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SIR,

Reply to Lliboutry's letter "Why calculated basal drags of ice streams can be fallacious"

Professor Lliboutry (1995) suggests that there is a deep layer of soft ice extruding ahead of the main glacier and that our study discovered the drag imparted by this deep ice, rather than the drag at the very bottom of the glacier. In this letter, we show that considerations of continuity do lead to inferred deep strain rates that are different from those at the surface, in line with Professor Lliboutry's suggestion, but the stresses associated with this extrusion flow are very small, too small to affect our analysis in an important way. Extrusion flow may be an attractive idea but it cannot account for the unusual result of calculated reverse basal drag.

In Whillans and Van der Veen (1993), we combined calculations of the effects of gravity with forces computed from strain rates measured at the glacier surface. The strain rates pertain to the upper, cold and strong part of the glacier. Forces on the bulk, the sides and base of a column through the glacier must sum to zero, thus the shear stress on the base of the column can be estimated. To our surprise, there are places on Ice Stream B, Antarctica, where the calculated basal drag enhances, rather than retards, glacier motion.

Lliboutry also suggests that deep ice can squeeze ahead, between the main body of ice above and the bed below, to the extent that the gross dynamics of the glacier is affected. Extrusion flow on a very large scale was discredited by Nye (1952) but the possibility of extrusion flow in restricted regions remains open. If it does occur, then faster ice at depth would exert a drag on the ice above in the opposite sense to that normally expected for glacier flow.

A full discussion of the possible importance of extrusion flow should consider the rate of deformation of the extruding layer, the cause and the forces involved. The cause may be large pressure on the up-glacier side of an obstruction and small pressure on the down-glacier side. The motion due to this pressure gradient can be modeled following the method of Hutter and others (1981), as has been done for cold over warm ice in Greenland (Whillans and Jezek, 1987). We doubt that significant extrusion can be predicted following such a line of investigation. However, let us set aside the question of the possible cause of extrusion and conduct a simpler analysis by estimating the magnitude of extrusion from measured strain rates and considerations of volume continuity. Using this estimate, we find in the next paragraph that this possibly forward-extruding ice can exert a drag of 7.4 kPa on the ice above. This is not sufficient to explain the anomalous basal drag of about –100 kPa calculated in the work under discussion.

The details of the calculation of the effect of extrusion flow follow.

KINEMATICS

Overall horizontal flow divergence must accord with along-flow changes in width and thickness of the ice stream and any mass imbalance. This regional horizontal divergence is given by the mean sum of horizontal normal strain rates, which is about $-1.0 \times 10^{-3}\,\mathrm{a}^{-1}$ (Whillans and Van der Veen, 1993; Hulbe and Whillans, 1994). Local values of horizontal divergence at the surface range from $-6.3 \times 10^{-3}\,\mathrm{a}^{-1}$ to $2.6 \times 10^{-3}\,\mathrm{a}^{-1}$. From these numbers, we use the round value of $5 \times 10^{-3}\,\mathrm{a}^{-1}$ for the fluctuation. There are no correlating fluctuations in width, thickness or mass balance, so there must be corresponding reverse normal strain rates at depth. These special strain rates are likely to occur in the deepest, warmest and hence softest ice. Suppose that the bottom 10% of the thickness contains this mobile ice. Then the reverse deep normal strain rates must be minus ten times the surface fluctuation, or about $\pm 5 \times 10^{-2}\,\mathrm{a}^{-1}$. This estimate is similar to Lliboutry's. (Such deep strain rates have little effect on the total budget of forces discussed in Whillans and Van der Veen (1993) because they occur in weak ice and perhaps because they lie beneath the applicable column for calculation.)

STRESSES

The stress that causes this strain rate is ± 74 kPa (in which the constitutive relation $\sigma_{ij}' = B\dot{\varepsilon}_e^{1/n-1}\dot{\varepsilon}_{ij}$, in the usual notation, is used, with $n = 3$, $\dot{\varepsilon}_e \approx 5 \times 10^{-2}\,\mathrm{a}^{-1} = \dot{\varepsilon}_{ij}$ and a rate factor of $B = 200\,\mathrm{kPa\,a}^{\frac{1}{3}}$ appropriate for ice near the melt temperature, as in Hooke (1981). Caution must be exercised because the constitutive relation has not been properly tested.) The value of ± 74 kPa is the special normal stress in the deep layer that might be pushing or pulling ice to or from another site.

STRESS GRADIENTS

This stress may vary from site to site. The distance to consider for this variation is 2000 m, the size of the regions of calculated negative drag. Thus, there can be gradients in normal stress of 2×74 kPa/2000 m. (This is a gradient in deviatoric normal stress. The full stress gradient is similar.)

DRAG ON TOP OF EXTRUDING LAYER

Supposing that the extruding layer is about 100 m thick (10% of the ice thickness), and that there is no friction on the underside of the extrusion, the extrusion is mechanically opposed by drag with the ice above. A simple budget of forces for this extruding layer yields a drag of 7.4 kPa between the extruding layer and the ice above it. This is the number noted earlier.

Being stimulated by Lliboutry's (1995) letter, and continuing to neglect a quantitative consideration for the cause, we also considered the possibility of faster

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extrusion. However, we find that this is inconsistent with measured mass imbalances. This new idea is that deep ice may squirt in time-pulses. Being faster than the extrusion considered above, episodic extrusion would be mechanically more important. It would cause a drop in ice-surface elevation over the site of origin of the extrusion and surface lifting over the destination. Measurements of relative vertical velocity obtained with precision GPS of an expanded grid on Ice Stream B, Antarctica (Hulbe and Whillans, 1994), do show important topographic changes, but the changes are not at sites that would account for the negative basal drag. Thus, not even episodic, non-steady extrusion can account for the reverse basal drag.

Since conducting the work discussed in Whillans and Van der Veen (1993), we have completed a much more extensive study of the region, using a grid expanded five-fold. The interpretation of these new results is given in Hulbe and Whillans (1994) and Hulbe (1994; which contains data tables). Based on this more extensive survey, our current view is that there are zones of ice of differing viscosity horizontally juxtaposed. Including appropriate horizontal variation in viscosity would lead to more sensible calculated basal drag. We propose that bands of special strength develop in ice after extreme simple shear (at the sides of up-glacier tributaries). The viscosity may vary according to the up-glacier origin of the ice.

Lliboutry's (1995) suggestion is very reasonable to the extent that he carries it. However, the stresses imparted by the envisioned extrusion are too small to explain the calculated backward basal friction. The reason the issue arises for Ice Stream B, Antarctica, could be that the ice stream is unusual or, alternatively, that the survey work was more thorough than on many other glaciers. The ice stream has such a simple geometry that unusual results cannot be attributed to uncertainties in ice thickness or width.

We thank Professor Lliboutry for raising this suggestion. It is good for Science to discuss possible oversights. We remain concerned that we may have overlooked some perfectly good explanation for the results and would welcome more suggestions, including further consideration of extrusion flow.

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SIR,

Analysis of satellite-altimeter height measurements above continental ice sheets

In a recent paper in the *Journal of Glaciology* the performance of three radar-altimeter "retracking" algorithms was investigated using simulated waveforms (Femenias and others, 1993). One of the techniques used was described as the Offset Center of Gravity (OCOG) method. There appears to be a misunderstanding about the function of this algorithm which, in itself, is not a retracking procedure but a means of determining the *amplitude* of the waveform. This amplitude is then used to find the position on the leading edge of the waveform which equals some percentage of the amplitude (e.g. 0.3, used by Partington and others (1991)). Furthermore, to reduce the effects of the leading edge on the amplitude estimate, each waveform sample is squared. The centre of gravity and waveform width that can be obtained from using the OCOG procedure were never intended to be used to calculate a retrack position in the way that they have been by Femenias and others (1993). Instead, they were designed to be used as part of a satellite onboard-tracking loop (such as is used for the ice mode of the ERS-1 altimeter). A complete and correct description of a retracking procedure using the OCOG algorithm is given in Bamber (1994). It should be noted that using the waveform width and centre of gravity to find the retrack point on the waveform and using the amplitude to "threshold" retrack it give very different results.

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