IMAGE INVENTORY USING THE WAVELET TRANSFORM

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1. The Need for a Multi-Scale Analysis

Today, large astronomical plates are digitised with fast scanners, leading to images with about 10^9 pixels. This amount of information permits astronomers to apply various computer vision techniques to get inventories of the objects on the plates.

Many kinds of vision models have been implemented. The most classical one is based on the detection of the edges; we have applied this (Bijaoui et al. 1978) to astronomical imagery. We choose the Laplacian of the intensity as the edge line. As this function is the sum of the second partial derivatives of a noisy function, we need to smooth and to threshold it. The results are independent of large scale variations, such as the ones due to sky background. No previous background mapping is necessary; this permits real time analyses. The resulting procedure is very fast, requiring small memory sizes. Many false detections exist if we do not want to miss threshold objects. The accuracy of the magnitudes is not sufficient. However, the main disadvantage lies in the difficulty of getting an available object classification: astronomical sources are not recognized from their edges, but from their intensity profiles.

Many reduction procedures have been built using a model in which the image is the sum of a slowly variable background with superimposed small scale objects (Stobie 1986; Slezak et al. 1988b). The first step needs to build a background mapping (Bijaoui 1980). For that purpose we need to introduce a scale: the background is defined in a given area. Many statistical estimators derived from the local histogram of intensity are used: mode, median, result from a model, etc.

The resulting background map is subtracted. Each pixel which has a significant intensity is considered to belong to a real object. A cross-correlation with the star profile is done in order to optimize the detection of these objects. A threshold is computed from the distribution of the intensity pixels. An image labelling is performed (Rosenfeld 1969), producing positions, magnitudes and pattern parameters.

Generally, this procedure leads to quite accurate detection and recognition. The computations are fast and require little memory. The model works very well for sparse fields; but for richer ones, a detection may correspond to many objects. The background map is done at a given scale: larger objects are removed. The smoothing is only adapted to the star detection, not to extended objects. The analysis does not take into account the wings of the objects. The classification

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allows us to separate stars from galaxies but not to recognize the galaxy type.

An improvement of the previous model is done with the introduction of a radial profile for each source (Le Fèvre et al. 1986; Slezak et al. 1988a). An astronomical object is associated with a point-like structure; we have thus only to detect the local maxima. The radial profile of the object contains the main information on the source. The method is similar to the previous one up to the image labelling, which is replaced by a maxima detection followed by the determination of the radial profile. The quality of the measurements is increased, and the derived pattern parameters permit a gain in the separation between stars and the galaxies.

The defects of this procedure lie in the impossibility to describe complex structures. The method is adapted to quasi stellar sources on a slowly varying background.

In fact, the vision models we have used on many set of images failed to accomplish complete analysis because they are based on a single scale for the adapted smoothing and for the background mapping. The observation of sky images furnishes many examples for which we see a small star embedded in a larger structure, itself embedded in a larger one, and so on. A multiscale analysis allows us to get a background adapted to a given object and to optimize the detection of different size objects. This is the reason why we became interested in the use of the Wavelet Transform.

2. The Continuous Wavelet Transform

Morlet-Grossman's (Grossmann & Morlet 1985) definition of the continuous wavelet transform for a $1D$ signal $f(x) \in L^2(R)$ is:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) \psi^*\left(\frac{x-b}{a}\right) dx$$

where z^* designs the conjugate of z, $\psi(x)$ is the analyzing wavelet, a (> 0) is the scale parameter and b is the position parameter. This is a linear transformation which is convenient for numerical computations, statistical analysis and astronomical understanding of the results. The wavelet transform is covariant under translations: the analysis does not depend on the origin of the coordinate frame. It is the general property of convolution operators. It is also covariant under dilatations: this is the property which gives its originality to the wavelet transform. We get a mathematical microscope the properties of which do not change with the magnification. Our vision model thus decomposes the image into the scale space, allowing us to detect objects of different sizes.

We can restore $f(x)$ from its transform by the formula:

$$f(x) = \frac{1}{C_\psi} \int_{0}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{a}} W(a,b) \psi\left(\frac{x-b}{a}\right) \frac{da\, db}{a^2}$$

where:

$$C_{\psi} = \int_{0}^{+\infty} \frac{|\hat{\psi}(\nu)|^2}{\nu} d\nu$$

The wavelet function must have a null mean ($\hat{\psi}(\nu) = 0$). This transform is a set of pass-band filterings.

In the Fourier space, we get:

$$\hat{W}(a,\nu) = \sqrt{a}\,\hat{f}(\nu)\,\hat{\psi}^*(a\nu) \quad .$$

When the scale a varies, the filter $\hat{\psi}^*(a\nu)$ is only reduced or dilated, keeping the same pattern. The Fourier space is scanned with a filter the band of which is proportional to the frequency.

3. The Discrete Wavelet Transform

Littlewood-Paley's decomposition (Littlewood & Paley 1931) furnished a nice approach for this frequency scan. We start from a signal which is perfectly observed in the frequency band $[-\frac{1}{2},\frac{1}{2}]$. The Shannon sampling step is 1. By low and high pass filterings, we separate the information into two parts. The sampling step is now 2 for the two resulting signals. We iterate on the low frequency part, leading to two signals sampled with a step 4, and so on. By this analysis, the information is well described by the successive high frequency parts. These correspond to the discrete wavelet transform with a special wavelet function, the difference of two sine cardinal ones.

The discrete wavelet transform is generally not performed by a simple discretization of the continuous transform. The classical algorithm (Mallat 1989) is a generalization of Littlewood-Paley's decomposition, but it is performed in direct space. It corresponds also to an extension of the classical Haar Transform.

The discrete wavelet transform can be processed by many algorithms (Bijaoui 1991). The constraints we put on the transform result from the chosen strategy. For stars, and generally astronomical sources are quite isotropic sources, no direction is privileged. Thus we choose an isotropic wavelet. We need to connect fields from different scales. The redundancy is not critical, but we need it to restore an image from the transform. Finally, we need also to have a fast algorithm. These constraints led us to use the *Algorithme à trous* (Bijaoui 1991; Holdschneider et al. 1989), which results from the difference between two B-spline interpolations. $B_3(x)$ is close to a Gaussian function and the results are quasi-isotropic. With $B_5(x)$ the discrepancy to the Gaussian is very faint, and the interpolation and the wavelet can be considered as isotropic.

This algorithm computes a new image for each dyadic scale. We can reduce the sampling scale by scale, in the case of a pyramidal algorithm.

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4. A Multiscale Vision Model

After applying the wavelet transform on the image, we have to extract, measure, and recognize the significant structures. The wavelet space is a 3*D* one. An object has to be defined in this space.

In the first step, we do an image segmentation scale-by-scale in the wavelet space. An object could be defined from each labelled fields without taking into account the interscale relationships, called neighbourhoods. We can restore an image of these objects from the known wavelet coefficients, but this restoration does not use all the information.

Secondly, we link the labelled fields from one scale to the following one. That leads to building a tree of neighbourhoods, from the largest scale to the smallest one. After this operation we can say if a large scale field contains smaller ones, which contain still smaller ones, and so on.

The image is a set of connected trees, corresponding to different objects. We could define an object as one tree, but must beware of reducing the number of objects in too high a manner. A small star may belong to a small nebula, the tree corresponds to the nebula, and we do not consider the star if we take into account only the connected tree. This is the reason why we define an object as a subtree resulting from the image segmentation in the wavelet space.

Let us consider now an object, such as it was defined. It corresponds to a field D in the wavelet space. It is fully determined from its wavelet coefficients $w_0^{(i)}(k,l)$. We have to restore an intensity distribution $c^{(0)}(k,l)$, such that its wavelet transform has the same coefficients in D. The restoration algorithm is an extension of the classical Van Cittert's deconvolution algorithm (Burger & Van Cittert 1932).

This algorithm provides an image for each object. It is easy to compute from each of them any kind of parameters: mean position, total intensity, pattern parameters, etc.

5. Conclusion

While we have not done enough experiments to claim that the resulting measurements would be more accurate than the ones derived from other models, the vision model resulting from the wavelet transform allows us to detect, to measure and to recognize an object as complex as available. The procedure does not introduce any prior information on the stellar profile or on the scale of the background variations; this is very important for automated procedures.

Finally, our experiments show that the quality of the detection is very good with this procedure. An experiment on the SA57 field gives a dispersion of less than 0.08 for the magnitudes of about 23 - 24 (compared to careful interactive processing). Using a very different approach, Coupinot et al. (1992) also obtained accurate measurements from the wavelet transform. The main disadvantage lies in the amount of data used. The algorithm à trous leads to an increase in the data by the number of scales. In our experiments we used 4 - 5 scales, but this increase is too high for large astronomical images. We thus are now examining a way to reduce this data amount with a pyramidal transform.

The wavelet information is kept only on a few coefficients; this leads not only to data compression but also to data fusion. This last operation is essential for the comparison of many images observed under different conditions (Bijaoui & Giudicelli 1991).

This vision model may be improved using the stellar profile. In the wavelet space, we can recognize the wavelet images connected to star-like objects. This procedure is more complicated and we have used it only for image restoration (Starck & Bijaoui 1993).

References

Bijaoui, A., Lago, G., Marchal, J and Ounnas, C., 1978. 'Traitement des Images et Reconnaissance des Formes', ed. INRIA, p. 848.

Bijaoui, A., 1980. *Astron. Astrophys.*, **84**, 81.

Bijaoui, A., 1991. *Ondelettes et Paquet d'Ondes*, ed. INRIA, p. 115.

Bijaoui, A. and Giudicelli, M., 1991. *Exper. Astronomy*, **1**, 347.

Burger, H.C. and Van Cittert, P.H., 1932. *Z. Physik*, **79**, 722.

Coupinot, G., Hecquet, J., Aurière, M. and Futually, R., 1992. *Astron. Astrophys.*, **259**, 701.

Grossmann, A. and Morlet, J., 1985. 'Mathematics and Physics, Lecture on recent results', World Scientific Publ.

Holdschneider, M., Kronland-Martinet, R., Morlet, J. and Tchamitchian, P., 1989. In 'Wavelets', ed. J.M. Combes et al., Springer Berlin, p. 286.

Le Fèvre, O., Bijaoui, A., Mathez, G., Picat, J.P. and Lelièvre, G., 1986. *Astron. Astrophys.*, **154**, 92.

Littlewood, J. and Paley, R., 1931. *Jour. London Math. Soc.*, **6**, 230.

Mallat, S., 1989. *IEEE Trans. Patt. Anal. Mach. Intell.*, **11**, 674.

Rosenfeld, A., 1969. In 'Picture Processing', Academic Press New York, p. 127.

Slezak, E., Bijaoui, A. and Mars, G., 1988a. *Astron. Astrophys.*, **200**, 1.

Slezak, E., Mars, G., Bijaoui, A., Balkowski, C. and Fontanelli, P., 1988b. *Astron. Astrophys. Supp. Ser.*, **74**, 83.

Stark, J.L. and Bijaoui, A., 1993. 'Progress in Wavelet Analysis and Applications', eds. Y. Meyer and S. Roques, Editions Frontières Gif/Yvette, p. 447.

Stobie, R.S., 1986. 'Pattern Recognition Letters', p. 317.