Chapter 3 is a comprehensive treatment of the gamma function, with all the relevant results covered such as its characterisation by Bohr-Mollerup and Wielandt. True to the title of the book, the authors choose to define gamma as an infinite product, adopting the Weierstrass approach. In any account of Γ, Stirling's approximation formula $\Gamma(x + 1) = \sqrt{2\pi x}\,(x/e)^x$ is unavoidable and in this book it is proved by Laplace's method. The digamma function ψ and the beta functions are discussed and then nice applications to summation of series are given.

Chapter 4 treats products related to prime numbers and partitions. It culminates with Jacobi's triple product identity: if $x \neq 0$ and $|q| < 1$, then

$$\prod_{k=1}^{\infty}\left(1 - q^{2k}\right)\left(1 + xq^{2k-1}\right)\left(1 + \frac{q^{2k-1}}{x}\right) = \sum_{k=-\infty}^{\infty} x^k q^{k^2}.$$

This identity spawns a number of other identities, including the celebrated Euler's pentagonal number theorem $\prod_{n=1}^{\infty}\left(1 - q^{2n}\right) = \sum_{n=-\infty}^{\infty}(-1)^n q^{n(3n-1)/2}$ and Gauss's triangular number theorem $\displaystyle\prod_{n=1}^{\infty}\frac{1 - q^{2n}}{1 - q^{2n-1}} = \sum_{n=0}^{\infty} q^{n(n+1)2}$, which concern pentagonal and triangular numbers respectively. Identities between products and series have an interpretation in number theory and applications to partitions.

Chapter 5, ‘Epilogue', gives a perspective on some further work on infinite products. As a curiosity it should be mentioned that in this chapter one product for $\sin z$ which generalises Euler's product appeared as an examination question on the Cambridge Mathematical Tripos in 1904.

The last chapter is a summary of products for miscellaneous constants, elementary functions and products involving Γ and ζ, arranged in neat tables. Exercises of various difficulty test the reader's understanding of each section.

This delightful book is strongly recommended for students who want to know more about infinite products and teachers who teach courses that involve the subject. The authors have succeeded in their aim to spark a flame to study the beautiful field of infinite products.

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MARTIN LUKAREVSKI
Department of Mathematics and Statistics,
University "Goce Delcev",
Stip, North Macedonia
e-mail: *martin.lukarevski@ugd.edu.mk*

Visual complex analysis (25th anniversary edition) by Tristan Needham, pp 720, £40 (paper), ISBN 978-0-19-286892-3, Oxford University Press (2023)

All that aficionados of the first edition of *Visual complex analysis* (VCA) need to know is that, although the main body of the text of this second (25th anniversary) edition is largely unchanged except for a few corrections and updates, there have been significant improvements in layout and organisation. The page size is bigger, which means that the 500 or so diagrams are clearer and larger. They also now all have explanatory captions which are sufficiently detailed to make it feasible to navigate the book just by using the figures as stepping-stones. A standard numbering system for sub-sections, equations and figures has been added; the index has been expanded and the bibliography updated.

For those not familiar with VCA, it offers a unique take on introductory complex analysis by putting geometrical intuitions to the fore. In the author's own words, "The basic philosophy of this book is that while it often takes more imagination and effort to find a picture than to do a calculation, the picture will always reward you by bringing you nearer to the Truth" (p.252) and, "As with other new ideas in this book, we have not attempted to present the arguments in rigorous form; "insight", not "proof", is ever our watchword" (p.471). As a quick example, the complex derivative $f'(z) = |f'(z)|e^{i \arg f'(z)}$ is envisaged as locally effecting an "amplitwist", amplifying by $|f'(z)|$ and twisting through $\arg f'(z)$; the fact that (infinitesimally) small squares map to small squares is a manifestation of the Cauchy-Riemann equations. The inspiration for this type of approach came from Needham's careful reading of Newton's *Principia* and his realisation that Newton's geometrical arguments were a radical new representation of limiting processes rather than merely a translation of earlier Newton-Leibniz calculus algorithms into geometrical form.

Compared with traditional texts on complex analysis, the distinctive features of VCA are the order and relative emphases placed on each theme, the greater than usual input from physics, the relaxed attitude to rigour, and some original and innovative material. Thus, about a quarter of VCA is devoted to the excellent Chapter 3 on Möbius transformations and inversion and Chapter 6, by far the most lucid introduction that I have seen to non-Euclidean geometries and the way that their relevant isometries may be described using Möbius transformations. This contrasts with Chapter 9 on Cauchy's formula and its applications — the heart of a traditional text — which occupies just 25 pages, Cauchy's theorem having made its main appearance as late as p.467! Regarding order, winding numbers are introduced before complex integration and placed in the context of a more general topological argument principle before being applied to Rouché's theorem, the maximum-modulus theorem, and the Schwarz-Pick lemma. The impact of physics is most apparent in the closing Chapters 10-12 which look at complex functions as vector fields, $H(z)$, with a central role played in visualisation by the Pólya vector field, $x \to H(z)$. The final chapter deals with harmonic functions in $\mathbb{R}^2$ with applications to fluid flow and electrostatics, including the method of images, Riemann's mapping theorem, and Poisson's formula for the solution of the Dirichlet problem. There are just over 300 interesting and enticing exercises at the ends of chapters, many tying-up loose ends in the text or exploring alternative approaches.

VCA is sumptuously produced and written in a compelling and wholly engaging style. There is so much to savour and enjoy on every page and the reward of a fresh insight in every figure. By the end, my personal "party bag" included the diagrams in Chapter 5 which render the Cauchy-Riemann equations obvious, Liouville's theorem proved via Schwarz's lemma, the condition for equality in the inequality $\left|\int_L f(z)\,dz\right| \leqslant \int_L |f(z)|\,.\,|dz|$, the author's use of his own concept of complex curvature to explain Newton's famous result that elliptical orbits only occur for kr and k/r^2 laws of force, and the formula $\oint_L \bar{z}\,dz = 2i$ (area enclosed by L) which makes it crystal-clear that the integral of the non-analytic function $\bar{z}$ is path-dependent. I also like the author's one word, "Done", to signify the completion of arguments!

Sir Roger Penrose ends his Foreword to this 25th anniversary edition with the words, "… I am sure that readers, over a broad range of relevant knowledge—from those with no prior experience of complex analysis to those already experts—will gain greatly from the charm, distinct originality, and visual clarity of the arguments presented here". Peter Shiu ended his very enthusiastic review of the first edition of VCA in the March 1999 Gazette (pp.182-183) with the sage advice, "If your budget limits you to buying only one mathematics book in a year then make sure that this is the one that you buy." Tristan Needham ends his Preface to the new edition with the toast, "Cheers! I raise my glass to the *next* 25 years!". This will take us to the golden anniversary of VCA and, rather presciently, the parcel containing my review copy weighed in at a golden 1.618 kg!

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Mastering calculus through practice by Barbara de Holanda, Maia Teixeira and Edmundo Capelas de Oliveira, pp 348, EUR 53.49 (paper), ISBN 978-3-030-95823-7, Springer Verlag (2023)

Calculus is understood in this book as watered-down mathematical analysis, not putting much value on rigour. To master calculus properly would mean to go much further in the direction of analysis and to be able to solve more difficult problems. This book contains solved exercises from the standard course in calculus, with topics functions, limits, derivatives and integrals in Chapters 2 to 5 respectively. The first chapter, entitled 'Preliminaries', is the largest, comprising one third of the book; it consists of high school mathematics. The authors put much focus here on elementary geometric exercises, unrelated to themes from calculus, which might be more suitable for a pre-calculus book. Chapter 6, 'Brief Recap', reviews the preceding material. The last chapter contains the solutions.

With few exceptions, most of the exercises could be found in standard calculus textbooks; they are simple and straightforward, rather than challenging and thought-provoking tough nuts to be cracked.

The solutions are not always the best, and some of them are actually wrong. For instance, exercise 1.31 is: 'Let $n \in \mathbb{N}$. Show that, for $n > 2$, the inequality $n^3 > 3n + 5$ is valid'. This is 'proved' by the 'method of exhaustion in contrapositive statement' so that the inequality $n^3 \leqslant 3n + 5$ is checked for $n = 0$, 1, 2. And that's it. The authors claim that the result is proved since direct statement and contrapositive statement are logically equivalent. It is elementary logic that if you want to use the indirect method of proof you must show that $n^3 \leqslant 3n + 5 \Rightarrow n \leqslant 2$.

Some solutions are baffling. For instance, exercise 6.5 asks for the area A of the region bounded by the curve $y(x) = \sqrt{4x + 1}$ and the straight lines $x = 0$, $y = 0$ and $x = 12$. The worked solution uses the trapezium rule with division of the interval $[0, 12]$ in three subintervals with the points $x_0 = 0$, $x_1 = 2$, $x_2 = 6$ and $x_3 = 12$ to calculate the sum of the areas of the three trapeziums, which yields 56 as the approximation of $A = \int_0^{12} \sqrt{4x + 1}dx = 57$. But then the readers will be perplexed by the false claim that considering 12 trapeziums, all having the same width, would give the inequality $56 \leqslant A \leqslant 58$.

In the preface the authors write that they avoid the traditional method of ordering the exercises in their order of difficulty, but they don't seem to offer any alternative.

That there is no logical order in their selection is shown by exercise 4.42 which is to determine the equation of the tangent to the curve $y = 9\sqrt{x - 1}$ at its inflection point. Then, after two pages, exercise 4.53 asks how to define an inflection point. In fact Exercise 4.42 is clearly a misprint; the given solution $x = 1$ is an inflection point for the function $y = 9\sqrt[3]{x - 1}$.

There is unnecessary repetition. Exercise 2.30 asks for the minimum value of $z = x^2 + y^2$ for real x and y given that $2x + 3y = 16$. Then 2.38 is the same exercise for $2x + 3y = 12$ with only slightly changed wording.