Destouches insists that u is not the usual wave function, even though as a function it depends on the arguments P, T of a four-dimensional continuum. It is pointed out that P and M are not identical, that P is merely the argument of the function u. Finally, however, when it comes to writing down equations for u, they turn out to be Schrödinger equations, with (usually undetermined) non-linear terms added. In spite of some effort on the part of this reviewer, no clear physical theory or model emerged. This is, of course, a very subjective reaction, and other readers may be more successful.

P.G. Bergmann, Syracuse University

Statistical Estimates and Transformed Beta-Variables, by Gunnar Blom. Wiley, New York, 1959. 176 pages. $5.00.

This book deals with a variety of statistical topics, such as estimation of parameters, transformations of variables, order statistics and use of probability paper (and various plotting rules) in estimation theory. The author shows the connection of these and other topics by use of the probability integral transformation, which leads to a uniform distribution in the domain (0, 1), and hence that the cumulative frequency distribution of order statistics leads to the "Beta" distribution.

The first part gives a generalization of R.A. Fisher's theory of the asymptotic variance of estimates of parameters and of the Cramér-Rao inequality which provides lower bounds for the variances of these estimates. The second part deals with the properties of transformed Beta variables. In the third part, the properties of "nearly best linear estimates" for a two-parameter distribution are studied.

In contrast to the modern trend the mathematical treatment does not obscure the statistical aim. The book is clearly written and the theoretical results are immediately applied to various distributions. In addition to the well-trodden path of the normal distribution, the author considers the distributions of extreme values which now play an increasing role in statistics (the so-called Weibull distribution is an example of this type). The notation is perhaps unnecessarily complicated. Let $F(x, \alpha)$ be the probability function of a continuous random variable x with a parameter α. The inverse, the probability point $x = s$, may be written $x(F, \alpha)$. Instead the author writes $F(x, \alpha) = u$ and denotes the inverse by $x = g(u, \alpha)$.

A relatively easy and quick method for estimating parameters is the well known graphical procedure using probability paper. The problem of how best to plot the observations in probability paper so as to arrive at the best estimates of the parameter or parameters of the distribution function from which

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we are sampling is dealt with by Blom.

Let $x_{(i)}$, $i = 1, \ldots, n$ (where n is the sample size), be the ordered sample values. The vertical axis of the probability graph paper is labelled P, the horizontal axis is the ordered $x_{(i)}$'s. Some well known plotting methods are

(a) Plot $P_i = \frac{i}{n+1}$ against $x_{(i)}$,

(b) Plot $P_i = \frac{i - \frac{1}{2}}{n}$ against $x_{(i)}$.

Blom considers P_i's of the form $P_i = (i - \alpha)/(n - \alpha - \beta + 1)$ where $0 \leq \alpha$, $\beta \leq 1$ are constants chosen to give the resulting estimates various properties. If sampling from symmetrical distributions, for example, it turns out that if the criterion of "nearly unbiased" and "high efficiency" of estimates is desired, then $\alpha = \beta$; for example, the normal distribution requires $\alpha = \beta = 3/8$, that is, plotting of $P_i = (i-3/8)/(n+1/4)$ against $x_{(i)}$, leads to estimates of the mean and variance which are "nearly unbiased" and have high efficiency, relative to the usual estimates, viz. $\bar{x}$ and S_x^2. The author also discusses various choices of α, β when sampling from skewed distributions.

However, the discussion of other considerations that for example arise in time series data (return periods etc.) which give preference to the classical method (a) above is not given.

The question of the plotting position is closely connected with the general problem of estimating location and scale parameters by use of linear functions of order statistics. This method requires the knowledge of the expectations and the covariances of the order statistics. In general the latter can only be obtained by numerical integration. Therefore the author studies the construction of "nearly best" linear estimates for the two parameters, a method which lacks some of the mathematical elegance of best estimates but does not require the knowledge of the covariances and therefore facilitates the estimation.

A well-selected bibliography increases the value of the book. It is well organized and is therefore appropriate as a text-book. However, an index is missing.

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