Proceedings of the Edinburgh Mathematical Society (2005) **48**, 253–255 ©
Printed in the United Kingdom

BOOK REVIEWS

DOI:10.1017/S0013091505214815

MANDELBROT, B. *Gaussian self-affinity and fractals—globality, the Earth, 1/f noise, and R/S* (Springer, 2002), ix+654 pp., 0 387 98993 5 (hardback), £46 (US$59.95).

This, the third volume of Mandelbrot's 'Selecta', covers a wide range of topics related to self-affinity, thought of as the direct or statistical resemblance of small parts of shapes to their whole under affine transformations. Gaussian fractals, that is fractal shapes generated by some Gaussian random process, provide many examples of self-affine fractals and motivate their study. The bulk of the book comprises reprints of the author's papers written between 1965 and 1988. The papers have been edited to aid the coherence of the book, with the English clarified in places and the notation and terminology unified and updated, but with the meaning essentially unchanged.

After a short preface, there is a substantial 'Overview of fractals and multifractals', updating the author's views on topics ranging from the historic roots of fractals to the diversity of fractals in mathematics and the sciences at the start of the twenty-first century.

Parts I and II of the book, 'Advances in old but open topics' and 'Broader continuing issues', comprise a series of essays published here for the first time. They provide novel slants on matters such as the definition of self-affine functions, Weierstrass-type functions, and a versatile family of fractals generated by 3-interval piecewise linear figures which has a remarkable 'phase diagram' of fractal curves with different characteristics. One chapter discusses the recent proof that the exterior boundary of a planar Brownian path has dimension 4/3, confirming the author's remarkable 1982 conjecture, but also including a new conjecture, that the dimension of random walk clusters is 5/3. The 'Broader continuing issues' concern the ubiquity of self-affine scaling in nature, and historical and personal recollections. An interesting section brings to light an insightful but forgotten article written by John Venn in 1888, which included a remarkable diagram of a random walk or Brownian motion approximation.

Parts III and IV bring together papers from around 1970 on fractional Brownian motion and fractional Gaussian noise; several of these were coauthored with van Ness or Wallis. Most were originally published in hydrological journals, which may be why such concepts, now central in statistics, physics, finance and many other areas, took a while to be fully appreciated by the broader mathematics and science communities. Part V concerns fractional Brownian surfaces, introducing an algorithm for 'multi-temporal' Brownian functions, which, in the subsequent papers, is used to model the Earth's relief and turbulence.

In the mid 1980s Mandelbrot introduced a construction he called a 'self-affine fractal cartoon', a graph obtained by repeated self-substitution of a simple piecewise linear figure. Graphs constructed in this way have an underlying simplicity whilst at the same time a richness and diversity that makes them widely applicable. The papers in Part VI investigate dimensional, multifractal and other aspects of these cartoons, highlighting, for example, the 'anomaly' of differing values of box and Hausdorff dimension that is typical of self-affine, as opposed to self-similar, fractals.

The final part, Part VII, collects together papers from the early 1970s on R/S analysis and its applications. This statistical procedure was introduced to analyse long-range dependence, although, as the author has noted more recently, there are processes, known as meso-diffusive processes, where R/S analysis fails to detect long-term correlations. Again, this work had its origins in hydrology, but the subsequent papers include applications to areas as diverse as sunspot activity and the Chandler wobble of the Earth's pole.

The book ends with a very substantial bibliography, of which over 10 pages detail the author's own publications.

Written in Mandelbrot's unique, thought-provoking style, these papers and commentaries contain a wealth of ideas which will appeal to mathematicians, statisticians, scientists and economists alike. It is a book both for dipping into and for detailed study, making readily accessible seminal papers which contain ideas that are as relevant now as when first published. The substantial new contributions provide ample evidence that that the subject is very much alive and that the father of fractal geometry continues to brim with ideas that will ensure the subject's vitality for many years to come.

K. FALCONER

DOI:10.1017/S0013091505224811

JAMES, G. AND LIEBECK, M. *Representations and characters of groups*, 2nd edn (Cambridge University Press, 2001), 0 521 00392 X (paperback), £24.95 (US$35.95), 0 521 81205 4 (hardback), £80 (US$120).

This book provides an introduction from a character-theoretic point of view to the representation theory of finite groups over the complex numbers. Starting from only the most basic group theory and linear algebra, it covers a wide range of topics, from pure representation and character theory, through abstract group theory, to the foreign field of molecular vibrations. The expected reader is either a relatively advanced undergraduate or a beginning graduate student.

The contents of the book fall roughly into three parts. Chapters 1–11 introduce the basic notions and theorems of group representation theory. After recalling a little preliminary material from abstract algebra, the authors present group representations, the group algebra FG, Maschke's theorem and Schur's lemma. The theory is very well illustrated throughout by a number of examples, and the reader is encouraged too by a liberal number of exercises at the end of each chapter. The difficulty of these exercises varies, but all reinforce or provide further examples to the material in each chapter. Complete answers are provided at the back of the book.

Chapters 12–24, which form the heart of the book, are concerned with character theory. After defining characters and their inner products, the character table of a group is introduced and orthogonality relations studied. The authors supply the reader with many techniques and methods for calculating characters of a given group. These range from lifting characters from normal subgroups, through restriction and induction, to using arithmetic properties of characters. A discussion of real representations and real characters is included not only as an adjunct to the complex theory, but to illustrate the use of character theory in abstract group theory through the Frobenius–Schur count of involutions and the Brauer–Fowler theorem. Again there are many examples throughout the text and exercises for the reader at the end of each chapter.

Chapters 25–32 illustrate and apply character theory in a variety of interesting examples. Character tables of groups of order pq and of some p-groups are calculated, as are those for the simple group of order 168 and the general linear groups $GL(2,q)$ with entries in any finite field $\mathbb{F}_q$. By the end of these chapters, the reader has the character tables for all groups of order less than 32 and of all the simple groups of order less than 1000 (there are five non-abelian