SLOW AND FAST DIFFUSION IN ASTEROID-BELT RESONANCES: A REVIEW

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Abstract. This paper reviews recent advances in several topics of resonant asteroidal dynamics as the role of resonances in the transportation of asteroids and asteroidal debris to the inner and outer solar system; the explanation of the contrast of a depleted 2/1 resonance (Hecuba gap) and a high-populated 3/2 resonance (Hilda group); the overall stochasticity created in the asteroid belt by the short-period perturbations of Jupiter's orbit, with emphasis in the formation of significant three-period resonances, the chaotic behaviour of the outer asteroid belt, and the depletion of the Hecuba gap.

1. Introduction

This paper reviews the latest advances recorded in the study of the asteroidal mean-motion resonances and the recent discovery of the importance of numerous two and three-period resonances. Following a common usage, we call inner belt, the inner part of the main belt (from 2.0 to 2.9 AU), and outer belt, the domains beyond the 2/1 resonance (i.e. beyond 3.3 AU). The ring between them (from 2.9 to 3.3 AU) will be referred to as outer main-belt. In secs. 2 and 3, we review the role of resonances in the transport of asteroids and asteroidal debris to the inner and outer solar system, and compare results from numerical experiments to the known dynamics of these resonances. In particular, we discuss the contrasts of the depleted 2/1 resonance (Hecuba gap) and the high-populated 3/2 resonance (Hilda group). Sec. 4 gathers some significant results on chaos and resonance in the outer belt. At last (sec.5), we discuss the stochasticity generated by the short-period perturbations of Jupiter's orbit in the asteroid belt, viz. the significant 3-period (or 3-body) resonances and the role of the Great Inequality in the depletion of the central part of the Hecuba gap.

Almost all results in this review came from numerical simulations and analyses. However, it is worth emphasizing that semi-analytical and analytical theories of the main resonances are fundamental in allowing a correct understanding of the results obtained from numerical analyses; a thorough review of these theories was recently published by Michèle Moons (1997) and the reader is referred to it. We also mention that a new paper by Moons et al. (1998), did complement that review in several respects. Moons's review also included a detailed account of the advances recorded before 1996 so that we can restrict ourselves, here, to the discussion of those achieved in the past three years.

2. Fast-Diffusion Inner-Belt Resonances

The fast diffusion observed in several inner-belt resonances plays an important role in the transfer of asteroids and asteroid debris to the inner solar system and,

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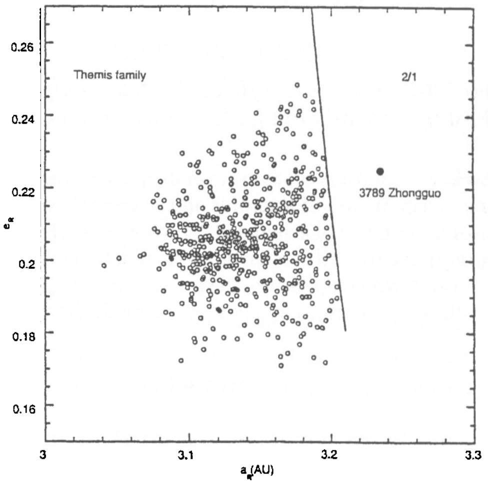

Fig. 1. Themis family together with the leftmost separatrix of the 2/1 resonance. Axes: Resonant proper elements (taken from Morbidelli *et al.*, 1995).

TABLE I
Fast diffusion of asteroids injected in resonances

Resonance	$< a >$ (AU)	Neighboring Families	Half-Life (Myr)	Impact Sun (%)	expelled (%)	Survivors (%)
4/1	2.06					
ν_6 (*)	2.1	Vesta	2.3	79.1	11.8	1.8
3/1	2.50	Vesta, Nysa, Maria	2.4	71.0	27.7	0.3
5/2	2.82	Dora, Gefion, Koronis	0.5	7.3	91.3	0

(*) at $i = 7^{\circ}$

in particular, to the neighborhood of the Earth. This entirely new fact became to appear when the study of the 4/1 and 3/1 resonances, in the frame of the restricted elliptic 3-body problem, showed paths allowing an asteroid to evolve in a short timescale (less than 1 Myr) to very high eccentricities (Wisdom, 1982, 1983; Ferraz-Mello and Klafke, 1991; Klafke *et al.*, 1992; Moons and Morbidelli, 1995). Later, simulations with more complete models, including other outer planets, showed trajectories starting in these regions and falling into the Sun (Farinella *et al.*, 1993, 1994a, 1994b; Morbidelli and Moons, 1995). By the same time, several asteroid families were shown to be intersected by resonances. The best-known example is Themis family (see fig. 1) whose proper elements show a distribution clearly cut by the left border of the 2/1 resonance (Morbidelli *et al.*, 1995). The reconstruction of the boundaries of the set of orbits originated at the fragmentation of the parent body showed that many objects may have been injected into resonances.

What happened to the missing asteroids in each case? An extended investigation

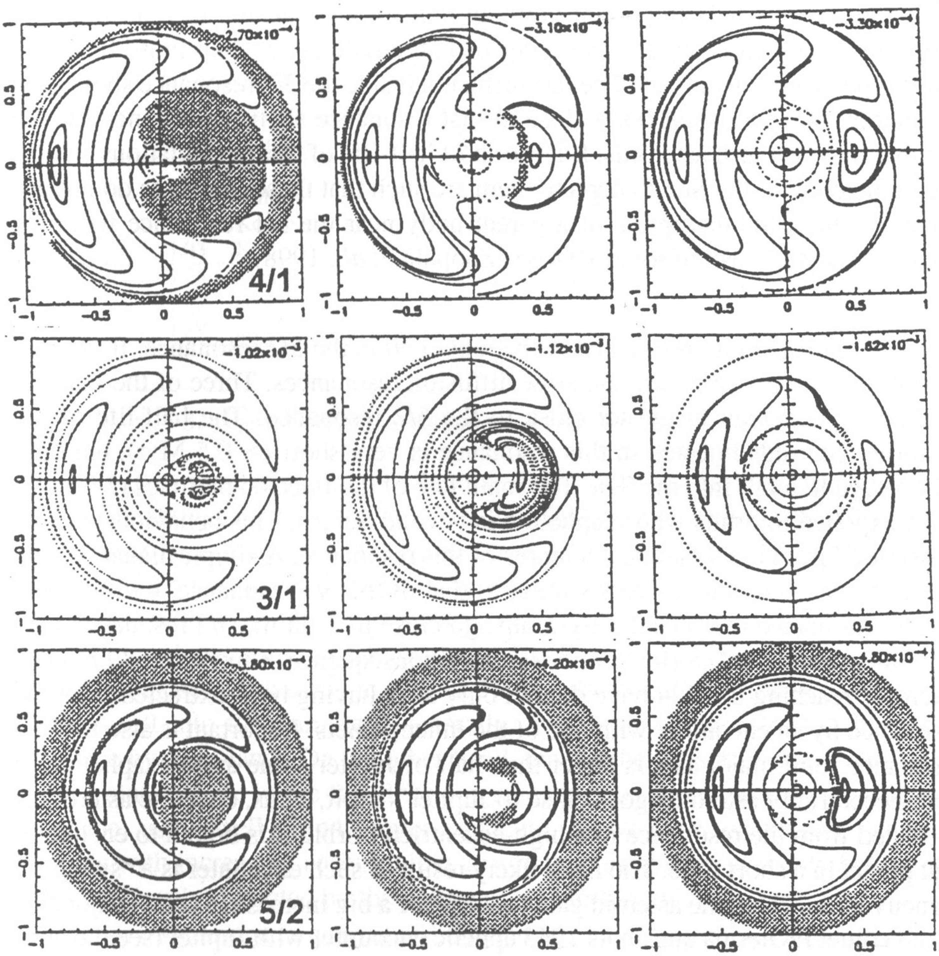

Fig. 2. Surfaces of section of the planar 3-body model of some inner belt resonances. *On top:* 4/1-resonance; *middle:* 3/1 resonance; *bottom:* 5/2 resonance. Coordinates: $x = e.\cos(\varpi - \varpi_{\rm Jup})$, $y = e.\sin(\varpi - \varpi_{\rm Jup})$. (adapted from Klafke *et al.*, 1992).

was conducted by Gladman *et al.* (1997) involving 1500 particles with initial conditions chosen in the intersection of the ejection fields of several asteroid families by the main resonances. Three of the families are close to the 3/1 resonance (see Table I). From a total of 393 fictitious asteroids showing an active interaction with this resonance, 279 terminated their evolution falling into the Sun (more than one third of them fell directly in the Sun, without being previously extracted from the resonance by encounters with the inner planets), 108 of them were expelled from the asteroid belt to orbits with aphelions situated beyond Saturn, and a few of them had an impact on one of the planets. The half-life of these populations ($\approx$ 2.4 Myr) is in good agreement with the timescale of the large eccentricity jumps seen in simulations using a restricted elliptic model (about 0.5 Myr; see Ferraz-Mello

& Klafke, 1991). It is important noticing that, in general, the median lifetimes found by Gladman *et al.* are one order of magnitude lower than those based on Öpik-type evolution models. The fast diffusion inside the 3/1 resonance shows that the disruption of a parent body close to that resonance shall rapidly increase the number of near-Earth asteroids. But the short half-life of these populations and the high efficiency of the Sun to deplete them are such that these increases do not last for long times; the disruption of a parent-body near the 3/1 resonance will only cause a "shower" lasting some 10 Myr (Zappalà *et al.*, 1998).

The next important resonance in the inner belt is the 5/2 resonance. It lies in a transition zone between fast and slow diffusion resonances. Three of the families studied by Gladman *et al.* are close to the 5/2 resonance. The half-life of the fictitious asteroids injected in this resonance is very short ($\approx$ 0.5 Myr), but only 28 evolved to a fall into the Sun. The main fate of the injected particles (349) was to be expelled to orbits whose aphelion is beyond Saturn. This behaviour may be understood from the dynamical features of this resonance. A simple planar 3-body model reveals a dynamics very similar to that of the 3/1 resonance, but, while in the 3/1 resonance the eccentricity jumps go close to 1, in the 5/2 resonance they may only reach $e \approx 0.8$ (fig. 2 *bottom*). As a consequence, almost all 28 fictitious asteroids reaching the Sun have done it only after having been extracted from the resonance by an encounter with one of the inner planets. Important is also that the resonance semi-major axis is larger than half of Jupiter's one and the aphelion of high-eccentricity asteroids goes close to Jupiter's orbit. Thus, once the asteroid is extracted from the resonance in a high-eccentricity orbit, it is bound to encounter that planet in a short time. The most likely result of such encounter is a "swing-by maneuver" in which the asteroid gains energy for a big increase in semi-major axis as did comet P/Oterma during its 1963 aphelic encounter with Jupiter (see the orbit in Carusi *et al.*, 1984). This explains the huge proportion of fictitious asteroids driven to orbits with aphelions beyond Saturn.

To complete the panorama of the fast-diffusion inner-belt resonances, we shall also mention the diffusion in the main branch of the secular resonance ν_6 (resonance 1:1 between the main component of the motion of the asteroid perihelion and g_6; g_6 is the second most important component of Jupiter's perihelion motion), and the 4/1 mean-motion resonance. Fictitious asteroids from the Vesta family that could have been injected into the ν_6 resonance show fates very similar to those of the 3/1 resonance (see Table I). The diffusion in the 4/1 resonance was not studied. However, as the main branch of the ν_6 resonance almost meets the 4/1 resonance at $i = 0°$, we may expect a diffusion at least as fast as that of the ν_6 resonance. We remind that the planar 3-body model of this resonance (without Saturn and ν_6) shows a dynamics in many points similar to that of the 3/1 and 5/2 resonance, but with many paths leading from lower eccentricities to $e = 1$ (see fig.2 *top*).

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TABLE II
Slow diffusion of asteroids injected in resonances

Resonance	$< a >$ (AU)	Neighbouring Families	Half-Life (Myr)	Impact Sun (%)	expelled (%)	Survivors (%)
8/3	2.71	Chloris	34	17.2	35.7	47.1
7/3	2.96	Koronis	19	0	56.6	42.3
9/4	3.03	Eos	>120	2.2	25.4	72.4
2/1	3.28	Themis	>100	6.5	22.9	69.9
3/2	3.97	none				
4/3	4.29	none				

3. Slow-Diffusion Resonances in the Outer Belts

High half-lives were observed by Gladman *et al.* in several resonances situated in the outermost rings of the main belt (7/3, 9/4, 2/1), as well as in the 8/3 resonance (in the inner belt, between the 3/1 and 5/2 resonances). Moreover, a very stable condition is known to exist in the 3/2 resonance, in the outer belt, where some 30 asteroids with diameters larger than 50 km, presumably primordial, are known. Table II reproduces some of the results of Gladman *et al.* These results are very different of those summarized in Table I. Now, only a small fraction of the injected asteroids fell into the Sun, and, generally, only after being extracted from the resonance by one planet. On the other hand, the number of survivors is very large (against almost none in the fast-diffusion resonances). Recent analyses of the 9/4 resonance, using well-suited proper elements, showed the existence of several asteroids with increasing eccentricities inside this resonance, probably belonging to the Eos family (bisected by this resonance) (Morbidelli *et al.*, 1995).

In these resonances, in general, 3-body models have very robust structures confining the eccentricity to some tenths (Yoshikawa, 1991; Yokoyama and Balthazar, 1992; Ferraz-Mello, 1994). The minimum models able to show some significant diffusion are 4-body models including both Jupiter and Saturn. Diffusion charts of the 2/1 and 3/2 resonances were computed with a 3-D Sun-asteroid-Jupiter-Saturn model by Nesvorný and Ferraz-Mello (1997), using Laskar's frequency map analysis. The contour lines of these charts (fig. 3)are lines of same $\log_{10}|\rho|$, where

$$\rho = \frac{f_2 - f_1}{f_1}$$

is the relative variation of the perihelion frequency. The two frequencies f_1 and f_2 come from Fourier spectra obtained in two overlapping time intervals of length $2\Delta t$ and whose medium point separation is Δt.

The diffusion chart of the 2/1 resonance (fig.3 *left*) is a composite of 2 parts: one ($e \geq 0.2$) obtained with a 31×51-points grid and $\Delta t = 133,333$ yr, and the other ($e \leq 0.25$) obtained with a 26×101-points grid and $\Delta t = 200,000$ yr. In both

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grids, $\Delta e = 0.01$. (In the domain where the two sets overlap, the one corresponding to the lower part is shown). The initial inclination is $I = 0$, the initial longitudes of the perihelion and ascending node are equal to those of Jupiter and the initial value of the critical angle

$$\sigma = 2\lambda_{\rm Jup} - \lambda_{\rm ast} - \varpi_{\rm ast}$$

is taken equal to zero. (λ denote mean longitudes and ϖ the longitude of the perihelion).

The frequency map of the 3/2 resonance (fig. 3 *right*) was obtained with a 51×51-points grid, $\Delta t = 200,000$ yr, and the same initial value of the angles as above, but, now,

$$\sigma = 3\lambda_{\rm Jup} - 2\lambda_{\rm ast} - \varpi_{\rm ast}.$$

The data used to draw these figures are the same presented as dot maps in Figs. 3 and 10 of Nesvorný and Ferraz-Mello (1997). In this paper, we are only showing the (a, e) maps for $i_0 = 0$; (a, i) maps showing the diffusion of orbits with initial inclinations up to 40° are also given in that paper.

The dispersion (r.m.s. of the time-variations) associated with each $|\rho|$ was determined from the statistical analysis of the corresponding values of $|\rho|$ obtained with two different values of Δt, in the strip $0.2 \le e \le 0.25$ (Ferraz-Mello *et al.* , 1998a). The results were time propagated using a random walk hypothesis. The levels corresponding to the lowest values of $|\rho|$ have a dispersion of the order of 10^{-1} in 1 Gyr. The dispersion increases almost linearly with $|\rho|$ (that is, exponentially with $\log_{10}|\rho|$). In the level corresponding to $\log_{10}|\rho| = -2.5$, it may reach the unity in a time not larger than 1 Gyr.

The interpretation of these results is not obvious. In a one-dimensional random walk, a dispersion equal to 1 means a 30 percent probability that the value $|\rho| = 1$ is reached (that is, a 100% change in the frequency value). The level corresponding to a dispersion reaching 1 in about 1 Gyr appears in black in the charts of fig.3. In all domains not encircled by the black strip (painted in green, yellow, orange, red) much higher dispersions are reached and escape from the resonance should be effective to deplete these domains in, at most, a few Gyr. In the domains encircled by the black strip (painted in blue), the fate of a solution is less evident. In the central part of the blue domains, the dispersion is very small: of the order of 10^{-1} in 1 Gyr.

In the 3/2 resonance, the low-dispersion area is large and we may expect that the number of solutions starting in its interior and reaching the outside, in the age of the solar system, is very small. Asteroids are not expected to remain there forever, but the escape times are at least one order of magnitude larger than the age of the solar system. In the 2/1 resonance, the low-dispersion area is formed by small disconnected domains; in this case, even a solution with an initial low dispersion can reach the zone of high dispersion and leave the resonance in, at most, some Gyr. These results are consistent with those of numerical simulations done

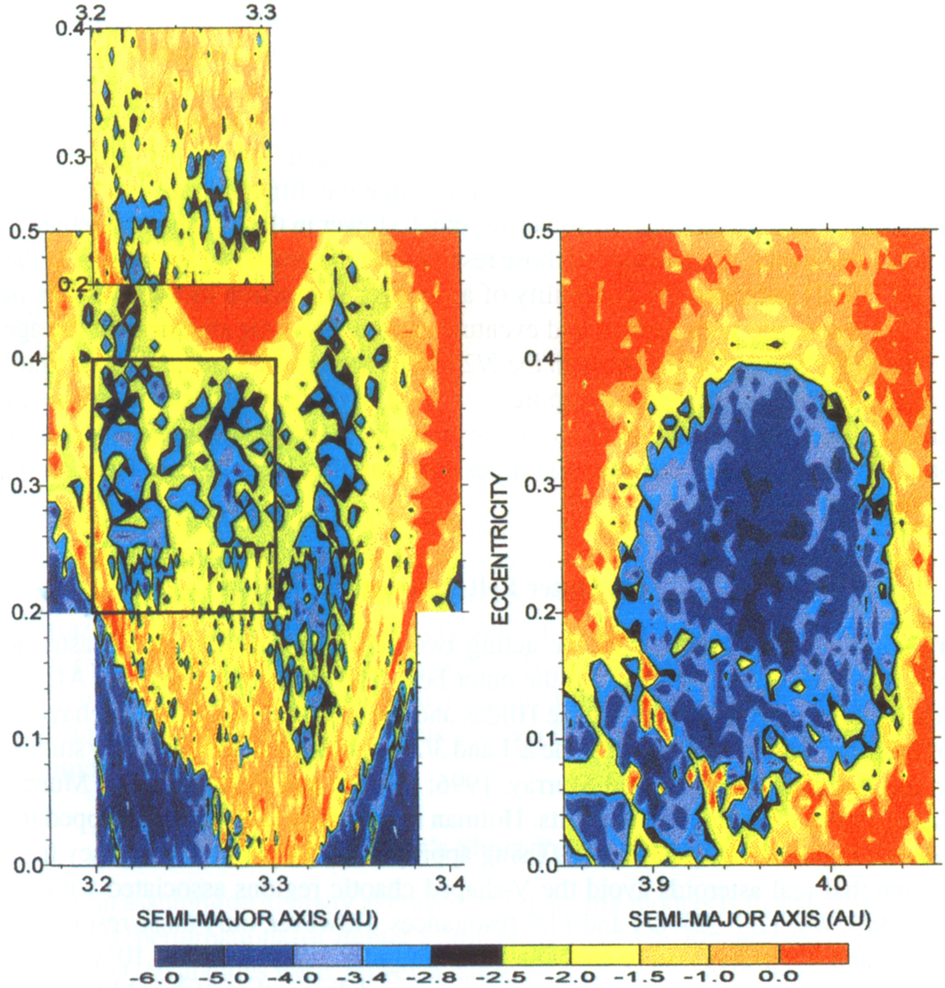

Fig. 3. Diffusion chart of the 2/1 (left) and 3/2 (right) resonances. The plotted parameter is the decimal logarithm of the relative variation of the perihelion motion, $\log_{10}|\rho|$, in 130,000 and 200,000 yr (see text). The inset on top left shows the chaoticity enhancement inside the area marked with a rectangle, when the Great Inequality period is forced to be half of the current one (see sec.5). (Adapted from Nesvorný and Ferraz-Mello, 1997).

by Morbidelli (1996), inside or near these domains, showing a large number of escapes in a 1-Gyr timespan. Some simulated solutions remained in the resonance, but were already showing an increased chaoticity at the end of the integration time.

The results obtained with Laskar's frequency map analysis led to conclusions similar to the ones obtained from the study of the maximum Lyapunov exponents and presented at the ACM-1993 Symposium (Ferraz-Mello, 1994 a, b): There is an extensive stochasticity in the 2/1 and 3/2 resonances, with Lyapunov exponents in the range $10^{-5} - 10^{-3.5}\mathrm{yr}^{-1}$ in the 2/1 resonance, and only $10^{-5.5} - 10^{-7}\mathrm{yr}^{-1}$ in the 3/2 resonance. Those results did unravel that the diffusion rate was the main difference between these two resonances, much slower in the 3/2 resonance than in the 2/1 resonance. An analysis of those results with the empirical formula of Lecar *et al.* (1992) indicated the possibility of a stronger interaction of the asteroids of the 2/1 resonance with Jupiter, and eventual escape, in a time shorter than the age of the Solar System, while those in the 3/2 resonance would, generally, need times larger than 10^{10} years before escaping. The great advantage of Laskar's frequency analysis over the calculation of Lyapunov exponents is their easier interpretation, and, moreover, shorter computation times allowing it to be done over large arrays of initial conditions.

4. Outer Belt Resonances

To complete the inventory of the acting two-period resonances of the asteroid belt, we have to include those of the outer belt. The domains beyond 3.3 AU are reasonably depleted, except for the Hildas and the Trojans. The chaotic behaviour of the asteroids situated between the 2/1 and 3/2 resonances was extensively studied in the last years (Holman and Murray, 1996; Murray and Holman, 1997; Murray *et al.*, 1998), with many new results. Holman and Murray (1996) have mapped the chaotic region in the (a, e)-space. Using appropriated proper elements, they have shown that real asteroids avoid the V-shaped chaotic regions associated with the 11/6, 9/5, 7/4, 12/7, 5/3, 8/5 and 11/7 resonances. However, only those resonances of 2^{nd} and 3^{rd} orders lead to encounters with Jupiter in less than 3×10^7yr. In the higher-order resonances the removal times are much larger than 10^8yr (Murray and Holman, 1997).

Fig.4 shows some calculations of the Lyapunov time (inverse of the maximum Lyapunov exponent) as a function of the asteroid semi-major axis. Results corresponding to three models are shown: (a) Jupiter in a fixed orbit; (b) Jupiter's orbit affected by the main long-period (secular) perturbations; (c) Jupiter's orbit affected also by short-period perturbations Their intercomparison shows that models including only the long-period perturbations of Jupiter's orbit are largely insufficient to give a correct picture of the actual chaoticity of the outer belt. Such chaoticity only appears when short-period perturbations are included. It is worth mentioning that the results obtained with a model fully including the action of the 4 outer planets is very similar to those obtained with model (c) (see Murray *et al.*, 1998).

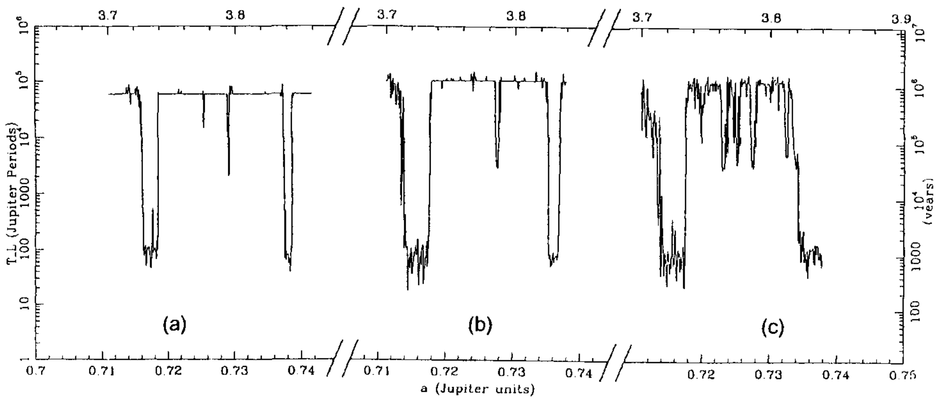

Fig. 4. The Lyapunov time as a function of semi-major axis in the outer asteroid belt in three 3-body models: (a) Jupiter in a fixed orbit; (b) Jupiter's orbit affected by the main long-period (secular) perturbations; (c) Jupiter's orbit affected also by short-period perturbations (adapted from Murray *et al.*, 1998).

In the outer belt, the 3/2 and 4/3 resonances are also found. The 3/2 resonance was already considered in the previous section. The 4/3 resonance was studied with the same tools used to study the 3/2 resonance, and a diffusion chart may be seen in Ferraz-Mello *et al.* (1998a). It is very similar to that of the 3/2 resonance, but with a smaller low diffusion area. However, Nesvorný and Ferraz-Mello (1997) have shown that this resonance is much affected by the outermost planets and a complete model, including also Uranus and Neptune, increases ρ by one order of magnitude.

5. The Role of Jupiter's Short-Period Inequalities

A common assumption to many averaged and mapping models is that short-period inequalities (perturbations) of Jupiter's orbit are neglectable. Resonances involving short-period inequalities were, however, since 1996, pointed out as responsible for the depletion of the inner parts of the Hecuba gap (Ferraz-Mello, 1996, 1997; Michtchenko and Ferraz-Mello, 1997; Ferraz-Mello *et al.* 1998 b). In this case, chaos is created by the overlap of the frequency $2n_{\rm Jup} - 5n_{\rm Sat}$ of the Great Inequality of Jupiter's motion, with the frequencies

$$f_\sigma + k_1 f_\varpi + k_2 g_6$$

(complex structure seen in fig.5); f_σ is the frequency of libration of the critical angle of the 2/1 resonance, f_ϖ the asteroid's perihelion motion and k_j are integers.

The best way to put this phenomenon into evidence is to play with the critical dependence of the Great Inequality period (currently $\approx$ 930 years) on tiny variations of the periods of Jupiter and Saturn. Simulations with the planetary orbits slightly

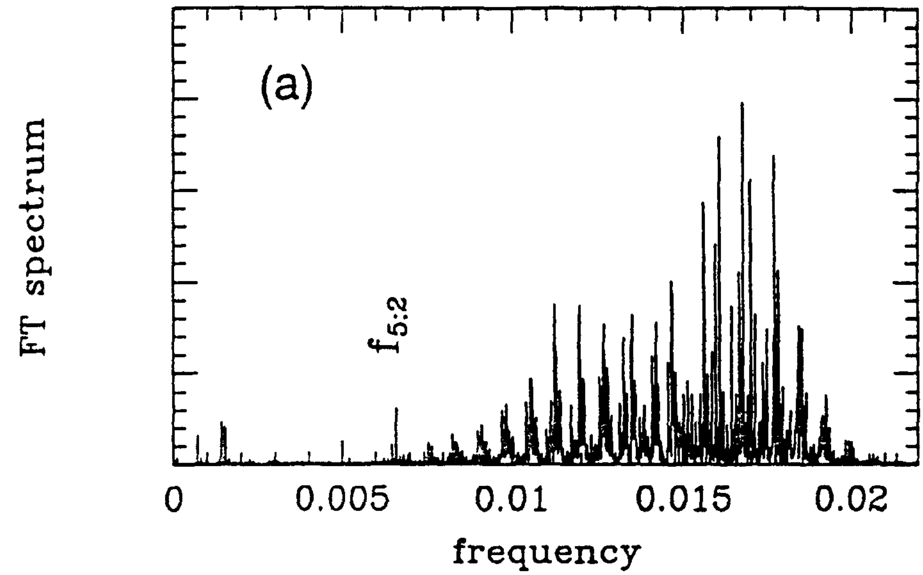

Fig. 5. FFT power spectrum of solutions starting at $e_0 = 0.3$, in the frame of a 4-body model including Saturn. Frequency unit is rad/yr. (Taken from Michtchenko and Ferraz-Mello, 1997).

modified and such that the Great Inequality period is close to the asteroids' libration period (about 440 years), in the middle of the frequency comb of fig.5, have shown a significant enhancement of the diffusion (see the inset in fig.3)

A similar confirmation was obtained by Henrard (1998), but with Saturn's semi-major axis decreased by 2.7 percent. In this case, the period and amplitude of the Great Inequality in longitude are much smaller than the actual ones and, as a consequence, the effects become barely visible. Henrard's work also confirmed the increase of the stochasticity (in a simplified model) at the exact beats of the Great Inequality frequency and the $f_\sigma + kf_\varpi$. A complete study of the dependence of the stochasticity on the period of the Great Inequality was later done by Roig and Ferraz-Mello (1999) with a symplectic mapping allowing fast simulations.

A series of recent papers has shown the role of all Jupiter's short-period inequalities in the formation of three-period (or three-body) resonances involving the periods of Jupiter, Saturn and the asteroid:

$$k_1 n_{\rm Jup} + k_2 n_{\rm Sat} + k_3 n_{\rm ast} \simeq 0.$$

(n denote mean-motions and k_j are integers; Murray *et al.*, 1998, Nesvorný and Morbidelli, 1998a,b). Many asteroids are trapped in these resonances and the study of the diffusion in their interior lead to estimate that they can remain trapped there from 100 Myr to some Gyr (Murray *et al.*, 1998). Fig.6 shows the Lyapunov exponents in the outer main belt, just in front of the 2/1 resonance. The peaks

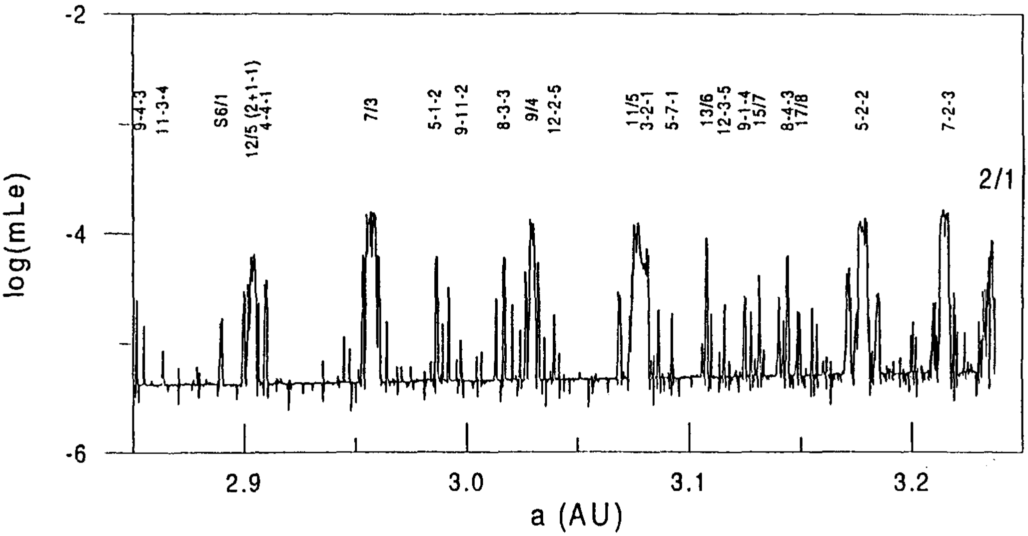

Fig. 6. Maximum Lyapunov exponents (in yr^{-1} units and logarithmic scale) of solutions starting from a regular grid in a (and $e_0 = 0.1$; $i_0 = 0$), in a model including the 4 outer planets. The main resonances are labeled on top of the corresponding peak (taken from Morbidelli and Nesvorný, 1998).

in this plot are identified with several 3-period resonances. These resonances are spanned by the combination of the main short-period perturbations in the motions of the asteroid and Jupiter. For instance, the 5–2–2 resonance is a combination of the perturbations of asteroid and Jupiter whose frequencies are $3n_{\mathrm{Jup}} - 2n_{\mathrm{ast}}$ and $2n_{\mathrm{Jup}} - 2n_{\mathrm{Sat}}$, respectively. Among the many asteroids found in these resonances, we mention (490) Veritas in the 5–2–2 resonance, whose chaoticity was first recognized by Milani and Farinella (1994) and is expected to allow an estimate of the age of Veritas family.

6. Conclusion

During many years, the success of KAM theory in Mathematics pervaded Astronomy and many results on the dynamics of Hamiltonian systems with 2 degrees of freedom were quickly translated into interpretations of the observed solutions of three-body models. Regularity was expected to be the rule and chaos the marvelous exception. Analytical models were constructed looking for resonances and their overlaps, while numerical simulations were aimed at representing longer and longer times, in the hope of seeing the less visible manifestations of chaos.

Now, the situation is reversed, and we see that chaos is the rule and regularity the exception. Problems as the "unexplained" Hecuba gap, only remained a puzzle for longtime because of insufficient modeling of Jupiter's motion; they started being solved when full 4-body models including Saturn, or 3-body models including the short- period perturbations of Jupiter's orbit, were used. These models allowed the global stochasticity inside the 2/1 resonance to be disclosed (Ferraz-Mello 1994a,b).

On the other hand, it became clear that it is enough to increase the number of planets acting on the asteroids to get an extremely dense network of significant chaos-generating resonances. The role of the short-period inequalities of Jupiter's motion on this phenomenon is well illustrated in fig.4. A similar comparison showing how the depletion rate of the Hecuba gap depends on the adopted model for Jupiter's motion, was given by Ferraz-Mello (1996, 1997).

We not only know the global stochasticity inside the classical resonances, but we find measurable resonances almost all over the asteroid belt. The puzzle is no longer to find a chaotic evolution able to drive an asteroid across the asteroid belt, but to find those asteroids that, like the Hildas, are able to remain in a bounded domain of the belt for times longer than the age of the Solar System.

The lesson to be learned is that in real-world dynamics we have to consider that our models, no matter how sophisticated, never consider all forces acting on the asteroids, and these forces may drive the asteroid to cross the barriers appearing in simplified models (*cf.* Ferraz-Mello, 1994b).

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