these temperatures are colder than the equilibrium temperature of both ice and pure water and of ice and air-saturated water. This raises the question of the meaning of temperate ice, which has also been considered by Lliboutry (1971). Soluble impurities, both air and salts of atmospheric origin, play a key role since their presence in the water phase lowers the equilibrium temperature. The effect of impurities on the thermal properties of ice and on the evaluation of temperature from bore-hole closure data is considered in Section 5. Using these results, it is shown in Section 6 that the familiar concept of ice at the pressure melting point is not well defined. (See also Lliboutry (1971).) A new definition of temperate ice is developed and the field results discussed in its context.

2. Experimental procedure

Field experiments were carried out on Blue Glacier, Mount Olympus, Washington, U.S.A. (LaChapelle, 1959; Allen and others, 1960), during the summers of 1969 and 1970. Bore holes were made by thermal drills similar to those described by Shreve and Sharp (1970) and were uncased. The holes were located along a 175 m transverse profile a short distance above the equilibrium line of the glacier, where the ice thickness is about 125 m. This is near the site of a bore-hole deformation study by Kamb and Shreve (1966). In most of the holes water stood up to the base of the permeable firn for a period of one to three months, and it was found that all such holes tended to close up in the course of time at essentially all depths. This phenomenon is distinct from the rapid near-surface closure encountered on some glaciers in the winter chill zone (for example, Raymond, unpublished, p. 170). In order to keep the holes open for other measurements, they were maintained at almost constant radius by "reaming" at intervals of about one week with a conical thermal drill, or "reamer". From the electrical power supplied to the reamer and its rate of progress, the average thickness of ice Δa removed from the bore-hole wall at any depth can be calculated. If a is the hole radius

$$H\Delta a = \frac{P}{2\pi a u} \tag{1}$$

where H (= 300 MJ m^{-3}) is the latent heat per unit volume of ice, P is the power applied to the reamer, and u is its speed.

3. Temperature evaluation from bore-hole closure rate in ice free of impurites

We will first assume that closure of the water-filled holes occurs primarily by freezing: a layer of ice is laid down on the hole wall as a consequence of heat being conducted outward into the surrounding ice mass. The temperature of the wall is fixed at a known value because the hole contains water. In Sections 3.1 and 3.2 the analysis is described by which the initial temperature can be inferred from the measured bore-hole closure rate, and it is then applied to the Blue Glacier data. The effect of impurities on the thermal properties of ice, which can be substantial, is not considered in this analysis but is postponed until Section 5. Other possible causes of bore-hole closure are discussed in Sections 3.3 and 3.4 and found to be unimportant by comparison with the refreezing process. This discussion justifies *a posteriori* the initial assumption of closure by freezing.

3.1. *Conductive heat transfer*

To evaluate the effect of heat conduction on bore-hole closure, consider as an appropriate model of the ice mass a region bounded internally by a circular cylinder of radius a. Suppose that the ice is initially at the temperature T_0 and that at time $t = 0$ the temperature of the boundary is fixed at T_b. It is therefore assumed that the bore-hole wall temperature T_b is established instantaneously, although in practice, of course, it takes a finite time to drill the hole. We shall see later that this introduces only a small error.

The thermal diffusion equation is

$$K_i \nabla^2 T = \rho_i c_i \frac{\partial T}{\partial t} \tag{2}$$

where K_i is the thermal conductivity of ice, ρ_i is its density, and c_i is its specific heat capacity. Usually a thermal diffusivity κ_i is defined by

$$\kappa_i = \frac{K_i}{\rho_i c_i}.$$

The appropriate solution giving the temperature T as a function of distance r from the axis and of time t is discussed by Carslaw and Jaeger (1959, p. 334):

$$T - T_b = (T_0 - T_b) \frac{2}{\pi} \int_0^\infty \exp(-\kappa_i s^2 t) \frac{N_0(sr) J_0(sa) - J_0(sr) N_0(sa)}{J_0^2(sa) + N_0^2(sa)} \frac{ds}{s} \tag{3}$$

where J_0 and N_0 are Bessel functions of the first and second kinds. If T_0 and T_b vary with z, the axial coordinate, Equation (3) is still valid as long as d^2T_0/dz^2 and d^2T_b/dz^2 are negligible. This amounts to neglecting conduction in the z direction during the period of measurements, and is a good approximation. The heat flux f_c conducted away from the bore-hole wall is

$$f_c = -K_i \left(\frac{\partial T}{\partial r}\right)_{r=a}. \tag{4}$$

It is convenient to define a dimensionless time $t^\star$ and a dimensionless flux $f^\star$ as follows

$$t^\star = \frac{\kappa_i}{a^2} t = \frac{K_i}{\rho_i c_i a^2} t, \tag{5}$$

$$f^\star = -\frac{a}{K_i(T_0 - T_b)} f_c. \tag{6}$$

Although $f^\star$ could be obtained in the general case from Equations (3) and (4), an asymptotic expansion, valid for large $t^\star$, is useful for our purposes:

$$f^\star = 2\left\{\frac{1}{\ln 4t^\star - 2\gamma} - \frac{\gamma}{(\ln 4t^\star - 2\gamma)^2} - \ldots\right\} \tag{7}$$

where γ ($= 0.5772 \ldots$) is Euler's constant.

If freezing due to conduction is the only important mechanism of hole closing,

$$H\Delta a = \int_{t_1}^{t_2} f_c \mathrm{d}t \tag{8}$$

where t_1 and t_2 are successive reaming times.

Combination of Equations (1), (6) and (8) gives

$$T_0 - T_b = -P/2\pi u K_i \int_{t_1}^{t_2} f^\star \mathrm{d}t \tag{9}$$

in which $f^\star$ is to be calculated from Equation (7). This result is used to determine the difference between initial glacier temperature T_0 and bore-hole wall temperature T_b. Because a, ρ_i and c_i enter into Equation (9) only logarithmically, via Equations (5) and (7), it is not necessary that they be known accurately.

3.2. *Application to the Blue Glacier data*

The theory will now be applied to determine glacier temperature using data from a bore hole called R1. Hole R1 was typical of the water-filled holes drilled in both 1969 and 1970. Its pertinent history is given in Table I,

TABLE I. HISTORY OF BORE HOLE R1

Date	*Operation*	*Interval*
14–16 July 1969	drill	
17 July 1969	ream	
		I
26 July 1969	ream	
		II
1 August 1969	ream	

where the two intervals during which closure rate was measured have been given the labels I and II. Before applying Equation (9) we need to discuss the evaluation of three of the quantities it contains: $f^\star$, u and T_b.

Let us first investigate the behavior of the dimensionless flux $f^\star$. The radius of the hole was 31 mm. For a thermal diffusivity κ_i of 1.1 mm² s⁻¹, Equation (5) implies that $t^\star = 1$ when $t = 0.25$ h. Consequently we expect the asymptotic form for $f^\star$, Equation (7), to hold soon after drilling and to be applicable to the above data. This is borne out by comparison of Equation (7) with a graphical form for $f^\star$ given by Carslaw and Jaeger (1959). The dependence of $f^\star$ on time is shown in Figure 1. This indicates that if several days have elapsed since drilling, $f^\star$ varies only slowly with time. This means that although the drilling of hole R1 required two days, the flux anywhere in the hole a few days later was almost the same as if the hole had been drilled instantaneously. Hence it is valid to apply the assumption of instantaneous drilling to the above data. Reaming could also be considered instantaneous. The values of $\int f^\star \mathrm{d}t$ for intervals I and II are 2.52d and 1.51d respectively.

We next consider the evaluation of the speed of the reamer u. The depth of the reamer was logged as a function of time on 26 July and 1 August, and its speed determined as a function of depth from a point-by-point differentiation of the data. Since different amounts of ice were removed on the two dates, plots of reamer depth against time will be different. However, it is possible to normalize one set of data to the other; this is shown in Figure 2, in which the data of 1 August are normalized to those of 26 July. Since the power was nominally the same (800 W), the normalization factor should be given by the ratio of the values of $\int f^\star \mathrm{d}t$ appropriate to the intervals I and II, but a factor 10% smaller was required. Possible sources of the discrepancy are differences in the reaming efficiency due to the different ice thickness removed, or error in electrical power determination. From Equation (1) it is found that the average thickness of ice removed on 26 July was 2.4 mm and that on 1 August was 1.3 mm. This is a small fraction of the mean hole radius of 31 mm. The closure rate averaged over depth is about 1.7 mm week⁻¹ and the corresponding flux is 0.84 W m⁻².

The temperature of the hole wall T_b was assumed to be the equilibrium temperature of ice and water:

$$T_b = -\beta p_w \tag{10}$$

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where $\beta = 0.0074$ deg/bar, and p_w is the "gauge" pressure in the water, which is the pressure measured relative to one atmosphere. A more complete discussion of the equilibrium temperature in the presence of air or other soluble impurities will be given in Sections 4 and 5. We point out here that Equation (10) is valid if the water contains just enough air to be saturated at a pressure of one atmosphere. The water level in bore hole R1 was 6.6 m below the snow surface when it was drilled. Although the distance between water level and snow surface decreased as ablation proceeded, the depth of the water remained constant; hence T_b was independent of time.

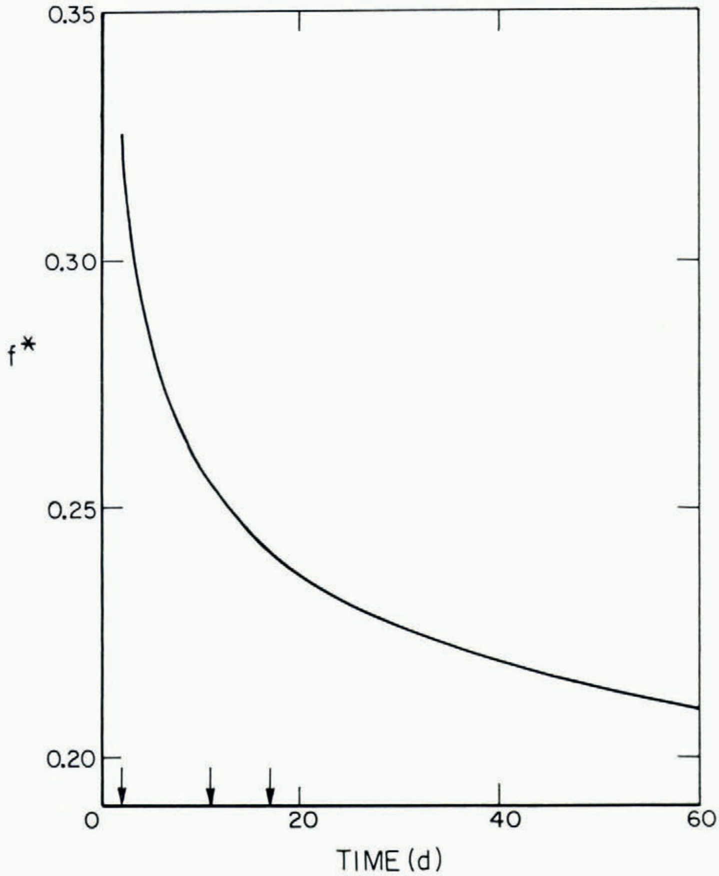

Fig. 1. Dimensionless flux f^* as a function of time in days since drilling. The arrows indicate reaming dates.

Equation (9) was used to determine the temperature difference $T_0 - T_b$ as a function of depth for intervals I and II. The thermal conductivity K_i was taken to be 2.1 W m^{-1} deg^{-1}. Because of the 10% discrepancy in the reaming data the results for the two intervals differ by that amount, but the temperature discrepancy is only 0.006° C. Consequently the results were averaged. Finally, with the temperature of the hole wall T_b determined by Equation (10), the glacier temperature T_0 was found. This is shown in Figure 3. It can be seen that the data are consistent with a linear temperature distribution, as shown by the solid line.

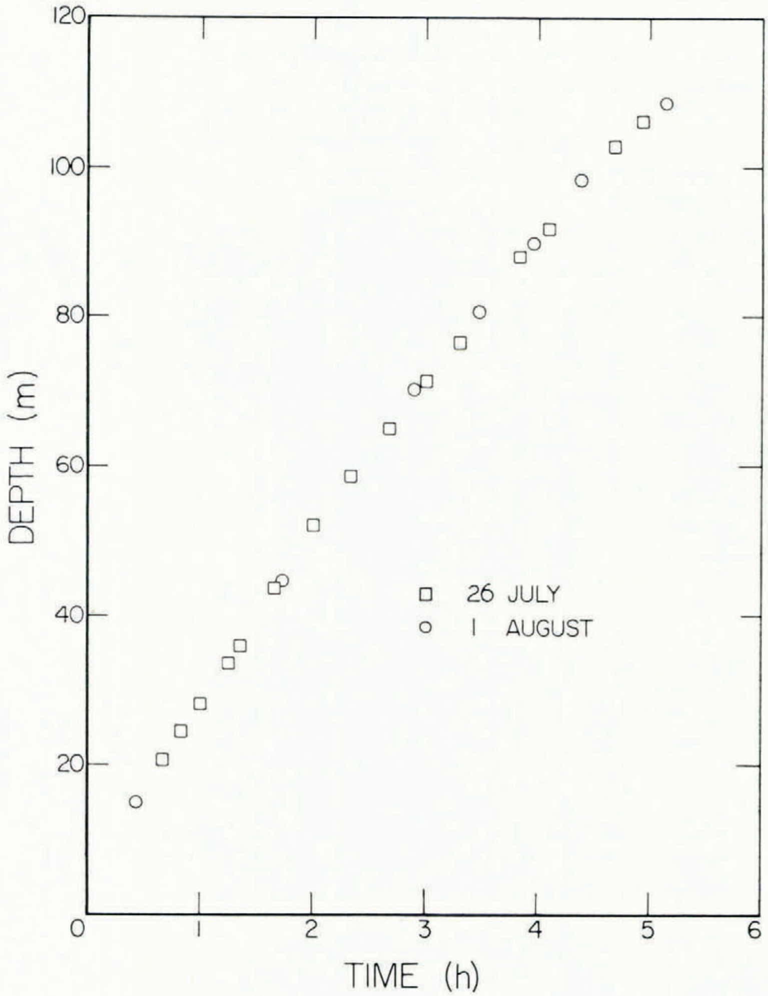

Fig. 2. Progress of the reamer in meters as a function of time in hours during two reaming operations. The data of 1 August are normalized to those of 26 July.

3.3. *Advective transfer of heat*

We now consider the effect that advection of heat by vertical flow of water may have on bore-hole closure or expansion. Since there is a temperature gradient $\mathrm{d}T_\mathrm{b}/\mathrm{d}z$ along the hole wall, such a flow will indeed transfer heat. Almost the identical problem is discussed in books dealing with heat exchanger design (for example, Kays, 1966, chapter 8). Except near the point where water enters the hole, $\partial T/\partial z$ at a given z is the same throughout the water as on the hole wall. Therefore the advected heat flux f_a at the wall can be calculated immediately:

$$f_\mathrm{a} = \frac{\pi a^2 \rho_\mathrm{w} c_\mathrm{w}\, v_0\, \mathrm{d}T_\mathrm{b}/\mathrm{d}z}{2\pi a}$$

$$= \frac{\rho_\mathrm{w} c_\mathrm{w}}{2} \frac{\mathrm{d}T_\mathrm{b}}{\mathrm{d}z} a v_0 \tag{11}$$

where ρ_w and c_w are respectively the density and specific heat capacity of water and v_0 is the average velocity of the water. From Equation (10) $dT_b/dz = 0.72 \times 10^{-3}$ deg m^{-1}. If the flow is upward, some supercooling must occur throughout the water, or ice must form there. If $v_0 = 10$ mm s^{-1}, for example, calculations indicate that the supercooling on the axis of the hole would have to be 0.02° C. If ice forms, f_a will probably be close to zero.

Equation (11) shows that the assumption that the observed flux of 0.84 W m^{-2} averaged over the depth is entirely due to conduction will be in error unless v_0 is much less than 18 mm s^{-1}. (Frictional heating at this velocity is negligible.) The possibility of water flow

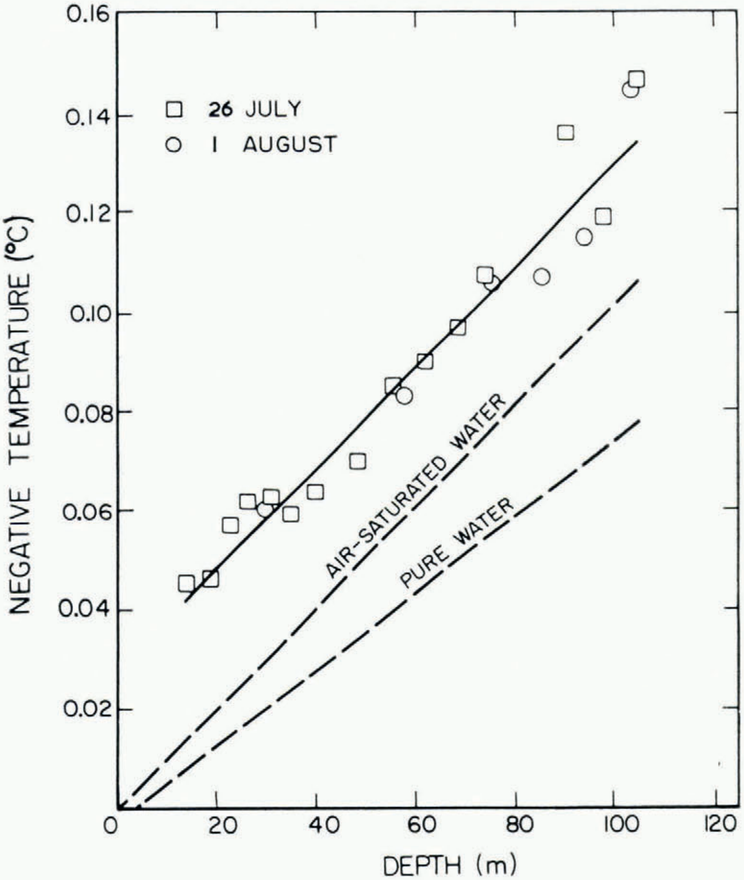

Fig. 3. Glacier temperature in negative degrees Celsius as a function of depth, evaluated from reaming data on two dates. The broken lines represent the equilibrium temperatures of ice and pure water and of ice and air-saturated water.

cannot be definitely ruled out since no direct measurements were made. However, the water surfaces in several holes were clearly visible, and they were completely calm, indicating that water flow, if any, was very small. Moreover, it is unlikely that the closure could be due to advection produced by a single source of water. This is because the changing slope of the data of Figure 2 indicates a variation of the closure rate with depth, while dT_b/dz is probably constant.

Water-filled holes were observed to close at a very slowly decreasing rate over a period of many weeks and over a distance of 150 m. It would be surprising if the glacier were to provide the necessary water flow conditions to do this by advection, particularly since there seemed to be no well-defined water table, as indicated by the fact that the water was much lower than the base of the permeable firn in a few holes. Intermittent flow might be expected, particularly in the downward direction, since at the bottom of the hole the hydrostatic pressure in the water exceeded that in the ice by about 0.8 bar. There is evidence that this occurred in a bore hole called X in 1970. Twelve weeks after drilling, the water level suddenly dropped, while reaming one day earlier indicated that the average closure rate had decreased by an order of magnitude.

We should also consider the possibility of convective instability of the water. This has been discussed by Krige (1939) in connection with the measurement of geothermal flux. The water will be stable if the gradient $\mathrm{d}T_\mathrm{b}/\mathrm{d}z$ is less than a critical value

$$\left.\frac{\mathrm{d}T_\mathrm{b}}{\mathrm{d}z}\right|_\mathrm{critical} = \frac{gb(T+273)}{c_\mathrm{w}} + B\,\frac{\kappa_\mathrm{w}\nu}{gba^4}$$

where b, κ_w and ν are respectively the thermal coefficient of volume expansion, the thermal diffusivity and the kinematic viscosity of water, g is the gravitational acceleration, and B ($=216$) is a numerical constant. This gives $\mathrm{d}T_\mathrm{b}/\mathrm{d}z|_\mathrm{critical} \approx 0.2\ \mathrm{deg\ m^{-1}}$ and since by Equation (10) $\mathrm{d}T_\mathrm{b}/\mathrm{d}z \approx 0.7\times10^{-3}\ \mathrm{deg\ m^{-1}}$, the water is stable.

3.4. *Deformation of the ice*

The possible effect of deformation of the ice on bore-hole closure or expansion is now considered. First let us estimate the axially symmetric deformation due to the pressure difference $\Delta p = p_\mathrm{w} - p_\mathrm{i}$ where p_w and p_i are respectively the hydrostatic pressures in the water and in the ice far from the hole. The closure rate has been calculated by Nye (1953) under the assumption that it is independent of main glacier flow. To investigate the validity of this assumption for bore hole R1, it is convenient to write the flow law of ice in a form analogous to that for a viscous liquid:

$$\sigma_{ij}' = \frac{A^n}{\tau^{n-1}}\,\dot{e}_{ij} \tag{12}$$

where σ_{ij}' and $\dot{e}_{ij}$ are components of the stress deviator and strain-rate tensors respectively, and $\tau^2 = \frac{1}{2}\sigma_{ij}'\sigma_{ij}'$. In a bore-hole experiment by Kamb and Shreve (1966) near the region of interest in Blue Glacier, Kamb (private communication) has obtained values for the flow law parameters A and n of 1.77 bar $\mathrm{a}^{1/n}$ and 5.25 respectively. The "effective viscosity" η associated with the flow law (12) is

$$\eta = \tfrac{1}{2}\,\frac{A^n}{\tau^{n-1}}. \tag{13}$$

Nye (1953) shows that the value of τ associated with bore-hole closure in the absence of main glacier flow is

$$\tau_\mathrm{c} = \left(\frac{a}{r}\right)^{2/n}\frac{|\Delta p|}{n}.$$

On the other hand, an estimate for the value of τ associated with main glacier flow (Nye, 1952) is

$$\tau_\mathrm{m} = F\rho_\mathrm{i}gh\cos\alpha_\mathrm{s}\sin\alpha_\mathrm{s} \tag{14}$$

where F is the ratio of hydraulic radius to channel depth, h is the vertical depth, and α_s is the surface slope.

We can compare τ_c and τ_m in bore hole R1. We have

$$p_\mathrm{w} = \rho_\mathrm{w}g(h - 6.6\ \mathrm{m})$$

because the water was 6.6. m below the surface. Also (Nye, 1953)

$$p_i = \rho_i g h \cos^2\alpha_s \tag{15}$$

where the surace slope α_s is about 13° at bore hole R1 and $\rho_i \approx 0.9$ Mg m^{-3}. This gives $\Delta p \approx 0.8$ bar at the bottom of the hole ($h = 105$ m). The factor F in Equation (14) is approximately 0.75 (Kamb, 1970, p. 705). Consequently,

$$\tau_c(r = a) = 0.15 \text{ bar},$$
$$\tau_m = 1.5 \text{ bar}$$

at the bottom of the hole. Clearly the conditions $\tau_m \ll \tau_c$ is not fulfilled, the effective viscosity in Equation (13) is strongly influenced by main glacier flow, and the assumption of independence of closure (or expansion) and main flow is inapplicable. In fact, since $\tau_m \gg \tau_c$ in the lower part of the hole, the effective viscosity there is essentially determined by the main flow and is constant (at a given depth):

$$\eta \approx \tfrac{1}{2}\frac{A^n}{\tau_m{}^{n-1}} = \tfrac{1}{2}\frac{A^n}{(F\rho_i g h \cos\alpha_s \sin\alpha_s)^{n-1}}$$

by Equations (13) and (14).

This result permits the expansion rate due to deformation, $\mathrm{d}a/\mathrm{d}t|_\mathrm{d}$, to be calculated. For constant viscosity we have

$$\left.\frac{\mathrm{d}a}{\mathrm{d}t}\right|_\mathrm{d} = \frac{a}{2\eta}\Delta p$$

(Nye, 1953). Hence near the bottom of hole R1

$$\left.\frac{\mathrm{d}a}{\mathrm{d}t}\right|_\mathrm{d} = a\Delta p\,\frac{(F\rho_i g h \cos\alpha_s \sin\alpha_s)^{n-1}}{A^n}$$

which gives an expansion rate of 0.14 mm week^{-1} at the bottom of the hole or 8% of the observed average closure rate of 1.7 mm week^{-1}. This would make the actual glacier temperature there about 0.004 deg colder than shown in Figure 3. This error is small compared with other uncertainties and decreases rapidly with decreasing depth. At a depth of 45 m, $\Delta p = 0$; hence $\mathrm{d}a/\mathrm{d}t|_\mathrm{d} = 0$ there. In the upper part of the hole calculations are more difficult because the condition $\tau_m \gg \tau_c$ does not hold. However, $\mathrm{d}a/\mathrm{d}t|_\mathrm{d}$ is negligible there because both τ_m and τ_c are small.

It has been suggested by M. F. Meier (private communication) that asymmetric closure of the bore hole might be significant, and this has been observed in holes located in regions of very high longitudinal strain-rate (C. F. Raymond, private communication). In the vicinity of hole R1 the surface longitudinal strain-rate is compressive and about 0.02 a^{-1}; the transverse strain-rate is about ten times smaller. Although the actual effect on bore-hole closure cannot be calculated without solving the proper flow problem, which is difficult, it is apparently negligible because the strain-rates are small. A strain-rate of 0.02 a^{-1} corresponds to a change in length of 0.012 mm week^{-1} in a length of 31 mm (the hole radius), while the observed average closure rate was 1.7 mm week^{-1}.

We conclude that while it is difficult to do accurate calculations, the contribution to the closure of bore hole R1 by deformation of the ice is negligible, except perhaps for a small effect near the bottom of the hole.

4. Simple models of a temperate glacier

In this section two simple models of a temperate glacier are considered and compared with the Blue Glacier temperatures obtained with the analysis of Section 3. Because soluble impurities are always present and because the state of stress is non-hydrostatic, it is not obvious what the temperature of a temperate glacier should be.

The simplest model for bulk temperate ice is a mixture of pure ice and pure water in equilibrium. Then the temperature is

$$\begin{aligned} T(\text{pure}) &= \delta - \beta p \\ &= -\beta p_i + 0.0024 \text{ deg} \end{aligned} \tag{16}$$

where δ is the triple point temperature ($+0.0098°$ C), $\beta = 0.0074$ deg/bar, p is the absolute pressure in the ice, and p_i is the corresponding gauge pressure (measured relative to one atmosphere). Strictly speaking, p as well as T should be measured relative to its value at the triple point, but this is small enough (6×10^{-3} bar) to be neglected in Equation (16) and in what follows. [It was also neglected in Equation (10).] Notice that at atmospheric pressure $T \neq 0°$ C because $0°$ C is defined to be the equilibrium temperature of ice and air-saturated water.

Photographs taken with a submersible bore-hole camera (Harrison and Kamb, to be published) indicate that air bubbles are probably present throughout the thickness of the glacier in the vicinity of bore hole R1. This suggests that a more realistic model of bulk temperature ice there might be a mixture of pure ice and air-saturated water. (It will be assumed that impurities are excluded from the ice lattice.) Since dissolved air at one atmosphere lowers the equilibrium temperature by $0.0024°$ C, and since the solubility is proportional to pressure, we have instead of Equation (16)

$$\begin{aligned} T(\text{air saturated}) &= \delta - \beta p - 0.0024\, p \\ &= \delta - \beta' p = -\beta' p_i \end{aligned} \tag{17}$$

where $\beta' = 0.0098$ deg/bar. Actually, because the solubilities of nitrogen and oxygen are unequal, the composition of the air in equilibrium with the liquid phase may change somewhat with depth. But Equation (17) should remain a good approximation.

In applying the models leading to Equations (16) and (17) to an actual glacier it is necessary to know the pressure in the ice p_i as a function of depth. One possibility is to take

$$p_i = \rho_i g h \cos^2 \alpha_s \tag{15}$$

which was done in Section 3.4. This represents the average compressive stress in a slab of ice deforming in pure shear, which is a reasonable model for the situation at hole R1. However, the stress in the slab is not pure hydrostatic, and since we are interested in the equilibrium temperature, it is better to use for p_i the maximum compressive stress (Kamb, 1961). This is easily shown to be

$$p_i = \rho_i g h\, (\cos^2 \alpha_s + \cos \alpha_s \sin \alpha_s). \tag{18}$$

For $\alpha_s = 13°$, which is the surface slope at hole R1, Equations (15) and (18) differ by a factor of 1.23, which is appreciable. Translated into a temperature prediction with the "air-saturated" model, Equation (17), for example, the difference is 0.02 deg at the bottom of R1.

With p_i given by Equation (18), the temperatures predicted by the model Equations (16) and (17) are shown as broken lines in Figure 3. It is seen that in the vicinity of bore hole R1 the Blue Glacier temperatures obtained from the analysis of Section 3 are somewhat colder than predicted by either model. The difference is 0.05 deg, averaged with depth, for the "pure" model and 0.03 deg for the "air-saturated" model.

5. Effect of impurities

In Section 3 Blue Glacier temperatures were evaluated with the assumption that the ice is impurity free. The results are so close to those predicted by the simple models of temperature glaciers that it turns out to be important to consider the effects of soluble impurities on the thermal properties of the ice. This will give new insight into the nature of temperate ice, and will suggest a possible improvement to the temperature evaluation of Section 3.

5.1. *Thermal properties*

Renaud ([1952], 1958) and Lliboutry (1971) have pointed out the importance of impurities in temperate ice. Here we are concerned with the effect of impurities on the thermal properties of bulk glacier ice. It is interesting that up to a point, at least, the problem is similar to that discussed in connection with sea ice, and we can refer to a paper by Schwerdtfeger (1963) for some details. Attention is focused on heat capacity because thermal conductivity and density, which are the other thermal properties entering into the thermal diffusion equation, are much less affected by impurities. Since liquid phase is present in bulk ice that is warmer than the eutectic temperatures of the impurities, the effective specific heat capacity c must contain three terms. The first is the direct contribution of the ice (assumed to be pure) and the second, which will be neglected, is that of the liquid. The third expresses the fact that temperature change must be accompanied by some phase change because the concentration of the solution in equilibrium with the ice depends on temperature. If heat is added, for example, some of it must melt ice to dilute the liquid phase, since the total amount of solid soluble impurities in solution is fixed. Consequently,

$$c = c_\mathrm{i}\left(1+\frac{L}{c_\mathrm{i}}\frac{\mathrm{d}w}{\mathrm{d}T}\right) \tag{19}$$

where c_i is the specific heat capacity of pure ice, L ($= 333$ kJ kg^{-1}) is the specific latent heat of pure ice and w is the fractional liquid water content of the bulk ice.

To evaluate $\mathrm{d}w/\mathrm{d}T$ we must find the relation between equilibrium temperature and w. This depends on pressure and on the air and the solid soluble impurity contents of the liquid phase. Assuming that air bubbles are in contact with the liquid phase, it was shown in the discussion of Equation (17) that the combined effects of pressure and air content lower the equilibrium temperature by $\beta' p$. The effect of solid soluble impurities is somewhat different in that no solid impurity phase is likely to be present at Blue Glacier temperatures, these being well above eutectic temperatures for all common impurities. The relation between equilibrium temperature and fractional salt content s of the liquid phase is linear. (See Schwerdtfeger (1963).) Since

$$s = \frac{\sigma}{w}$$

where σ is the fractional salt content of the bulk ice, the temperature is lowered by an amount $\alpha s = \alpha\sigma/w$, where α is a positive constant depending upon the solid impurity composition.* It is convenient to characterize the effect of solid impurities by the single parameter θ_m, where

$$\theta_\mathrm{m} = -\alpha\sigma. \tag{20}$$

Hence the temperature lowering by the solid soluble impurities is $-\theta_\mathrm{m}/w$. Therefore we finally obtain for the equilibrium temperature

$$T = \delta - \beta' p + \frac{\theta_\mathrm{m}}{w}. \tag{21}$$

* The present notation differs from Schwerdtfeger's:

$$\alpha \text{ (present)} = [-\alpha \text{ (Schwerdtfeger)}]^{-1}$$

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Except for the triple point correction δ (discussed in Section 4) and the pressure term, Equation (21) is the same as used for sea ice (Schwerdtfeger, 1963). Equation (21) should also contain a term allowing for curvature of the interface between ice and liquid, but in actual glacier ice it is probably much smaller than the θ_m/w term (Lliboutry, 1971).

It follows from Equations (19) and (21) that

$$c = c_i\left(1+\frac{\theta_t^2}{\theta^2}\right) \tag{22}$$

where

$$\theta = T - \delta + \beta' p \tag{23}$$

and the constant θ_t is given by

$$\theta_t = -\left(\frac{L\alpha\sigma}{c_i}\right)^{\frac{1}{2}} = -\left(-\frac{L\theta_m}{c_i}\right)^{\frac{1}{2}}. \tag{24}$$

It can be seen from Equations (23) and (17) that θ is the difference between the actual temperature and that in the "air-saturated" bulk ice defined by Equation (17), in which ice and air-saturated water are in equilibrium but no other impurities are present. The negative sign has been included in Equation (24) so that θ_t can be interpreted in terms of a possible glacier temperature.

From Equations (21) and (23) we see that the fractional liquid water content w is

$$w = \frac{\theta_m}{\theta}. \tag{25}$$

Hence θ_m can be interpreted as the θ at which the last of the solid phase disappears during melting. If w is large the effective thermal conductivity and density as well as the heat capacity become temperature dependent, but these complications are likely to be unimportant in glacier ice.

Although (22) is probably a reasonable approximation for bubbly ice, the exact form of the effective heat capacity must depend upon the details of the chemistry and hydrology of the liquid phase. These details are incompletely understood, although work by Nye and Frank (in press) is important. For example in deriving Equation (22) it has been implicitly assumed that the pressure in the liquid phase is constant during a temperature change. This is not obviously valid because phase change accompanies temperature change, and the specific volumes of ice and water are different. However, the pressure change should be negligible when the liquid phase is in intimate contact with air bubbles, as assumed here. When bubbles are not plentiful the effect could give rise to stresses in the ice. This might be significant if there is no hydraulic connection throughout the liquid phase and if, in addition, temperature change is rapid, so that there is not time for the stresses to relax.

5.2. *Application to Blue Glacier data*

To apply these considerations to the Blue Glacier data it is first necessary to estimate the parameter θ_m defined by Equation (20), since it characterizes the effect of solid soluble impurities. Some experimental data on the ionic salt content of glacier ice have been summarized by Langway (1967, p. 43). Renaud (1958) reports salt contents to be in the range from 1 to 5 parts in 10^6 by weight. We therefore assume that the fractional salt content σ of the Blue Glacier ice is likely to lie in the range from 0.5 to 10 parts in 10^6, and that an appropriate value of α is 55° C, which is characteristic of sea-water. By Equation (20)

$$-5.5\times 10^{-4}\ °\text{C} < \theta_m < -0.3\times 10^{-4}\ °\text{C}, \tag{26}$$

and by Equation (24) the corresponding range of θ_t is

$$-0.3\ °\text{C} < \theta_t < -0.07\ °\text{C}. \tag{27}$$

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The corresponding effective heat capacity c can now be estimated, assuming that the glacier temperature obtained with the analysis of Section 3 is approximately correct. We saw in Section 4 that this temperature is about 0.03 deg colder than the equilibrium temperature of ice and air-saturated water. According to the discussion of Equation (23) this means that $\theta = -0.03$ °C. Combining this with Equations (22) and (27) gives

$$6\, c_i < c < 100\, c_i \tag{28}$$

where c_i is the heat capacity of pure ice. This surprising result means that at the temperatures prevailing in Blue Glacier the effective heat capacity is dominated by the influence of minute quantities of solid soluble impurities. By Equations (25) and (26) the liquid water content corresponding to $\theta = -0.03$ °C should lie in the range

$$0.1\% < w < 2\%$$

although this result is extremely sensitive to error in θ.

Because of Equations (22) and (28) we should use the relation

$$K_i \nabla^2 T = \rho_i c_i \left(1 + \frac{\theta_t^2}{\theta^2}\right) \frac{\partial T}{\partial t}$$

rather than the simple diffusion equation (2) to evaluate initial glacier temperature from bore-hole closure data. This has not been done because the impurity content of the ice is not known, but it is possible to outline what must happen. Initially c will be approximately uniform but much larger than c_i, and so for a given initial temperature the presence of impurities should considerably speed up the rate of refreezing. As time proceeds c will increase everywhere, especially near the bore-hole wall. On the other hand, we have seen that at sufficiently large time the initial temperature evaluated with the simple diffusion equation depends only logarithmically on c; hence there is good reason to believe that it will not be grossly in error with the impurities present. The effect will be to make the actual glacier temperature somewhat warmer than found with the analysis of Section 3. However, since the bore-hole closure method actually measures the difference $T_0 - T_b$ between initial glacier and hole wall temperatures, and this is only of the order of 0.05 deg, the correction cannot be a large one. Because at bore hole R1 the hole wall temperature, Equation (10), is not greatly different from that of the "pure" temperature glacier model, Equation (16), the bore-hole closure method is almost a direct measurement of the difference between the model prediction and the actual temperature.

The effect of impurities on the effective heat capacity c and the uncertainty in the temperature of the bore-hole wall T_b are the main sources of uncertainty in the evaluation of the glacier temperature at hole R1. Actually both problems are related to impurities. The value of T_b assumed in Equation (10) requires that the water contain only enough air to be saturated at atmospheric pressure, while at depth it probably contains more air than this. (When a bore hole is melted the liquid phase is so dilute that the effect of the solid soluble impurities on T_b is negligible.) If it were air-saturated at all depths the factor $\beta = 0.0074$ deg/bar in (10) would be replaced by $\beta' = 0.0098$ deg/bar, making T_b 0.02 deg colder at the bottom of hole R1. As a result the actual glacier temperature would be colder than found in Section 3. Because the effects of large c and uncertain T_b are in the opposite sense, no refinement to the glacier temperature found in Section 3 seems justified. It is felt that the temperature indicated by the solid line in Figure 3 is accurate to 0.02 or 0.03 deg.

6. Definition of temperate ice

In Section 4 we considered the possibility of defining the temperature of temperate ice to be that of a mixture of ice and pure water, or alternatively, of ice and air-saturated water. Lliboutry (1971) suggests that ice be considered temperate when it contains liquid inclusions in which the concentration of salts is not too high. Temperate ice has also been defined as being that within 1 deg of its melting point (see Robin, 1967). Of these four definitions only the first three have any physical basis; the fourth conveys little information. The first two, although they are precise enough, are not applicable to real ice because they do not take into account the lowering of temperature by solid soluble impurities. The third definition is applicable to real ice but its usefulness suffers from a lack of precision. Can a definition be found that is free of these objections?

Our discussion of the refreezing of glacier bore holes suggest one way this might be done. However, the idea does not depend upon the existence of bore holes, glaciers, or even ice, since it is applicable to many materials. We adopt the point of view that the fundamental temperature-dependent properties of bulk ice are the thermal ones (effective thermal conductivity, density, and heat capacity) that occur in the diffusion equation, since they determine the thermal behavior. We can focus our attention on the effective specific heat capacity c, since it is spectacularly temperature dependent, while the other thermal properties are only weakly so. We had

$$c = c_i\left(1+\frac{\theta_t^2}{\theta^2}\right) \tag{22}$$

where c_i is the heat capacity of pure ice and θ is the temperature measured relative to that of ice and air-saturated water. The constant θ_t, which is defined in terms of the impurity content by Equation (24), is a kind of "transition temperature" above which the effective heat capacity does not behave like that of pure ice, but increases rapidly with temperature because phase change is becoming important. This is the familiar broadening of the melting point of a chemical compound by the presence of one or more soluble impurities. Since melting is actually a continuous process with temperature, the familiar term "ice at the pressure melting point" is ill-defined. This has been recognized by Lliboutry (1971).

It is logical to define ice to be temperate or warm if it is warmer than the transition temperature θ_t, and to be cold if it is not. At θ_t, $c = 2c_i$. In fact, if the transition temperature is defined to be that at which $c = 2c_i$, the definition is more general, being independent of the particular form of Equation (22) for c. From this point of view the transition temperature is that at which the direct and the phase change contributions to the effective heat capacity are equal. The transition temperature represents a change of regime as far as the solution of heat flow problems is concerned, because when θ is of this order the simple diffusion equation, and more general versions of it containing advective or other terms, break down. It is probably fair to say that this difficulty is often overlooked, and that this definition of temperate ice may draw attention to the problem.

We saw in Equation (27) that θ_t is likely to lie in the range

$$-0.3 < \theta_t < -0.07\ ^\circ\text{C}$$

while $\theta = -0.03\ ^\circ$C at all depths in bore hole R1. According to the new definition, therefore, Blue Glacier is temperate at R1, even though the borehole refreezes at all depths.

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