ON CURVES AND SURFACES
WITH PROJECTIVELY EQUIVALENT
HYPERPLANE SECTIONS

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ABSTRACT.   In this paper we describe projective curves and surfaces such that almost all their hyperplane sections are projectively equivalent. Our description is complete for curves and close to being complete for smooth surfaces. In the appendix we make some remarks on connections between the mentioned property of a projective variety and its adjunction properties.

Introduction.   In this paper we are concerned with the following question: how to describe the projective varieties such that almost all their hyperplane sections are projectively equivalent? We give the complete answer for curves and a partial one for smooth surfaces (in characteristic 0 both).

The question we are interested in was considered, for the case of surfaces, in 1925 by Guido Fubini and Gino Fano ([6, 4, 5]). The final results are contained in [5]. Fano gives the complete list of surfaces with projectively equivalent hyperplane sections (and arbitrary singularities); we consider only smooth surfaces, and our list is apparently superfluous: according to Fano, some of the surfaces therein should not have projectively equivalent hyperplane sections, but I did not manage to prove it, nor to follow the argument in [5]. For the case of curves in characteristic 0, our result is complete.

Nowadays this problem was considered by Edoardo Ballico [1] in arbitrary characteristic. Our method differs from that of [1]. For the case of curves our result is in accord with [1], for the case of surfaces in characteristic 0 our result strengthens Proposition 5.2 of [1].

In the appendix we prove a result concerning connections between projective equivalence of hyperplane sections, finiteness of monodromy group and the adjunction properties of a variety.

When the first draft of this paper was finished, I learned that Rita Pardini [9] had proved Fano results from [5] in full.

Notation and conventions.   Throughout the paper, the base field will be the field $\mathbf{C}$ of complex numbers. If $\mathcal{E}$ is a locally free sheaf over X, then $\mathbf{P}(\mathcal{E}) = \operatorname{Proj} \operatorname{Sym}(\mathcal{E})$.

If $p: X \longrightarrow Y$ is a nonramified covering, we will say that it is *split* if each connected component of X is mapped isomorphically on Y.

384

We will say that a surface $X \subseteq \mathbf{P}^n$ is a *scroll* if

$$\left(X, O_X(1)\right) \cong \left(\mathbf{P}(\mathcal{E}), O_{\mathbf{P}(\mathcal{E})|C}(1)\right)$$

for a smooth curve C and a rank 2 locally free sheaf $\mathcal{E}$.

If $(\mathbf{P}^n)^*$ is the dual to projective space $\mathbf{P}^n$ and $\alpha \in (\mathbf{P}^n)^*$, we denote by $H_\alpha \subset \mathbf{P}^n$ the hyperplane corresponding to α.

We say that a projective variety $X \subseteq \mathbf{P}^n$ is *linearly normal* if the linear system of its hyperplane sections is complete.

If $X \subseteq \mathbf{P}^n$ is a smooth projective variety of dimension d, the monodromy group of its hyperplane section is the monodromy group acting on $H^{d-1}(Y, \mathbf{R})$ as its smooth hyperplane section Y varies (*cf.* [3]).

Statement of results. Let $X \subseteq \mathbf{P}^n$ be a projective variety. We say that X satisfies *the* FF *condition* (named so after G. Fubini and G. Fano) if almost all hyperplane sections of X are projectively equivalent.

PROPOSITION 0.1. *If X is an irreducible curve not contained in a hyperplane, then* FF *condition is satisfied if and only if* $\deg X \leq n + 1$.

PROPOSITION 0.2. *If $X \subseteq \mathbf{P}^n$ is a smooth irreducible surface satisfying the* FF *condition, then X is either a rational scroll, or a Veronese surface $v_2(\mathbf{P}^2) \subset \mathbf{P}^5$, or its isomorphic projection, or a non-linearly-normal scroll with elliptic base.*

According to Fano [5], of all the surfaces listed in the above proposition, only linearly normal rational scrolls and $v_2(\mathbf{P}^2) \subset \mathbf{P}^5$ satisfy the FF condition.

Here is an amusing corollary to Proposition 0.2.

PROPOSITION 0.3. *If the surface $X \subseteq \mathbf{P}^n$ is a scroll with base of genus ≥ 2 or a linearly normal scroll with elliptic base, then almost all hyperplane sections of X are not linearly normal.*

1. **Preliminaries; the** FF **condition and monodromy.** Let $X \subseteq \mathbf{P}^n$ be a projective variety. For $\alpha, \beta \in (\mathbf{P}^n)^*$ denote by $\Phi_{\alpha\beta}$ the set of projective isomorphisms $\varphi: H_\alpha \rightarrow H_\beta$ such that $\varphi(X \cap H_\alpha) = X \cap H_\beta$.

PROPOSITION 1.1. *Assume that X satisfies the* FF *condition and X is smooth or $\dim X = 1$; if we denote by $Y = X \cap \mathbf{P}^n$ the generic hyperplane section, then the action of the monodromy group on $H^\bullet(Y)$ is induced by the action of a subgroup $G \subseteq \{g \in \mathrm{Aut}(\mathbf{P}^n) : gY = Y\}$.*

PROOF. The condition FF implies that there exists a Zariski open subset $U \subseteq (\mathbf{P}^n)^*$ such that, for $\alpha, \beta \in U$ we have $\Phi_{\alpha\beta} \neq \emptyset$ and H_α is transversal to X. Consider a fiber

space Φ over $U \times U$ such that $\Phi_{\alpha\beta}$ is the fiber over (α, β). Φ is a principal Γ-bundle, where $\Gamma = \{g \in \mathrm{Aut}(\mathbf{P}^n) : gY = Y\}$. If $Y = X \cap \mathbf{P}^{n-1} = X \cap H_\alpha$, consider the restriction of Φ to $\{\alpha\} \times U$. Each loop $\{\alpha_t\}(t \in [0;1])$ in U can be lifted to this restriction as the path $\{(\alpha_t; \varphi_t)\}$, where $\varphi_t \in \Phi_{\alpha,\alpha_t}$. It is clear that φ_1 induces in $H^\bullet(Y)$ the monodromy transformation corresponding to the loop $\{\alpha_t\}$. The proposition is proved.

COROLLARY 1.2.   *If a smooth variety $X \subseteq \mathbf{P}^n$ satisfies the* FF *condition, then the monodromy group of its hyperplane section is finite; if* dim X *is even, this group is trivial.*

PROOF.   Since the connected component $\Gamma_0 \subseteq \Gamma$ acts trivially in cohomology and Γ/Γ_0 is finite, the first assertion holds; the second assertion follows immediately from the first one and the Picard-Lefschetz theory.

The following proposition is quite similar to the main construction of [8], so we omit some details of the proof.

PROPOSITION 1.3.   *Let $X \subseteq \mathbf{P}^n$, $X \neq \mathbf{P}^n$ be a smooth projective variety, and $L \subseteq (\mathbf{P}^n)^*$, $L \cong \mathbf{P}^1$ a Lefschetz pencil of hyperplanes. Assume that there exist a Zariski open subset $U \subseteq L$ and $\alpha \in U$ such that for any $\beta \in U$ there exists a projective isomorphism $\psi_\beta : H_\beta \longrightarrow H_\alpha$ satisfying the following conditions:*

   i) $\psi_\beta(H_\beta \cap X) = H_\alpha \cap X$;
   ii) ψ_β is identity on $H_\alpha \cap H_\beta$.

Then X is a quadric.

PROOF.   Choose the homogeneous coordinates in $\mathbf{P}^n$ so that the equations of H_α (resp. the axis of L) are $x_n = 0$ (resp. $x_{n-1} = x_n = 0$). For $u \in \mathbf{C}$ denote by H_u the hyperplane defined by the equation $x_n = ux_{n-1}$, and denote by H_∞ the hyperplane $x_{n-1} = 0$.

Now for $\beta \in U$ denote by Ψ_β the set of projective automorphisms $\psi_\beta : H_\beta \longrightarrow H_\alpha$, such that $\psi_\beta(H_\beta \cap X) = H_\alpha \cap X$ and $\Psi|_{H_\alpha \cap H_\beta \cap X} = \mathrm{id}$. Consider a fiber space Ψ over U such that Ψ_β is the fiber over β. Let $\Gamma \subseteq \Psi$ be a quasi-section of Ψ over an open subset $U' \subseteq U$; the projection $\pi : \Psi \longrightarrow U$ induces a regular function u on Γ such that any $p \in \Gamma$ may be regarded as a linear isomorphism $f_p : H_u \longrightarrow H_0 = H_\alpha$; writing f_p^{-1} in the matrix form, we obtain regular functions $a_0, \ldots, a_{n-1}$ such that f_p^{-1} sends $(x_0 : \cdots : x_{n-1}) \in H_\alpha$ to the point

$$\left(x_0 + a_0(p)x_{n-1} : \cdots : x_{n-2} + a_{n-2}(p)x_{n-1} : a_{n-1}(p)x_{n-1} : u(p)a_{n-1}(p)x_{n-1}\right) \in X \subseteq \mathbf{P}^n.$$

Set $a_n = u \cdot a_{n-1}$. If S is the smooth projective model of Γ, then a_j's may be regarded as rational functions on S; not all of them are constant, because u is not constant and $a_n = u \cdot a_{n-1}$.

Now we can proceed as in [8, Section 3]: not all a_j's, $0 \leq j \leq n$, are constant, hence some of them must have poles. Assume that the maximal order of these poles equals m and is attained at the point $\xi \in S$; by [7, Lemma 3.1], for each $c \in \mathbf{C}$ and $x = (x_0 : \cdots : x_{n-1} : 0)$ there exists a map $h : \Delta \longrightarrow X \cap H_\alpha$, $h : t \longmapsto \left(\tilde{x}_0(t) : \cdots : \tilde{x}_{n-1}(t) : 0\right)$, where Δ is

the unit disk in the complex plane, such that $\tilde{x}_i(0) = x_i$ for $0 \leq i \leq n - 2$, $\tilde{x}_{n-1} \sim ct^m$ as t tends to 0. The point $\lim_{t \to 0} f_{h(t)^{-1}}\big(h(t)\big)$ is in X, and its homogeneous coordinates are

$$(x_0 + cb_0x_{n-1} : \cdots : x_{n-2} + cb_{n-2}x_{n-1} : cb_{n-1}x_{n-1} : cb_nx_{n-1});$$

as it is explained in [8], b_j's do not depend on c. Hence $X \cap H_{b_n/b_{n-1}}$ is a cone; since L is a Lefschetz pencil, this cone must be a quadratic cone and X must be a quadric.

COROLLARY 1.4.   *If $X \subseteq \mathbf{P}^n$ is not a linearly normal rational scroll nor the Veronese surface $v_2(\mathbf{P}^2) \subset \mathbf{P}^5$, then Proposition 1.3 holds with hypothesis (ii) replaced by "ψ_β is identity on $H_\alpha \cap H_\beta \cap X$".*

PROOF.   If $\deg X = d$, then the hypothesis implies that its generic linear section of codimension 2 contains at least $d + 1$ points in general position, hence "identity on the linear section of codimension 2" implies "identity on the projective space of the section", and the Proposition applies.

2. **Case of curves.**   In this section we prove Proposition 0.1. Assume that $X \subseteq \mathbf{P}^n$ is a curve for which FF holds, and that X is not contained in a hyperplane. We are to prove that $\deg X \leq n + 1$.

Let us apply Proposition 1.1. In our case the generic hyperplane section is a set of $\deg X$ points in $\mathbf{P}^{n-1}$, and the monodromy group consists of permutations of these points. According to [2], this group is the whole symmetric group; on the other hand, if $Y = X \cap \mathbf{P}^{n-1}$ is the generic hyperplane section, then no n points of Y belong to a hyperplane (we will call it the *generic position property*). Proposition 0.1 follows immediately from the above observations and the following lemma.

LEMMA 2.1.   *If there are $s > m + 2$ points in $\mathbf{P}^m$ such that no $m + 1$ of them belong to a hyperplane, then there is no automorphism of $\mathbf{P}^m$ that interchanges two of these points and leaves the rest $s - 2$ points fixed.*

PROOF.   If $s \geq m + 4$, there is nothing to prove since any projective automorphism of $\mathbf{P}^m$ fixing $m + 2$ points in general position must be identity. Hence we may assume that $s = m + 3$. Due to the generic position condition we may choose the homogeneous coordinates so that $p_1 = (1 : 0 : \cdots : 0)$, $p_2 = (0 : 1 : \cdots : 0), \ldots, p_{m+1} = (0 : \cdots : 0 : 1)$, $p_{m+2} = (1 : \cdots : 1)$. If $p_{m+3} = (x_0 : \cdots : x_m)$, then it follows from the generic position condition that $x_i \neq 0$ for all i, $x_i \neq x_j$ for $i \neq j$. Hence the automorphism $\varphi : \mathbf{P}^m \to \mathbf{P}^m$ that interchanges p_{m+2} and p_{m+3} should be defined by a diagonal matrix $\mathrm{diag}(x_0, \ldots, x_m)$; since $\varphi(x_{m+3}) = x_{m+2}$, we see that each of the x_j can be chosen to equal 1 or -1; this contradicts the generic position condition. The lemma and Proposition 0.1 are proved.

3. **Case of surfaces, part 1.**   We keep the notation of Section 1. Assume that $X \subseteq \mathbf{P}^n$ is a smooth surface for which the FF condition holds.

PROPOSITION 3.1. *If the generic hyperplane section of X is not a rational curve, then* $\Phi \longrightarrow U \times U$ *is a finite covering.*

PROOF. The fiber of Φ over $(\alpha, \beta) \in U \times U$ is isomorphic to $\{g \in \operatorname{Aut} H_\alpha : g(X \cap H_\alpha) = X \cap H_\beta\}$. Since the group of automorphisms of a smooth curve of genus > 1 or a polarized elliptic curve is finite, we are done.

PROPOSITION 3.2. *If the genus of the generic hyperplane section of X is greater than* 1, *then the covering* $p: \Phi \longrightarrow U \times U$ *is split.*

PROOF. Assume the contrary; then the covering $p^{-1}(U \times \{\alpha\}) \longrightarrow U \times \{\alpha\}$ is not split for the generic $\alpha \in U$. Hence, there exists a connected component $\Psi \subseteq p^{-1}(U \times \{\alpha\})$ such that $p: \Psi \longrightarrow U \times \{\alpha\}$ is a nontrivial covering. Thus, there exists a loop in U originating at (α, α), such that its lifting to Ψ defines a nontrivial automorphism of $X \cap H_\alpha$. Since any nontrivial automorphism of a Riemann surface C of genus > 1 acts nontrivially on $H^1(C, \mathbf{R})$, we infer that this loop defines a nontrivial element of the monodromy group of the hyperplane section $X \cap H_\alpha$. This contradicts Corollary 1.2.

PROPOSITION 3.3. *If* $X \subseteq \mathbf{P}^n$ *is a smooth surface for which the* FF *condition holds, then the genus of the generic hyperplane section of X is at most* 1.

PROOF. Assume the contrary, and let $\alpha \in U$ be a generic point. Since $\Phi \longrightarrow U \times U$ is split by Proposition 3.2, there exists a section $s: U \times \{\alpha\} \longrightarrow \Phi$, such that $s\big((\alpha, \alpha)\big) = \operatorname{id}_{H_\alpha}$. Set $\psi_\beta = s(\beta): H_\beta \longrightarrow H_\alpha$. Define, for the generic $x \in X$, the mapping $f: U \longrightarrow X \cap H_\alpha$ by the formula $\beta \longmapsto \psi_\beta(x) \in H_\alpha$. By Proposition 1.4 this mapping is not constant, hence $\overline{f(U)} = X \cap H_\alpha$. But this equality is impossible since $X \cap H_\alpha$ is not a rational curve. This contradiction completes the proof.

4. **Case of surfaces, part 2.** In this section we assume that $X \subseteq \mathbf{P}^n$ is a linearly normal smooth surface, that the condition FF holds for X and that the generic hyperplane section of X is an elliptic curve. We make use of the following important result of Zak [11]:

THEOREM 4.1 (F. L. ZAK). *If* $X \subseteq \mathbf{P}^n$ *is a smooth surface such that the monodromy group of hyperplane section of X is trivial, then X is either a scroll, or the Veronese surface* $v_2(\mathbf{P}^2)$, *or its isomorphic projection to* $\mathbf{P}^4$.

It follows immediately from this theorem and Corollary 1.2 that the assumptions of this section imply that X is $\mathbf{P}_C(\mathcal{E})$ embedded by the complete linear system $|O_{X|C}(1)|$, where $\mathcal{E}$ is a rank 2 locally free sheaf on the elliptic curve C. $H^0\big(O_{X|C}(1)\big)$ will be canonically identified with $H^0(\mathcal{E})$. Let us denote $\mathcal{L} = \det \mathcal{E}$.

If $s \in H^0\big(O_{X|C}(1)\big) = H^0(\mathcal{E})$, consider the homomorphism $f_s: \mathcal{E} \longrightarrow \det \mathcal{E}$ defined by the formula $\xi \longmapsto s \wedge \xi$.

PROPOSITION 4.1.   *If the section s defines a smooth hyperplane section of X, then the sequence of sheaves*

(1) $$0 \longrightarrow O_C \xrightarrow{s} \mathcal{E} \xrightarrow{f_s} \mathcal{L} \longrightarrow 0$$

is exact.

The proof is straightforward.

PROPOSITION 4.2.   *Consider an exact sequence of sheaves*

(2) $$0 \longrightarrow O_C \xrightarrow{s} \mathcal{E} \xrightarrow{f} \mathcal{L} \longrightarrow 0$$

where C is an elliptic curve, $\mathcal{E}$ is a locally free sheaf of rank 2, $\mathcal{L}$ is an invertible sheaf (hence, $\mathcal{L} = \det \mathcal{E}$), as an extension of L by O_C. The class of this extension in $\mathrm{Ext}^1(L, O_C)$ is determined, up to proportionality, by the linear subspace

$$\mathrm{Im}\big(H^0(C, \mathcal{E}) \to H^0(C, \mathcal{L})\big) \subseteq H^0(C, \mathcal{L}).$$

PROOF.   Since the sheaves are locally free and the underlying variety is a smooth curve, $\mathrm{Ext}^1(L, O_C)$ is canonically isomorphic to $H^1(C, \mathcal{L}^{-1})$ and, by Serre's duality, canonically dual to $H^0(C, \mathcal{L})$; the fundamental class of the extension (2) in $\mathrm{Ext}^1(L, O_C) = \big(H^0(C, \mathcal{L})\big)^*$ is

$$\delta: H^0(\mathcal{L}) \to H^1(O_C) \cong \mathbf{C},$$

where δ is the connecting homomorphism associated with the exact sequence (2). Hence this class is determined, up to proportionality, by $\mathrm{Ker}\,\delta = \mathrm{Im}\,f_*$.

Let us return to our surface X.

PROPOSITION 4.3.   *For generic hyperplanes $H_1, H_2 \subseteq \mathbf{P}^n$ there exists a projective automorphism $F: \mathbf{P}^n \to \mathbf{P}^n$, such that $F(X) = X$, $F(H_1) = H_2$, and F maps each line of the ruling of X into itself.*

PROOF.   The smooth hyperplane section of X defined by a section $s \in H^0(\mathcal{E})$ is projectively isomorphic to the curve C embedded by the linear system $|V_s|$, where

$$V_s = \mathrm{Im}\big(H^0(\mathcal{E}) \xrightarrow{(f_s)_*} H^0(\mathcal{L})\big).$$

It follows immediately, from the exact cohomology sequence associated with (1) and the ampleness of $\mathcal{E}$, that V_s has codimension 1 in $H^0(\mathcal{L})$.

Now if s and t are two generic sections of $\mathcal{E}$, then the hyperplane sections defined by s and t are projectively isomorphic if and only if there exists an isomorphism $\varphi: C \to C$ such that $\varphi^* L = L$ and $V_s = \varphi^* V_t$. Since the group of automorphisms of a polarized elliptic curve is finite, the FF condition implies that the hyperplanes $V_s \subseteq H^0(\mathcal{L})$ are the same for almost all $s \in H^0(\mathcal{E})$. By Proposition 4.2 this implies that the extensions

(1) are "congruent up to multiplication by a constant" for various s. Hence, for generic $s, t \in H^0(\mathcal{E})$ there exists an automorphism $g \colon \mathcal{E} \to \mathcal{E}$ and a constant $\lambda \in \mathbf{C}^*$ such that the diagram

$$
\begin{array}{ccccccccc}
0 & \longrightarrow & O_C & \overset{s}{\longrightarrow} & \mathcal{E} & \overset{f_s}{\longrightarrow} & \mathcal{L} & \longrightarrow & 0 \\
 & & \| & & g{\downarrow} & & \| & & \\
0 & \longrightarrow & O_C & \overset{\lambda t}{\longrightarrow} & \mathcal{E} & \overset{f_t}{\longrightarrow} & \mathcal{L} & \longrightarrow & 0
\end{array}
$$

is commutative. The automorphism g induces a projective automorphism $F \colon \mathbf{P}^n \to \mathbf{P}^n$ that maps X into itself and preserves the fibers of X over C. Translating all this into the geometric language, we obtain our proposition.

PROPOSITION 4.4.   *If $X \subseteq \mathbf{P}^n$ is a linearly normal scroll with elliptic base, then the FF condition does not hold for X.*

PROOF.   Assume the contrary. Then Proposition 4.3 applies. Since generic hyperplane section intersects each line of the ruling only once, the automorphism F of the above proposition fixes all the points of $H_1 \cap H_2 \cap X$. This contradicts Proposition 0.2. The proposition is proved.

5. Proof of Propositions 0.2 and 0.3.

To complete the proof of Proposition 0.2, we use the following fact: if the generic hyperplane section of a smooth surface face $X \subseteq \mathbf{P}^n$ has genus 0, then X is either a rational scroll, or $\mathbf{P}^2$, or the Veronese surface $v_2(\mathbf{P}^2)$, or its isomorphic projection. When put together with Propositions 3.3 and 3.4, this yields the required result.

To prove Proposition 0.3, observe that if a scroll X is isomorphic to $\mathbf{P}_C(\mathcal{E})$, where C is a curve and $\mathcal{E}$ is a locally free sheaf of rank 2, such that H is a hyperplane section of X, then $\big(H, O(1)\big) \cong (C, \det \mathcal{E})$, where H is a hyperplane section of X. Hence, if this hyperplane section were linearly normal, then almost all hyperplane sections would be projectively isomorphic to the curve C embedded by the complete linear system $|\det \mathcal{E}|$, contrary to Proposition 0.2. The proposition is proved.

6. Appendix. Finite monodromy groups and adjunction.

The FF property and finiteness of the monodromy group have to do with the adjunction properties of the variety.

PROPOSITION 6.1.   *Consider the following properties of a smooth projective variety $X \subseteq \mathbf{P}^n$, $\dim X = d > 1$:*

  i) Almost all hyperplane sections of X are projectively equivalent.
  ii) The monodromy group of hyperplane sections of X is finite.
  iii) If $p \neq q$, $p + q = d - 1$, then $h^{p,q}(X) = h^{p,q}(Y)$, where Y is a smooth hyperplane section of X.
  iv) $|K_X + Y| = \emptyset$.

Then the following implications hold:

$$\text{i)} \Rightarrow \text{ii)} \Leftrightarrow \text{iii)} \Rightarrow \text{iv)}.$$

If, in addition, $\dim X \leq 3$, then iii) $\leftrightarrow$ iv).

PROOF. The implication i) $\Rightarrow$ ii) is just Corollary 1.2; the equivalence of ii) and iii) is proved in [3, Exposé XVIII]. To prove that iii) $\Rightarrow$ iv), observe that iii) implies that

$$(3) \qquad h^{d-1}(X, O_X) = h^{d-1}(Y, O_Y).$$

Now the exact sequence

$$0 \longrightarrow O_X(-1) \longrightarrow O_X \longrightarrow O_Y \longrightarrow 0$$

together with Kodaira vanishing theorem, yields the exact sequence

$$(4) \quad 0 \longrightarrow H^{d-1}(X, O_X) \longrightarrow H^{d-1}(Y, O_Y) \longrightarrow H^d\big(X, O_X(-1)\big) \longrightarrow H^d(X, O_X) \longrightarrow 0.$$

Hence, (3) is equivalent to injectivity of the homomorphism $H^d\big(X, O_X(-1)\big) \longrightarrow H^d(X, O_X)$ from (4); by Serre duality this is equivalent to the equality

$$\dim |K_X| = \dim |K_X + Y|.$$

The latter equality holds if and only if $|K_X + Y| = \emptyset$. Indeed, the "if" part is obvious since $\dim |K_X| \leq \dim |K_X + Y|$, and to prove the "only if" part observe that $|K_X| \neq \emptyset$ implies the inequality $\dim |K_X + Y| > \dim |K_X|$, since the linear system $|Y|$ is movable.

To prove the last assertion observe that, if $2 \leq \dim X \leq 3$, property iii) is *equivalent* to the equality (3). The proposition is proved.

In the paper [10], A. J. Sommese gave a complete description of threefolds having property iv). Proposition 6.1 shows that [10] yields description of smooth threefolds with finite monodromy group of hyperplane section, as well. All the threefolds with FF property are among those from [10]; no doubt only few of the latter actually have the FF property.

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