

The Manchester Corporation Waterworks Life Annuities.

BY the courtesy of the Waterworks Committee of the Manchester Corporation, we are enabled to present to our readers a complete and authentic reprint of the report which Mr. A. J. Finlaison addressed to that Committee, in which will be found fully set forth the basis and results of his valuation of the life-annuity and term-annuity contracts of the Corporation. For purposes of reference, we also subjoin the rates for annuities on single lives in use by the Corporation up to October 1882, and the rates as amended at that date. In regard to the latter, we may observe that they correspond exactly with the annuity-rates adopted by the National Debt Office for many years (until, in fact, about twelve months ago, when they were revised), on the assumption that the Three Per-Cents were purchasable at par.—ED. J.I.A.

TABLE I.—Annuities on Single Lives.
Amount of annuity granted for every £100 invested at the under-mentioned ages.

Age last Birthday	MALES		FEMALES		Age last Birthday
	Old Rates (in force prior to Oct. 1882)	New Rates (in force since Oct. 1882)	Old Rates (in force prior to Oct. 1882)	New Rates (in force since Oct. 1882)	
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	
30	5 6 2	5 4 0	5 0 4	4 16 6	30
31	5 7 3	5 5 0	5 1 4	4 17 4	31
32	5 8 3	5 6 0	5 2 2	4 18 3	32
33	5 9 6	5 7 1	5 3 2	4 19 2	33
34	5 10 10	5 8 3	5 4 3	5 0 2	34
35	5 12 6	5 9 6	5 5 8	5 1 2	35
36	5 14 3	5 10 11	5 7 1	5 2 2	36
37	5 15 10	5 12 4	5 8 6	5 3 2	37
38	5 17 5	5 13 10	5 9 10	5 4 3	38
39	5 19 2	5 15 4	5 11 2	5 5 5	39
40	6 0 9	5 17 0	5 12 9	5 6 8	40
41	6 2 6	5 18 10	5 14 2	5 7 11	41
42	6 4 7	6 0 9	5 15 11	5 9 4	42
43	6 6 9	6 2 9	5 17 11	5 10 9	43
44	6 9 2	6 5 0	5 19 11	5 12 3	44
45	6 11 9	6 7 5	6 2 2	5 13 11	45
46	6 14 9	6 10 0	6 4 5	5 15 8	46
47	6 18 3	6 12 10	6 6 9	5 17 7	47
48	7 1 11	6 15 11	6 9 0	5 19 7	48
49	7 5 10	6 19 2	6 11 3	6 1 9	49
50	7 9 7	7 2 8	6 13 9	6 4 1	50
51	7 13 6	7 6 4	6 16 10	6 6 8	51
52	7 17 6	7 10 1	7 0 4	6 9 4	52
53	8 1 5	7 14 0	7 3 10	6 12 4	53
54	8 5 6	7 17 11	7 7 5	6 15 5	54
55	8 10 4	8 2 1	7 11 3	6 18 10	55
56	8 15 5	8 6 4	7 15 1	7 2 4	56
57	9 0 10	8 10 8	7 19 6	7 6 2	57
58	9 6 10	8 15 4	8 4 2	7 10 3	58
59	9 13 6	9 0 3	8 9 3	7 14 7	59
60	10 0 5	9 5 6	8 15 4	7 19 2	60
61	10 7 3	9 11 5	9 1 7	8 4 3	61
62	10 14 5	9 17 11	9 7 8	8 9 9	62
63	11 2 8	10 5 1	9 14 10	8 15 8	63
64	11 10 6	10 12 9	10 2 3	9 2 0	64
65	12 0 5	11 1 0	10 10 0	9 8 9	65
66	12 11 8	11 9 10	10 18 10	9 16 1	66
67	13 2 6	11 18 10	11 8 4	10 3 11	67
68	13 14 10	12 8 4	11 18 10	10 12 4	68
69	14 5 11	12 18 6	12 9 6	11 1 7	69
70	14 15 7	13 9 4	13 0 8	11 11 6	70
71	15 6 0	14 0 8	13 13 3	12 1 11	71
72	15 19 11	14 12 9	14 6 10	12 13 2	72
73	16 13 4	15 6 0	15 1 4	13 5 2	73
74	17 6 8	16 1 0	15 17 4	13 17 10	74
75	18 5 7	16 18 0	16 14 3	14 11 3	75

Annuities under the old rates were payable either half-yearly or quarterly, at the option of the annuitant; those granted under the new rates are payable half-yearly. Under both systems, annuities were apportionable up to the date of death.

TABLE II.—Annuities on Single Lives.
Cost of an annuity of £1 at the under-mentioned ages.

Age last Birthday	MALES		FEMALES		Age last Birthday
	Old Rates (in force prior to Oct. 1882)	New Rates (in force since Oct. 1882)	Old Rates (in force prior to Oct. 1882)	New Rates (in force since Oct. 1882)	
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	
30	18 16 9	19 4 6	19 18 5	20 14 5	30
31	18 12 11	19 1 0	19 14 6	20 10 9	31
32	18 9 5	18 17 4	19 11 3	20 7 0	32
33	18 5 4	18 13 5	19 7 9	20 3 2	33
34	18 0 11	18 9 4	19 3 6	19 19 4	34
35	17 15 6	18 5 1	18 18 7	19 15 5	35
36	17 10 0	18 0 8	18 13 6	19 11 6	36
37	17 5 2	17 16 1	18 8 7	19 7 7	37
38	17 0 7	17 11 5	18 4 2	19 3 6	38
39	16 15 9	17 6 7	17 19 9	18 19 4	39
40	16 11 2	17 1 8	17 14 9	18 15 0	40
41	16 6 6	16 16 8	17 10 2	18 10 6	41
42	16 0 11	16 11 3	17 5 0	18 5 11	42
43	15 15 5	16 5 9	16 19 2	18 1 1	43
44	15 9 8	16 0 0	16 13 5	17 16 2	44
45	15 3 8	15 13 11	16 7 3	17 11 0	45
46	14 16 8	15 7 8	16 1 5	17 5 8	46
47	14 9 4	15 1 1	15 15 7	17 0 1	47
48	14 1 9	14 14 3	15 10 0	16 14 4	48
49	13 14 3	14 7 4	15 4 8	16 8 5	49
50	13 7 4	14 0 3	14 18 11	16 2 3	50
51	13 0 7	13 13 3	14 12 4	15 15 10	51
52	12 13 11	13 6 5	14 4 11	15 9 2	52
53	12 7 9	12 19 9	13 18 1	15 2 4	53
54	12 1 7	12 13 3	13 11 3	14 15 3	54
55	11 14 10	12 6 10	13 4 5	14 8 2	55
56	11 8 0	12 0 6	12 17 11	14 0 11	56
57	11 1 2	11 14 4	12 10 9	13 13 8	57
58	10 14 0	11 8 2	12 3 7	13 6 3	58
59	10 6 8	11 1 11	11 16 4	12 18 10	59
60	9 19 7	10 15 7	11 8 1	12 11 3	60
61	9 13 0	10 9 0	11 0 3	12 3 6	61
62	9 6 6	10 2 2	10 13 1	11 15 8	62
63	8 19 7	9 15 1	10 5 3	11 7 9	63
64	8 13 6	9 8 0	9 17 9	10 19 9	64
65	8 6 4	9 1 0	9 10 5	10 11 11	65
66	7 18 11	8 14 0	9 2 9	10 4 0	66
67	7 12 4	8 7 6	8 15 2	9 16 2	67
68	7 5 6	8 1 1	8 7 5	9 8 4	68
69	6 19 11	7 14 9	8 0 4	9 0 7	69
70	6 15 4	7 8 7	7 13 5	8 12 10	70
71	6 10 8	7 2 6	7 6 4	8 5 4	71
72	6 5 0	6 16 8	6 19 5	7 18 0	72
73	6 0 0	6 10 9	6 12 8	7 10 11	73
74	5 15 4	6 4 8	6 6 0	7 4 0	74
75	5 9 5	5 18 4	5 19 8	6 17 4	75

Annuities under the old rates were payable either half-yearly or quarterly, at the option of the annuitant; those granted under the new rates are payable half-yearly. Under both systems, annuities were apportionable up to the date of death.

MR. FINLAISON'S REPORT.

19, OLD JEWRY, LONDON,
17 July 1883.

*The Chairman and Members of the Waterworks Committee
of the Corporation of the City of Manchester.*

GENTLEMEN,—In accordance with your request I have made an examination of the engagements of the Waterworks Committee on account of Life Annuities and Annuities for Terms of Years as they are reported to have stood on 31 December 1882.

The power conferred on the Corporation for the grant of Life Annuities dates from the Waterworks Act 1847, but no money appears to have been borrowed in this form until the year 1859. Thenceforth the amount of the loans repayable by means of annuities has gradually increased until the year 1882, when no less than £203,057 was received as the consideration for Life Annuities, and £3,679 for annuities for Terms of Years; a total of £1,384,098 having altogether been borrowed up to 31 December 1882 in return for the grant of annuities of all descriptions.

The Act of 1847 prescribed, to a certain extent, the rates by which annuities were to be fixed, by a statement that reference was to be observed to the prices charged for similar annuities by the Commissioners for the Reduction of the National Debt.

The Life Annuities granted by the National Debt Commissioners are divisible into two classes—one designed for the circumstances of persons of the position of depositors in Savings Banks, and the other for the fundholder of the Perpetual Annuities which constitute the bulk of the National Debt.

The Tables in force in 1847 for Savings Banks Life Annuities were extremely favourable to the purchaser, since they were founded upon the high rate of interest of $3\frac{3}{4}$ per-cent, without discrimination of sex, women being allowed to obtain life annuities at the rates computed to be applicable to men.

The maximum limit of a Savings Bank Life Annuity was, however, in 1847, £20 a-year. Under the Act 10 Geo. IV, c. 24, no limit is placed upon the maximum amount of life annuity; but the rate allowed to women is considerably below that for men. In the year 1847, however, the price of Stock was so low, that according to the terms of the Statute, which provide that the amount of annuity to be granted should be dependent upon the market prices on the day of contract, an intending purchaser had an opportunity of obtaining a Life Annuity calculated to return as high a rate of interest as $3\frac{3}{4}$ per-cent per annum. The Life Annuity Rates under the Act of 10 George IV, cap. 24, have been unchanged since 1847 as far as regards the probabilities of life, but the rate of interest has been subject to considerable variation; and in recent years has been lowered in proportion to the rise in price which has been established upon the money market.

The terms which regulate the grant of Savings Banks Life Annuities were first modified in 1853, when the rate of interest computed to be allowed to purchasers was reduced to $3\frac{1}{4}$ per-cent per annum, and the exceptional advantages given to women were withdrawn.

The maximum amount of a Savings Bank Life Annuity was raised however, in 1853 to £30, and in 1863 to £50 a-year. In 1882, a further reduction in the rate of interest for these Annuities was made to 3 per-cent per annum, under the direction of the Lords of the Treasury, in consequence of further advances in the market prices of Stock; and by this means the rates for the two descriptions of life annuities granted by the Government were brought into close agreement.

The Life Annuity Rates of the Manchester Corporation from 1859 to 1882 have been regulated by Tables stated to have been founded upon the whole of the statistics published in a Report of Mr. John Finlaison, Actuary to the National Debt Commissioners, printed by order of the House of Commons in 1829.

Many lives chosen as remotely as the year 1693, as well as the nominees of several Annuity Schemes started in the first half of the 18th century, would therefore influence the annuity-values comprised in the Tables; but the whole of the above-mentioned nominees were excluded from the Tables which formed the foundation of the rates for the grant of life annuities by the Government.

The experience of the National Debt Commissioners of the duration of annuities since 1829, particularly of those dependent upon male lives, has amply confirmed the wisdom exercised in the rejection of the statistics of the older schemes.

The life annuities granted by the Corporation are payable up to the day of death of the nominee, and the periodical instalments fall due half-yearly or quarterly, as the case may be. But the values in the Tables which regulate the grant of the annuities make no allowance for the fractional portion of the annuity instalment payable on the death of an annuitant, and they are so computed as to show the value of a life annuity, the first yearly payment of which is calculated to become due twelve months from the age at which it is granted, and at yearly intervals afterwards.

A sum of about £65,000 should have been added to the amount of £1,333,410 paid to the Corporation for Life Annuities up to 31 December 1882, on account of the apportionment of annuity instalments due for the intervals between the days appointed for the periodical payments and the date of the annuitant's death, in addition to the gain by the annuitant from the receipt of instalments, quarterly or half-yearly, as the case may be; and a further sum of also about £65,000 should have been added to the total purchase-money, on account of the probable greater vitality of the annuitants than the rate involved in the tables—that is to say, if no higher rate of interest than the 3 per-cent per annum, at which the tables are ostensibly calculated, were to be estimated as payable to the purchasers of annuities.

The number of life annuities stated to have been granted up to 31 December 1882 is 1,988, and their annual amount is

placed at £140,977. 5s. 2d. Of these, 309, of the yearly amount of £24,624. 11s. 8d., have ceased. A balance of 1,679 in number, and £116,352. 13s. 6d. in annual amount, remained, therefore, as a liability at 31 December 1882.

In accordance with the instructions supplied by the Committee, and the suggestion contained in the letter of Mr. Alderman King, appended thereunto, I have made a valuation of the whole of the existing annuities, upon the assumption that the Table under which they have been granted represented the value, at 3 per cent, of the annuities at the date upon which they commenced to accrue.

The result of such valuation, in full detail, for every description of life annuity, either for single or joint lives, is as follows:—

A.

Present value of Annuities granted for Single Lives:—	£
£55,779 a-year to Males	408,880
£46,334 a-year to Females	424,051
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	832,931
Present value of £14,139 a-year granted for Joint Lives,	143,316
Present value of £100 a-year Deferred Life Annuities ...	900
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Total	<u>£977,147</u>

For the information, however, of the Committee, I also give the result of a valuation at 3 per cent by the same Tables, but with a proper allowance for the proportionate part of the annuity due up to the date of death, as well as for the quarterly and half-yearly payment of instalments:—

B.

Single Lives:—	£
Present value of Annuities to Males ...	434,541
Do. do. to Females ...	444,845
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	879,386
Present value of Annuities granted for Joint Lives ...	148,929
Present value of Deferred Annuities	919
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Total	<u>£1,029,234</u>

In order to form a judgment of the significance of the above-mentioned liability at 31 December 1882, it is necessary to make an account of the amount of cash receipts and payments for life annuities, and to agree upon a rate of interest which would represent the rate at which the balance of money in hand, from time to time, has been utilized.

It is suggested, in the letter of Mr. Alderman King, that such rate of interest should be 4 per-cent per annum, payable half-yearly.

The total amount received for life annuities up to 31 December 1882, is stated at £1,333,410. If this sum has been invested at compound interest, at 4 per-cent, from half-year to half-year, as it had been received, principal and interest together would have amounted to £1,648,382 at 31 December 1882.

Similarly, had the amount of £599,043, paid, from time to time, for annuity instalments and expenses, been accumulated at interest at 4 per-cent per annum, a sum of £719,470 would have been arrived at by 31 December 1882.

The balance between the two above-mentioned sums of £1,648,382 and £719,470, or £928,912, consequently, represents the fund in the hands of the Corporation at 31 December 1882, on the assumption that the sums raised by means of life annuities have been utilized at 4 per-cent per annum.

It will be seen that a sum of £928,912 is less, by £48,235, than £977,147—the total value of the life annuities according to the Valuation marked A, and less, by £100,322, than £1,029,234—the result of the Valuation marked B.

A correct judgment of the result of the transactions in life annuities upon the assumption of the utilization of the sums raised in such manner, at a rate of interest of 4 per-cent, will, however, be better obtained from a valuation at 4 per-cent by a table of mortality that would correspond with the death-rate that will probably take effect among the life annuitants remaining upon the books of the Corporation.

The tables of mortality available for such a valuation are very limited in number; but those which have been adopted by the Government as the basis of the charge made by them for life annuities may be accepted as a criterion, from which an inference of the true state of affairs would be obtainable. A valuation, in full detail, of every description of life annuity by the Government Tables, at 4 per-cent, has consequently been made with the following outcome:—

C.

Present value of Annuities granted for Single Lives:—	£
£55,779 to Males	436,032
£46,334 to Females	453,597
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	889,629
Present value of £14,139 a-year, granted for Joint Lives,	148,786
Present value of £100 a-year Deferred Life Annuities . . .	860
	<hr/>
Total	<u>£1,039,275</u>

It will be seen, from the above statement, that a greater liability is shown, according to a 4 per-cent valuation, by the tables adopted

by the Government, than that disclosed under the Valuation marked B, made at 3 per-cent, by the tables which have formed the basis upon which the grant of life annuities by the Corporation has been regulated.

There are good grounds for supposing, however, that the measure placed upon the length of female life in the Government Tables is somewhat greater than has actually prevailed among such annuitants; but the modification which could be made in the results of the valuation, on account of the view taken of female mortality, would not reduce the liability to an extent that would, in any degree, bring it into a state of equilibrium with the amount of £928,912, which has been already stated as the sum which represents the utilization of the funds at 4 per-cent.

The rate of interest at which the sums received for the grant of life annuities have, in effect, been raised, is, however, the practical question for consideration.

Without entering upon very elaborate calculations, it would be impossible to affirm that an estimate of such rate of interest had been made, within very small fractional limits. In fact, the practical results of the comparatively large grants of recent years should be disclosed, to some extent, by the lapse of time, before the rate could be ascertained with precision. But, from careful computations which have been made, I am of opinion that such rate of interest will be found to approximate to $4\frac{3}{4}$ per-cent per annum.

In addition to the sums raised by means of the grant of life annuities, an amount of £50,688 has been borrowed by the Corporation in return for the grant of annuities for terms of years.

The Table No. IV shows that the annuities granted in return for advances repayable in a fixed term of years are founded upon a rate of interest of 3 per-cent per annum, where the instalments become due annually, at an interval of one twelvemonth from the date of the advance.

In practice, however, annuity payments are made more frequently than once a-year, and instalments commence before the lapse of twelve months from the date of purchase. The rate of interest paid to the purchasers of annuities for terms of years has consequently been above 3 per-cent. After bringing into account a sum of £256 for expenses, the result of the following valuation, at 4 per-cent, indicates that the rate of 4 per-cent in question is very nearly the exact rate of interest.

*Value, at 4 per-cent, of current Annuities for Terms of Years
at 31 December 1882.*

£1,681	1	0	payable half-yearly	.	.	.	£14,464
2,419	11	4	payable quarterly	.	.	.	16,710
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£4,100	12	4					£31,174
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The total amount received for term annuities has been already stated as £50,688. If this sum had been invested at 4 per-cent

per annum compound interest, payable half-yearly, it would have amounted to £64,506 by 31 December 1882.

On the other hand, £27,101 has been paid in annuity instalments, and £256 for expenses. These sums, accumulated half-year by half-year, at 4 per-cent, up to 31 December 1882, would have amounted to £33,078.

The balance between £64,506 and £33,078, or £31,428, consequently represents the fund in the hands of the Corporation, on the assumption that the balances in hand from time to time have been utilized at a rate of interest of 4 per-cent.

The value of the outstanding liabilities, at 4 per-cent, has been shown to be £31,174, or £254 less than the sum of £31,428, which represents the fund in hand. The balance of £254, compared with the amount involved, infers a rate of interest which would differ by so small a fraction from 4 per-cent that it is unnecessary to enter into further calculations on the subject.

I have been invited to remark upon the system which has been pursued, in the allotment into interest and redemption of capital of the annual charge for annuities.

In the case of annuities for terms of years granted so as to return to the purchaser a particular rate of interest, it would be a matter of mere arithmetical skill to analyze each instalment of annuity into its component parts of principal and interest, so that, at the termination of the contract, the total of the amounts periodically allotted as the repayments of principal should sum to the exact amount originally paid for the annuity.

But in the case of life annuities, no similar analyses of each separate account is possible. It is true, that the order of the deaths of a considerable body of life annuitants can be predicted within narrow limits of the actual facts; but there are no means by which the date of the deaths of individuals can be arrived at until such occurrences have actually taken place. No means, consequently, exist for dealing separately with life annuity accounts during the period of their currency. The expedient of considering such accounts in the mass must consequently be had recourse to; and further, if a precise division of interest and redemption of capital is the object of research, the true rate of mortality that prevails among the annuitants under observation should be available for the purposes of the calculation, as well as the exact rate of interest at which the annuities were in reality granted.

In the case of a body of life annuitants in existence, the exact rate of interest payable on the capital advanced by them, and the rate of mortality to which they will be subject, are, within certain limits, matters of estimate and opinion.

It is probable, however, that the duration of the life annuities granted by the Corporation will not materially differ from that assumed for annuities in the Tables which have been used by the Government. On the assumption that such will prove to be the case, and that money is of the value of 4 per-cent per annum, it has been in effect stated, in the Valuation marked C, that the outstanding life annuities represent a debt of £1,039,275; and

it has further been shown that, at the same rate of interest, the outstanding term annuities were worth £31,174—so that the two descriptions of annuities together represent an obligation of £1,070,449.

The total amount received for the purchase of annuities from the commencement of transactions is £1,384,098. It would therefore appear that £313,649—that is to say, £1,384,098, less £1,070,449—represents the amount of debt virtually redeemed, if the balance of capital outstanding is assumed to carry interest at 4 per-cent. But, as already stated, it is computed that the capital raised by the grant of life annuities has cost more than 4 per-cent; so that, although £1,384,098 would represent, with a close approach to accuracy, the amount of debt outstanding at 4 per-cent, £313,649 would be no more than the representation of the abatement to be made from the original purchase-money to reduce the total sum borrowed to the balance which would correspond to an outstanding capital debt at 4 per-cent on the Annuity Account.

The only known method by which the amount of capital debt outstanding from time to time on the Annuity Account is to be computed, is by means of a systematic valuation, such as has had to be made for the purpose of the present Report. And the readiest approximation which could be made, for the intervals between such valuation, would be, by an estimate of the present worth of the engagements, at any time, at a similar average value to that shown by the valuation previous to the calculation. An estimate of the amount of debt paid off from time to time could thus be obtained, and, by inference, the amount of interest payable on the balance of capital determined, inasmuch as such portion of the amount applied to the payment of annuities that did not represent a repayment of capital, would be in the nature of interest.

The system of calculation above described would give no more than approximate results, but a division of the total annual charge for annuities made on such a principle, would be a closer representation of the actual state of affairs than the method by which interest has been charged on the full amount paid for an annuity during the whole period of its currency.

It should be remembered, however, that while the present system of calculation has involved an allotment of interest on a proportion of the consideration money for annuities, which has in fact been repaid before such allotment, in the instalments of the annuities, the rate of interest which has been assumed has been less than the rate at which the capital has been borrowed. The result has therefore been that, whilst too low a rate of interest has been adopted, it has been computed upon too large an amount of capital, and thus to a certain extent the divergent assumptions have balanced each other. It is probable, however, that the adoption of the true rate of interest would more than balance the effect of the assumption made with respect to the amount of outstanding capital, and it would consequently follow that out of the total amount hitherto paid in annuity instalments, too little has been charged to revenue for interest, and too much has been charged to capital as a redemption of debt.

The Tables of rates for the grant of Life Annuities in force since October 1882 places the cost of Annuities on Female Lives at amounts which will return a rate of interest to the purchaser of no higher than 3 per-cent per annum, but, in my judgment, the charges made under the table for male lives will allow a rate of interest a fraction above 3 per-cent per annum, particularly at ages above 65 years. In consideration of the question as a whole, and having reference further to a similar rate of expense to that hitherto incurred in connection with the grant of annuities, I beg leave to recommend that a separate account should be kept of sums received for annuities under the new rates, and that interest should be reckoned upon the capital so raised at the rate of $3\frac{1}{2}$ per-cent per annum; the amount raised in this manner being reduced from time to time by the overplus of the amount paid in annuity instalments above interest at $3\frac{1}{2}$ per-cent on the balance of capital thus computed to be outstanding and unredeemed.

The amount of interest as above, at $3\frac{1}{2}$ per-cent per annum, is computed to include the usual expenses of the grant of annuities and should be charged to "revenue", while the balance of the annuity instalments should be placed to the account of "capital."

In conclusion, I beg leave to state that the voluminous statements and classifications of the annuities which have been necessary for the purposes of the enquiry have been skilfully prepared by the officers of the Committee, who have promptly supplied all the information necessary for the examination.

I am, Gentlemen,

Yours faithfully,

(Signed) ALEXR. J. FINLAISON,
Actuary to the National Debt Commissioners.
