To the Editor, *The Mathematical Gazette*

DEAR SIR.—The point made by Mr. Dunn at the end of his article " Tessellations with Pentagons " (*Gazette* LV, No. 394 (December 1971), pp. 366–9) about the reversal of the basic condition for a pentagon to be a tessellating cell by having "2 of the angles adding up to 360° and the other 3 to 180°" is interesting. Outlined below is a method of constructing such a pentagon, which will necessarily be re-entrant.

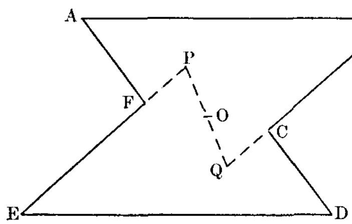

FIG. 1

Draw any re-entrant par-hexagon $ABCDEF$ (that is, a hexagon with three pairs of parallel sides), and produce any pair of parallel sides, EF and BC say, by equal distances inward to points P, Q respectively, as in Figure 1. Join PQ.

Because a par-hexagon enjoys point symmetry about its centre O, we have here that $PO = OQ$. We have thus divided the hexagon into 2 congruent pentagons (the one can be obtained from the other by a half-turn about O), which can now be used to tessellate a plane. It will be noticed that because EP and BQ are parallel,

$$\angle FPQ = \angle PQC;$$

therefore, in the pentagon $ABQPFA$,

$$\text{reflex } \angle FPQ + \angle PQB = 360°,$$

and it is easily seen that the other three angles of the pentagon sum to 180°. (It is well-known that any par-hexagon can be used to tessellate a plane surface.)

All that the above construction really does is to divide the parallel sides EF and BC *externally* in the *same* ratio. It is interesting to note that if we divide the parallel sides of any par-hexagon *internally* in the same ratio, as in Figure 2, we have the first type of pentagons, with 2 adjacent angles supplementary, here angles FPQ and BQP. The pentagons so formed are (i) re-entrant or (ii) convex according as the par-hexagon is re-entrant or convex.

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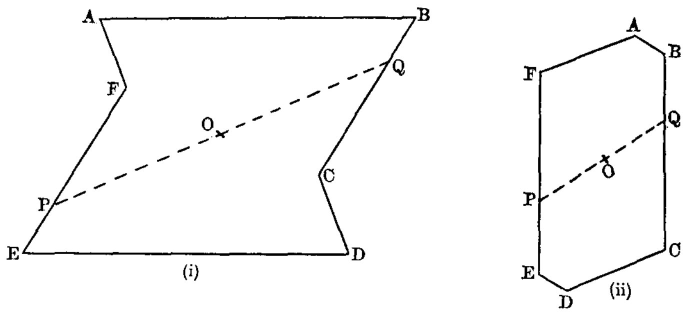

FIG. 2

It should also be noted that as P and Q travel round the sides of the par-hexagon, provided always that the line segment PQ is wholly contained within the hexagon, a whole class of pentagonal cells is generated, each of which can be used for tessellation.

Yours faithfully,

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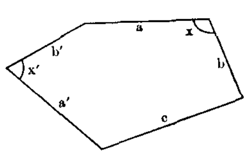
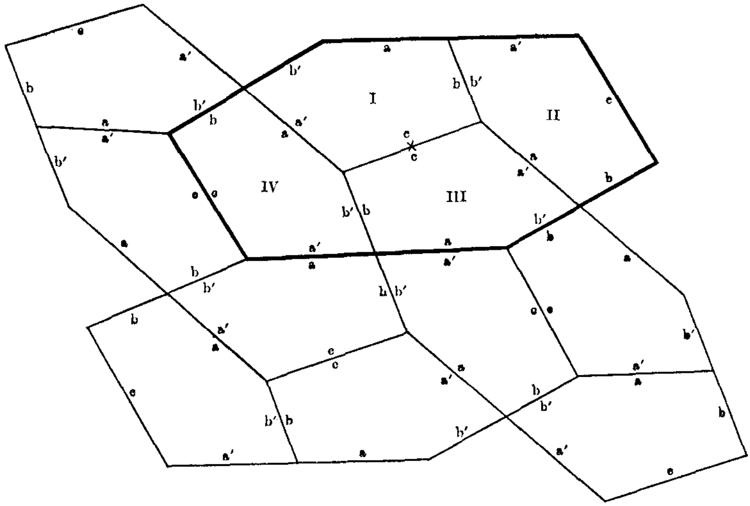
[Mr. Eba also draws attention to Mr. Dunn's second class of tessellating pentagons, of which a specimen is reproduced below. In the original article it is stated that " each hexagon is made up of four pentagons, two of which are mirror images of the other two ". Mr. Dunn points out that he here uses the description " mirror images " to describe pentagons related by an opposite isometry, but not of course by a single reflection. His point is that this tessellation includes congruent non-regular asymmetrical pentagons (I and III in the figure) and the same pentagons " turned over " (II and IV), not just rotated in the plane. He remarks " I think this is fairly unusual ". Mr Eba adds that I and III are obtained from each other by half-turn about the centre of the hexagon (marked with a cross in the figure), as are II and IV. The construction of this tessellation can be recommended as an instructive and entertaining exercise.



The fundamental pentagon: $a = a'$, $b = b'$ and $x + x' = 180°$.



Tessellation of pentagon forming a pattern of interlocking hexagons.

Finally, we are grateful to Mr. Eba for pointing out that the statement in the last line of p. 368, that the pentiamond has " all the sides equal " is, of course, incorrect; actually, three of the sides are 1 unit in length and the other two are 2 units each. D.A.Q.]

GLEANINGS FAR AND NEAR

The following comments on relative motion were made at the trial of Reginald Tom Hinks for the murder of his father-in-law on 1st December, 1933, and are quoted by F. Tennyson Jesse in "Comments on Cain", 1948.

When Dr. Scott-White, a witness for the defence, said he thought the bruise more consistent with a moving object striking a stationary object than a stationary head being met by a moving object, the learned Judge merely asked "Why?" Dr. Scott-White replied: "May I put it this way? Would you rather I hit you on the head with an ink-pot or would you rather fall on the ink-pot?" To which the learned Judge replied: "So long as the strength of the blow is the same I don't think it would matter."

(per Mr. A. B. Manning)