Epidemiology and Infection

cambridge.org/hyg

Letter to the Editor

Cite this article: Zhao S, Ran J, Yang G, Cao P (2020). Attach importance to the procedure of deriving reproduction numbers from compartmental models: Letter to the editor in response to '*Seasonality of the transmissibility of hand, foot and mouth disease: a modelling study in Xiamen City, China*'. *Epidemiology and Infection* **148**, e62, 1–2. https://doi.org/10.1017/S0950268820000588

Received: 21 January 2020
Accepted: 24 February 2020

Author for correspondence:
Shi Zhao, E-mail: zhaoshi.cmsa@gmail.com

Attach importance to the procedure of deriving reproduction numbers from compartmental models: Letter to the editor in response to '*Seasonality of the transmissibility of hand, foot and mouth disease: a modelling study in Xiamen City, China*'

Shi Zhao[1,2], Jinjun Ran[3], Guangpu Yang[4,5] and Peihua Cao[6]

[1]JC School of Public Health and Primary Care, Chinese University of Hong Kong, Hong Kong, China; [2]Shenzhen Research Institute of Chinese University of Hong Kong, Shenzhen, China; [3]School of Public Health, Li Ka Shing Faculty of Medicine, University of Hong Kong, Hong Kong, China; [4]Department of Orthopaedics and Traumatology, Chinese University of Hong Kong, Hong Kong, China; [5]SH Ho Scoliosis Research Lab, Joint Scoliosis Research Center of Chinese University of Hong Kong and Nanjing University, Hong Kong, China and [6]Clinical Research Centre, Zhujiang Hospital, Southern Medical University, Guangzhou, China

To the Editor

We read with interests Huang *et al.*'s article '*Seasonality of the transmissibility of hand, foot and mouth disease: a modelling study in Xiamen City, China*' [1]. Huang *et al.* modelled the long-term transmission dynamics of hand, foot and mouth disease (HFMD) in Xiamen, China. However, we suspected that there was an inconsistency between the model formulation and the effective reproduction number, R_{eff}, in [1]. As such, we highlighted the importance of following the next generation matrix (NGM) analytic procedure to find reproduction numbers in compartmental models.

The NGM approach is widely adopted to derive the reproduction numbers in modelling infectious disease epidemiology [2–9]. With this approach, we re-analysed the 'SEIAR' model in [1], and found the effective reproduction number, denoted by r_{eff} in this letter to distinguish from R_{eff} in [1], in Eqn (1).

$$r_{\text{eff}} = \beta S \cdot \frac{\omega}{\omega + dr} \cdot \left[\frac{p}{\gamma + dr + f} + \frac{(1-p)k}{\gamma' + dr} \right]. \tag{1}$$

Here, all notations have the same definitions as in Huang *et al.* [1]. We found that r_{eff} was different from that in Huang *et al.* [1]. For a sufficiently small value of the human natural death rate (dr), we have an approximated version of r_{eff} as in Eqn (2), and it has an analytic form closer to the R_{eff} in [1].

$$\lim_{dr \to 0^+} r_{\text{eff}} = \beta S \cdot \left[\frac{p}{\gamma + f} + \frac{(1-p)k}{\gamma'} \right]. \tag{2}$$

It was notable that the denominator of the first fractional term in the brackets, i.e. [·], has an additional term 'f' in Eqn (2), compared to the R_{eff} in [1]. This difference implied that the HFMD-induced fatality rate (f) was neglected in Huang *et al.* [1]. Furthermore, according to Table 1 in [1], the value of f was set to be (0.03% =) 0.0003 that appears relatively small compared to the main 'removing' rates (γ and γ') of HFMD infections. This setting suggested that the R_{eff} in [1] could be treated as a reasonable simplification from its theoretical version. Hence, this inconsistency between the model formulation and the reproduction number was unlikely to affect the main conclusions.

In conclusion, we call for caution in deriving and simplifying reproduction numbers from compartmental models.

Data. No data or material was used in this work.

Acknowledgements. The authors would like to acknowledge anonymous colleagues for helpful comments, and Ying Huo for proofreading.

Authors' contributions. All authors conceived the study, carried out the analysis, discussed the results, drafted the first manuscript, critically read and revised the manuscript, and gave final approval for publication.

Financial support. PC was supported by the National Natural Science Foundation of China (Grant Number: 81903406) and the China Postdoctoral Science Foundation (Grant Number: 2017M620378).

CAMBRIDGE UNIVERSITY PRESS

Conflict of interest. The authors declared no competing interests.

Ethical standards. The ethical approval or individual consent was not applicable.

Consent for publication. Not applicable.

Disclaimer. The funding agencies had no role in the design and conduct of the study; collection, management, analysis and interpretation of the data; preparation, review or approval of the manuscript; or decision to submit the manuscript for publication.

References

1. **Huang Z *et al.*** (2019) Seasonality of the transmissibility of hand, foot and mouth disease: a modelling study in Xiamen City, China. *Epidemiology and Infection* **147**, e327.
2. **Brauer F and Castillo-Chavez C** (2001) *Mathematical Models in Population Biology and Epidemiology*. New York, USA: Springer.
3. **Earn D *et al.*** (2008) *Mathematical Epidemiology*. Berlin: Springer.
4. **Grassly NC and Fraser C** (2008) Mathematical models of infectious disease transmission. *Nature Reviews Microbiology* **6**, 477.
5. **Keeling MJ and Rohani P** (2011) *Modeling Infectious Diseases in Humans and Animals*. Princeton, USA: Princeton University Press.
6. **Diekmann O, Heesterbeek J and Roberts MG** (2009) The construction of next-generation matrices for compartmental epidemic models. *Journal of the Royal Society Interface* **7**, 873–885.
7. **Diekmann O, Heesterbeek JAP and Metz JA** (1990) On the definition and the computation of the basic reproduction ratio R0 in models for infectious diseases in heterogeneous populations. *Journal of Mathematical Biology* **28**, 365–382.
8. **Heffernan JM, Smith RJ and Wahl LM** (2005) Perspectives on the basic reproductive ratio. *Journal of the Royal Society Interface* **2**, 281–293.
9. **Van den Driessche P and Watmough J** (2002) Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences* **180**, 29–48.

https://doi.org/10.1017/S0950268820000588 Published online by Cambridge University Press