MATTER EFFECTS IN NEUTRINO PROPAGATION

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ABSTRACT. Conditions and dynamics of the resonant neutrino conversion are described. We discuss the applications of the effect to the solar neutrinos as well as present status of the conversion inside the Sun. The influence of different matter density perturbations on the conversion is considered, and in this connection the possible effects of parametric and stochastic enhancement of the influence are remarked.

1. Introduction

Resonant neutrino conversion is the process of nonreversible transformation of one neutrino species into another one (in a system of mixed neutrinos) during the propagation through the matter with monotonously changing density. The transformation occurs continuously according to density change, and mainly, in the resonance layer (the layer in which the density varies in definite limits about the resonance value) [1,2].

According to definition the key points are : 1) mixing of neutrinos - the transitions occur between mixed neutrino components; 2) interactions of neutrinos with matter - the elastic forward scattering is essential [3]; 3) resonance (crossing of the resonance layer); 4) change of density, moreover the density should change slowly to satisfy the adiabaticity condition. In the first part of review we consider these items and then describe dynamics of conversion (for more details see reviews [4-6]).

Conditions of resonance conversion are fulfilled for the Sun in a wide region of neutrino parameters (mass difference $\Delta m^2 = m_2^2 - m_1^2$ and mixing angle, θ) [1]. The conversion (ν_e - ν_μ, ν_e - ν_τ) diminishes signals in the solar neutrino experiments : ν_μ or ν_τ are sterile at the low energies with respect to charged currents interactions. The effect depends on neutrino energy, and consequently the conversion distorts the energy

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spectrum of the electron neutrinos. Conversion may solve the solar neutrino problem [1]. It can ensure up to 15-fold suppression of the argon production rate in Cl - Ar -experiment, and rather independent suppression of ν e-scattering signal (Kamiokande-II). In a view of existing data and forthcoming Ga - Ge -experiment - results the present status of resonant conversion will be discussed.

In the third part of the review we consider the influence of different matter density perturbations on resonant conversion. Such a consideration has two aspects. Firstly, it enables to understand the sensitivity of the conversion effects to the model of the Sun and to evaluate the possibility of measuring the solar parameters with neutrino data. Secondly, if the perturbations depend on time then variations of ν_e -flux are induced via the resonant conversion. The parametric and stochastic enhancement of perturbation effects will be remarked.

2. Resonant neutrino conversion

2.1. CONDITIONS OF RESONANT CONVERSION

2.1.1. *Mixing.* The necessary condition of resonant conversion in a system of two neutrinos, for example ν_e - ν_μ, is the mixing of these neutrinos. Mixing implies the interaction, which transforms ν_e into ν_μ. In the simplest case the mixing (so called vacuum mixing) is induced by the nondiagonal mass terms of the Hamiltonian. Now ν_e and ν_μ which are called the flavor states or eigenstates of the weak interactions turn out to be the coherent mixtures of ν_1 and ν_2 -- the states with definite masses m_1 and m_2 (eigenstates of the mass matrix) :

$$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$$
$$\nu_\mu = -\sin\theta\, \nu_1 + \cos\theta\, \nu_2 \tag{1}$$

Here θ is the mixing angle. Flavor states themselves have no definite masses.

2.1.2. *Refraction.* Conversion is stipulated by the interactions of neutrinos with matter, and precisely -- by the elastic forward scattering, which is reduced to the appearance of the refraction indexes, n_e and n_μ, for neutrino waves [3] :

$$(n\text{-}1) \propto G_F N/k$$

Here G_F is the Fermi constant, N is the concentration of particles in medium, k is the momentum of neutrino. Matter influences on the evolution of mixed neutrinos if the indexes are different $n_e \neq n_\mu$. So, the conversion takes place in the transparent for neutrinos mediums,

being nonsymetric with respect to the mixed components. The influence is described by the refraction length, l_0, defined as the distance over which an additional phase difference between ν_e an ν_μ waves due to interactions becomes 2π [3]. The l_0 determines spatial scale of matter effects. For ν_e - ν_μ -system one has

$$l_0 = \frac{2\pi}{k\,|n_e\text{-}n_\mu|} = \frac{2\pi}{\sqrt{2}\;G_F\,N_e} = \frac{2\pi m_N}{\sqrt{2}\;G_F\,\rho^{eff}} \tag{2}$$

where $N_e = \rho.Y_e/m_N$ is the electron concentration, $\rho^{eff} = \rho.Y_e$ is the effective density, m_N is the nucleon mass. The corresponding width of matter :

$$d_0 = l_0{\cdot}\rho = \frac{2\pi m_N}{\sqrt{2}\;G_F} = 3.5\;10^9\;g/cm^2$$

is the universal constant, which in fact fixes the field of applications of the phenomena : the width d $\geq$ d_0 is needed.

Matter effects can be described equivalently in terms of potentials, V_e and V_μ, in which ν_e and ν_μ are moving : $V \propto (n\text{-}1)k \propto G_F N$ [7]

2.1.3. *Neutrino eigenstates in matter.* Effective mixing in matter. Effective (dynamical) mixing is used for the description of neutrino evolution [3]. In matter this mixing is introduced with respect to the neutrino eigenstates in matter, ν_{1m} and ν_{2m}, which are determined as the eigenstates of the total Hamiltonian, including the neutrino interactions (i.e. potentials V_e and V_μ) :

$$\nu_e = \cos\,\theta_m\,\nu_{1m} + \sin\,\theta_m\,\nu_{2m}$$
$$\nu_\mu = -\sin\,\theta_m\,\nu_{1m} + \cos\,\theta_m\,\nu_{2m} \tag{3}$$

θ_m is the mixing angle in matter. The eigenstates ν_{im} have definite energies (eigenvalues) as well as definite phase and group velocities. In vacuum ν_{im} coincide with ν_i and $\theta_m = \theta$. Matter changes mixing, and moreover θ_m depends on matter density. As it can be shown at $\rho^{eff} \neq 0$ the transitions $\nu_1 \div \nu_2$ takes place and hence ν_i are no more the eigenstates of the Hamiltonian : $\nu_{im} \neq \nu_i$.
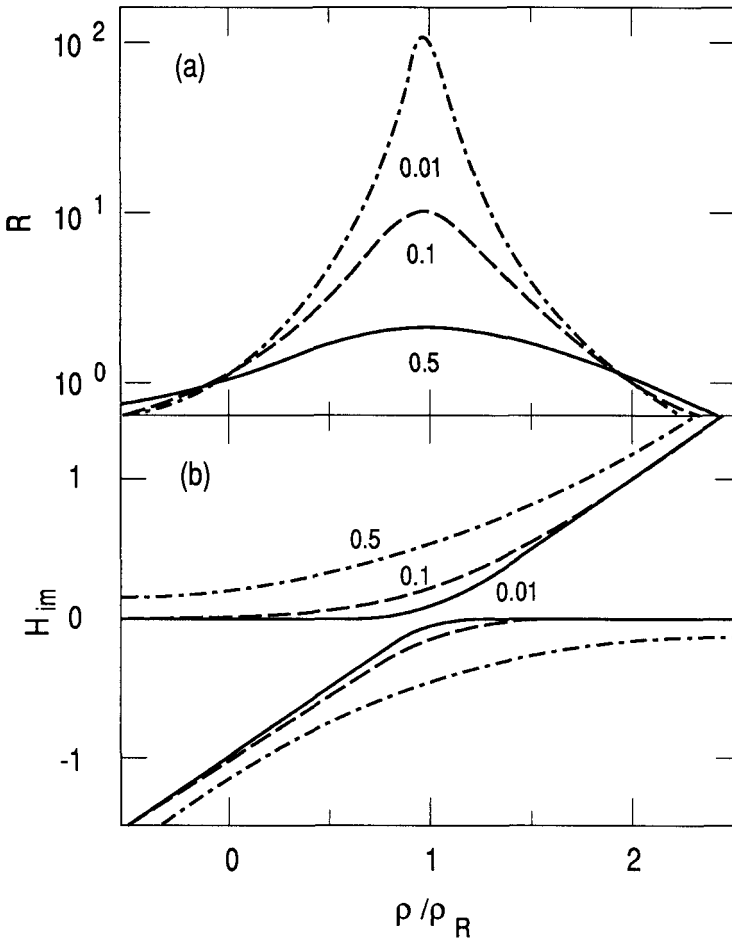
Figure 1. The dependence of (a) the resonance factor
$R = sin^2 2\theta_m / sin^2 2\theta$ and (b) level energies in units
$\Delta m^2.(2k\ cos2\theta)^{-1}$ on the effective density for different value
of $sin^2 2\theta$ (numbers of the curves).

2.1.4. *Resonance*. The dependance of mixing parameter, $sin^2 2\theta_m$, on
the density has a resonance character [1] (fig. 1). At

$$\rho_R = \frac{m_N\ cos\ 2\theta}{2\sqrt{2}\ G_F} \cdot \frac{\Delta m^2}{E} \qquad (4)$$

where $\Delta m^2 = m_2^2 - m_1^2$ and E is the energy of neutrino, this parameter
reaches the maximum : $sin^2 2\theta_m = 1$ for arbitrary small θ. The ρ_R is
called the resonant density. Half width of resonance at half height is
proportional to the vacuum mixing :

$$\Delta\rho_R = \rho_R \cdot tan\ 2\theta \qquad (5)$$

The width of resonance layer, $2\Delta\rho_R$ fixes the scale on which resonant conversion occurs.

Parameter $sin^2 2\theta_m$ defines (as $sin^2 2\theta$ does in vacuum) the depth of neutrino oscillations. Therefore in matter with resonant density the depth is maximal.

Resonance condition (4) can be written as

$$l_\nu = l_0 \cdot cos\ 2\theta \qquad (6)$$

where $l_\nu = 4\pi E/\Delta m^2$ is the vacuum oscillation length. According to (6) in resonance (at small θ) period of system (l_ν) itself coincides with period of external medium (l_0).

The value $\rho = \rho_R$ is the specific point in the dependance of θ_m on ρ. When ρ varies from $\rho \gg \rho_R$ to zero, the angle θ_m diminishes from $\pi/2$ to θ. At $\rho = \rho_R$ the $\theta_m = \pi/4$.

2.1.5. *Flavor changing of neutrino eigenstates.* Mixing angle θ_m determines according to (3) a flavor, or ν_e-, ν_μ -content, of the neutrino eigenstates, ν_{im}. Propagating through a matter with varying density the ν_{im} change their flavors. If θ is small and ρ varies from $\rho \gg \rho_R$ to $\rho \ll \rho_R$, the flavors of ν_{im} change almost completely. For example, ν_{2m} coincides practically with ν_e at $\rho \gg \rho_R$ and with ν_μ at $\rho \ll \rho_R$. Mainly a flavor changes in the resonance layer : $\rho_R - \Delta\rho_R < \rho < \rho_R + \Delta\rho_R$.

2.1.6 *Level crossing.* Cabibbo [8] and Bethe [7] had given the interpretation of the resonance in terms of eigenvalues H_{im} i.e. the level energies corresponding to ν_{im} (fig. 1b). Potentials, V_e and V_μ, are proportional to a density, and consequently the total energies of ν_e and ν_μ, H_e and H_μ, are the linear functions of ρ. At resonance they cross :

$$H_e(\rho_R) = H_\mu(\rho_R) \qquad (7)$$

Mixing of ν_e and ν_μ rejects level crossing : the eigenvalues $H_{im}(\rho)$, are not equal but their splitting is minimal in resonance (fig. 1b). In the resonance layer when ρ increases the curve $H_{1m}(\rho)$ goes from the line $H_e(\rho)$ to the line $H_\mu(\rho)$, the $H_{2m}(\rho)$ vice versa -- from $H_\mu(\rho)$ to $H_e(\rho)$.

In resonance the difference of the potentials $V = V_\mu - V_e$, compensates the difference of level energies related to masses. As the V for neutrinos and antineutrinos have an opposite sign the resonance as

well as resonant conversion in a given medium takes place for neutrinos or antineutrinos only, depending on the respective signs of Δm^2, V and $\cos 2\theta$.

2.1.7. *Adiabaticity*. [1,2,9,10]. Adiabaticity implies a slowness of density change. If ρ changes adiabatically (slowly), then the transitions between the eigenstates ν_{1m} - ν_{2m} can be neglected; the admixtures of ν_{im} in a given ν -state conserve; the system has a time to adjust itself to external condition (density) variations. The condition of adiabaticity can be written as

$$\frac{d\theta_m}{dr} \ll \frac{d\varphi}{dr} \equiv |H_{2m} - H_{1m}| \tag{8}$$

where φ is the phase difference between ν_{1m} and ν_{2m} waves $(d\theta_m(\rho)/dr \propto d\rho/dr)$. The condition is the most crucial in resonance, where the level splitting is minimal [1,2] :

$$2\Delta r_R \geq l_m^R \tag{9}$$

Here $\Delta r_R = l_0.\tan 2\theta$, $[l_\rho \equiv (d\rho/dr)^{-1}.\rho]$ is the spatial half width of the resonance layer; l_m^R is the oscillation length in resonance : $l_m^R = l_\nu/\sin 2\theta$. According to (9) at least one oscillation length should be obtained in resonance layer.

The degree of adiabaticity violation is determined by the adiabaticity parameter [4] :

$$æ_R = \frac{\Delta r_R}{l_m^R} = \frac{\sin^2 2\theta}{4\pi \cos 2\theta} \cdot \frac{\Delta m^2}{E} \cdot l_\rho \tag{10}$$

At $æ_R \leq 0.1$ the violation is strong and the transitions $\nu_{1m} \to \nu_{2m}$ become essential.

So far we have considered main conditions of resonance conversion : 1) mixing, 2) resonance (level crossing), 3) adiabaticity (see also the general discussion in [1]).

2.2. DYNAMICS OF RESONANT CONVERSION.

Resonant neutrino conversion is, in fact, the change of the flavor of neutrino state on the adiabatic (or weakly nonadiabatic) crossing of resonance layer. The decomposition of given ν -state over ν - eigenstates, ν_{im}, and the change the flavors of ν_{im} themselves are all we need to trace the conversion. The dynamics is reduced to the change of the admixtures of ν_{im} in $\nu(t)$.

To describe the process of conversion the survival probability, P(t), i.e. the probability to find the neutrino of initial type in a given moment t, is used.

One can single out three different regimes of conversion 1) nonoscillatory, 2) oscillatory adiabatic, and 3) nonadiabatic.
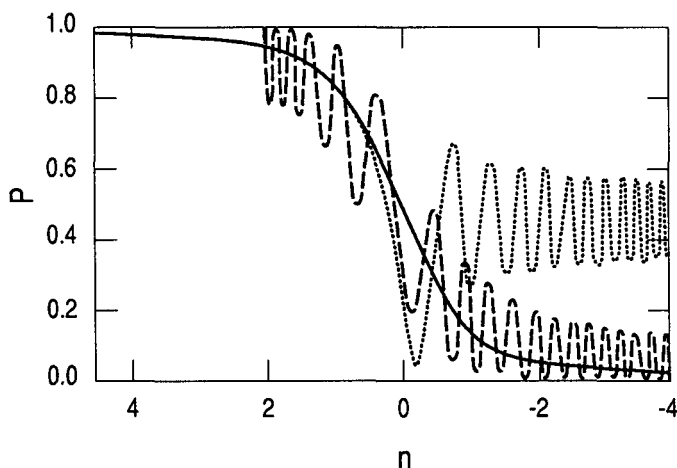

Figure 2. Spatial picture of resonant neutrino conversion. The dependence of survival probability on the distance from the resonance layer : $n = (\rho - \rho_R)/\Delta\rho_R$ for nonoscillatory transformation (solid line), oscillatory adiabatic transition (dashed line), adiabaticity violation regime (dotted line).

2.2.1. *Nonoscillatory transition* [1,2]. Let neutrinos, for definiteness ν_e, are produced at $\rho \gg \rho_R$ [more precisely at $(\rho - \rho_R)/\Delta\rho_R \gg 1$], and the adiabaticity is satisfied during the propagation. Then in the initial moment according to the first condition ($\rho \gg \rho_R$) the mixing angle equals $\pi/2$ and hence the neutrino state coincides practically with ν_{2m} : $\nu(t_0) = \nu_e \cong \nu_{2m}$. According to the second condition (adiabaticity) $\nu(t)$ will coincide with ν_{2m} throughout the propagation. Consequently, $\nu(t)$ changes its flavor together with ν_{2m}; the flavor of $\nu(t)$ follows density variations, like the flavor of ν_{2m} does (fig.2). If the final density is zero (or $\ll \rho_R$), then at the exit one has $\nu(t_f) = \nu_{2m}(\rho = 0) = \nu_2 = \nu_e \sin\theta + \nu_\mu \cos\theta$ and then the survival probability is [1] :

$$P = P(\nu_e \rightarrow \nu_e) = sin^2\theta \qquad (11)$$

The smaller vacuum mixing, the stronger transformation of the neutrino type into another one can be achieved in contrast with vacuum oscillations.

2.2.2. *Adiabatic oscillatory transition* [2]. let the adiabaticity condition
is fulfilled but the neutrinos are produced not far from thr resonance
layer : $|\rho_0 - \rho_R|$ < several $\Delta\rho_R$. Then $v(t)$ contains the comparable
admixtures of both eigenstates and moreover these admixtures will
conserve. As $v(t)$ does not coincide with one of the eigenstates the
flavor of $v(t)$ oscillates. Oscillations superpose on the conversion
(fig.2), but the averaged probability changes according to density
variations [2,9,10] :

$$\bar{P} = \frac{1}{2} - \frac{1}{2} \cos 2\theta_m(\rho_0) \; \cos 2\theta_m(\rho) \tag{12}$$

when ρ_0 approaches ρ_R, the transformation becomes weaker : P
increases. At $\rho_0 = \rho_R : P = 1/2$. (In the adiabatic regime not only P but
also the depth of oscillations, $A_p = \sin 2\theta_m(\rho_0) . \sin 2\theta_m(\rho)$ depends on
the initial and final densities only and does not depend on density
distribution in the intermediate points.

2.2.3. *Adiabaticity violation regime*. If the adiabaticity is violated, the
admixtures of neutrino eigenstates in a given $v(t)$ change. Let neutrinos
are produced far from the resonance layer ($\rho_0 \gg \rho_R$), then $v(t_0) = v_e$
consists of v_{2m} predominatingly. But the admixture of v_{1m} appears in
$v(t)$ in the course of propagation. Consequently the probability $P(t)$
begins to oscillate and the transition becomes weaker, than in
nonoscillatory case (fig.2). The averaged probability can be written now
as

$$\bar{P} = \frac{1}{2} + (\frac{1}{2} - P_{21}) \cos 2\theta_m(\rho_0) \; \cos 2\theta_m(\rho) \tag{13}$$

where P_{21} is the probability of v_{2m} - v_{1m} transition At small θ and at
the values of ρ_0 and ρ_f beyond the resonance layer, the P_{21} can be
evaluated using the well known Landau-Zener result [12,13]. It gives the
probability of transition between two levels under linear with time
perturbation, which mixes these levels and induces their crossing :

$$P_{21} = exp(-\pi^2 \ae_R) \tag{14}$$

There is a lot of improvements and generalizations of formula (14) in
literature [14].

2.2.4. *Analogies*. There are many analogies of resonant conversion in
different fields of physics. Let remark two of them.

The system of mixed neutrinos is similar to the weakly coupled
oscillators (for example, two pendulums) [15]. Matter effect is

equivalent to a change of pendulum eigenfrequences ω_1 and ω_2. Resonance corresponds to the equality $\omega_1 = \omega_2$. Now the conversion is nonreversible transmission of oscillations from one pendulum to another one under slow changing of frequences from $\omega_1 \ll \omega_2$ to $\omega_1 \gg \omega_2$.

Another analogy is the electron spin flip in the rotating magnetic field [16]. The states with the projections 1/2 and -1/2 correspond to ν_e and ν_m.

2.3. DIFFERENT REALIZATIONS OF RESONANT NEUTRINO CONVERSION.

The general conditions : mixing, level crossing, adiabaticity may have many realizations even for neutrino systems [11]. Depending on properties of neutrino states are mixed one can single out three types of conversion.

2.3.1. *Flavor conversion* (considered above). Neutrino states with different flavors but the same helicities are involved, for example n_{eL} - $n_{\mu L}$. In the course of conversion the flavor changes, and the helicity conserves. In general all three known neutrino species, ν_e, ν_μ, ν_t, are mixed. Such a system has three resonances but in usual medium (electrons, nuclei) in the lowest order of the perturbation theory the ν_μ and ν_τ interact equally and only two level crossings are possible. In (ν_e - ν_μ) and (ν_e - ν_τ) -chanels [17]. If $\Delta m_{21}^2 = m_2^2 - m_1^2$ differs from $\Delta m_{31}^2 = m_3^2 - m_1^2$ sufficiently (which realizes for mass hierarchy, m1 $\ll$ m2 $\ll$ m3), then the corresponding resonances are splitted on the density scale. In this case crossing of resonances can be described independently -- three neutrino task is reduced to two neutrino task (see reviews [4-6].

2.3.2. *Spin conversion* [18]. Conversion takes place in a system of left and right components composed the same dirac neutrino : ν_{eL} and ν_{eR}. Mixing is induced by the interaction of neutrino magnetic moment, μ_ν, with magnetic field, B [the exchange (mixing) energy is $2\mu_\nu B$]. Both level splitting and level crossing are stipulated by the refraction in inhomogenious matter. Now helicity is changed and flavor conserves.

2.3.3. *Spin-flavor conversion*. Both flavor and helicity of neutrino state are changed, for example ν_{eL} - $\nu_{\mu R}$ [19]. Mixing is induced by the interaction of the nondiagonal (transition) magnetic moment, μ_ν, with magnetic field. Now the mixing parameter is

$$"sin\ 2\theta" = (2\mu_\nu B)\cdot E/\Delta m^2 \qquad (15)$$

Mass difference and refraction (potentials) give the contributions to level splitting. Level crossing is related to the change of density.

Considered conversion types differ by the dependences of the adiabaticity parameter, $æ_R$, on the neutrino energy. One has $æ_R \sim 1/E$ for the flavor conversion (10), $æ_R$ = constant for the spin precession and

$$æ_R = (2\mu_\nu B)^2 \cdot E / (\pi \Delta m^2 l_\rho) \qquad (16)$$

for the spin flavor conversion. In the last case at B = const, for example, $æ_R$ is proportional to E.

3. Resonant conversion of solar neutrinos. Present status

3.1. EFFECTS OF FLAVOR CONVERSION

Neutrinos, ν_e, produced in the center of Sun, cross the layer of matter with monotonously decreasing density. Resonant flavor conversion results in suppression of the initial ν_e -flux, which depends on Δm^2, $\sin^2 2\theta$ and the energy of neutrinos :

$$F(E, \Delta m^2, \sin^2 2\theta) = P(E, \Delta m^2, \sin^2 2\theta) \cdot F_0(E)$$

Here F_0 is the flux without conversion. The suppression factor p (averaged survival probability) as the function of $E/\Delta m^2$ has a bath-like shape [1] (fig. 3).

Moreover there are two specific energies in $P(E/\Delta m^2)$ dependence : E_c and E_a. 1). The energy of resonance turning on, E_c, is fixed via the resonance condition (4) by the central density ρ_c :

$$E_c / \Delta m^2 = m_N / (2\sqrt{2} G_F \rho_c) \qquad (17)$$

For neutrinos with E < E_c there is no resonance (level) crossing. 2). The energy of adiabaticity violation, E_a, is determined by adiabaticity condition $æ_R(E_a) = 0.5$:

$$E_a / \Delta m^2 = l_\rho \cdot \sin^2 2\theta / (2\pi \cos 2\theta) \qquad (18)$$

(see (9, 10)). When E increases the adiabaticity parameter diminishes : $æ_R \sim 1/E$ (l_0 = const. in a wide region of ρ), the adiabaticity violates stronger and P increases (E > E_a). The energy E_a is proportional to $\sin^2 2\theta$ -- with decreasing vacuum mixing the baths become narrower.

If E rises, the resonance layer shifts to the surface of the Sun. In the region of resonance turning on ($E = E_c \pm$ several $.\Delta E_R$) the oscillatory adiabatic transition takes place , here $\Delta E_R = E_c.\tan 2\theta$, ΔE_R is the width of resonance in energy scale. At ($E_c +$ several. ΔE_R) $\div E_a$ the conditions for the nonoscillatory transformation are fulfilled and $P = \sin^2\theta$. At $E > E_a$ neutrinos convert in adiabaticity violation regime (see sect. 2.2.3., (13, 14). The mutual position of the suppression bath and the neutrino spectrum depends on Δm^2, and if the spectrum falls on the edges of the bath, it distorts.
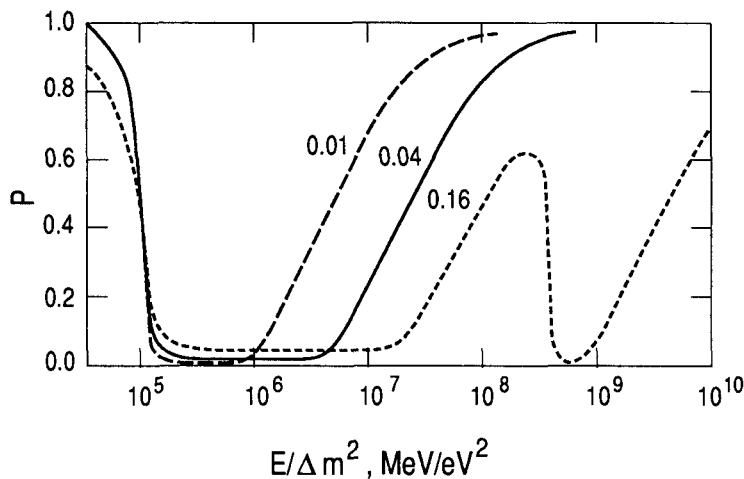

Figure 3. Suppression baths of flavor conversion. The dependences of suppression factors on $E/\Delta m^2$ for different values of $\sin^2 2\theta$ (numbers on the curves). The suppression pit for 3ν-mixing (an example, dashed line).

Two remarks are in order. 1). The averaging of P over the ν - production region results in the smoothing of the left edges of baths. 2) For three neutrino mixing the suppression factor P(E) is the superposition of two baths, which are shifted on the axis E one with respect to another by factor of $\Delta m^2_{13} / \Delta m^2_{12}$ (fig. 3).

3.2. SPIN-FLAVOR CONVERSION AND SPIN CONVERSION

In contrast with flavor conversion for spin flavor case [19] (1) mixing parameter "$\sin^2 2\theta$ is not constant : it depends on the magnetic field and the neutrino energy (15), (2) effective density is $\rho^{eff} = \rho (Y_e - Y_n)$ for ν_e-$\bar{\nu}_\mu$, where Y_n is the number of neutrons per nucleon, (3) the dependence of the adiabaticity parameter on the neutrino energy differs from that of flavor case. If $B(r) \sim \rho^\alpha$ and $\alpha < 0.5$, then æ$_R$ rises with increasing E and the adiabaticity becomes better (or restores). This results in rather complicated shape of suppression pits, which in general

differs from flavor conversion baths (especially on the bottom and nonadiabatic edge) fig.4 [20]. P (E) depends on the magnetic field profile, which is essentially unknown. In some cases (B(r)) the shape of pit may coincide with that of the flavor conversion and the problem appears to distinguish these two cases.

For spin conversion the energy splitting is $\Delta H = G_F(\rho(Y_e - Y_n/2)/m_N$ and there is no level crossing inside the Sun. Moreover the effect does not depend on neutrino energy. It can be evaluated from the spin-flavor results by the transition $\Delta m^2/E \rightarrow 0$ (P at the largest values of $E/\Delta m^2$ in fig.4) [20].
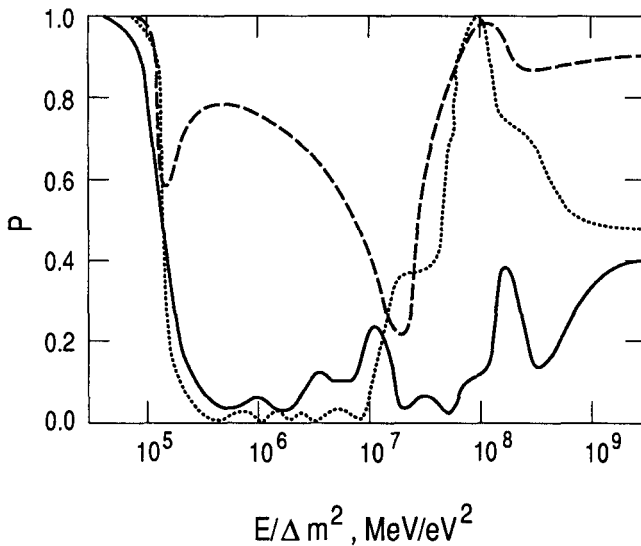

Figure 4. The suppression pits for spin-flavor conversion. The dependences of suppression factor on $E/\Delta m^2$ for different distributions of the magnetic field. Maximal strengh of field $B_1 = 10^6 - 10^7$ Gauss in the center of the Sun and $B_0 = 10^4$ Gauss in the convective zone were used (for more details see [20]. $\mu_v. = 10^{-11} \mu_B$.

3.3 PRESENT STATUS OF RESONANT NEUTRINO CONVERSION

The suppression factor for [37]Ar-production rate in Cl-Ar-experiment due to flavor conversion is

$$R_{Ar}(\Delta m^2, sin^2 2\theta) = \frac{1}{Q_{Ar}^{SSM}} \int dE \sigma(E) F^0(E) P(E, \Delta m^2, sin^2 2\theta) \qquad (19)$$

where $Q^{SSM}_{AR} = \int dE \sigma(E) F^0(E)$, $\sigma(E)$ is the cross-section, F^0 (E) is the neutrino flux on the Earth in the standard solar model (SSM) [21]. The equation R_{Ar} (Δm^2, $sin^2 2\theta$) = c = const determines lines of 2f equal

suppression [1,2,4-6] (or ISOSNU lines [22] on Δm^2-$\sin^2 2\theta$-plot (fig.5a). Consequently, Davis's data [23], $R^{exp}_{AR} = (0.2 \div 0.5)$ can be explained in terms of resonant flavor conversion if the values of Δm^2 and $\sin^2 2\theta$ find between ISOSNU lines $c_{min} = R^{exp}_{min} = 0.2$ and $c_{max} = 0.5$. The system of three neutrinos is described by two points on Δm^2 - $\sin^2 2\theta$ - plot and as can be shown [24], to achieve the observed suppression of Q_{Ar} at least one point should be placed between ISOSNU lines $c_{min} = R^{exp}_{min} = 0.2$ and $c_{max} = \sqrt{R^{exp}_{max}} = 0.7$.

The suppression of ν e-signal in Kamiokande-II experiment [25] which is sensitive to the high energy part of boron neutrino spectrum only, is rather independent on the suppression of Q_{Ar} (fig.5b) [24]. Preliminary Kamiokande-II result with 1σ errors [25] excludes some part of loops in fig.5b. i.e. definite values of Δm^2 and $\sin^2 2\theta$.

The suppression factor of Ge-production rate in Ga-Ge experiment, R_{Ge}, is introduced similarly to (19). According to fig.5c. at $R_{Ar} = 0.2 \div 0.5$ any result for R_{Ge} from 0.04 to 0.98 is possible and moreover 1σ Kamiokande-II limit restricts this region of prediction very weakly.

The above consideration (fig.5) corresponds to fixed parameters of standard solar model. In fact, one should consider the central temperature, T_c, abundances, X,Y,Z and others as free parameters and carry out the multidimensional analysis, which includes both solar and neutrino parameters. In that time the crucial signature of resonant conversion, which enables to distinguish its effects from the astrophysical ones, is the distortion of continuous energy spectra from individual reactions (pp-, ^{8}B-decay). The distortion is described by suppression bath (fig.3) and depends on Δm^2 and $\sin^2 2\theta$.

Spin-flavor conversion may result in strong suppression of both ^{37}Ar-production rate and number of ν e-scattering events as well as in the variety of the energy spectrum distortion, if the transition magnetic moment $\mu_\nu > 10^{-11} \mu_B$ (μ_B is the Bohr magneton).

Possible anticorrelations of solar neutrino flux and 11-years solar activity [23] can be explained immediately by the magnetic field variations via the spin-flavor conversion. But it is not excluded that the change of ν -flux is related to the variations of density distribution which influences on flavor conversion (see sect.4).
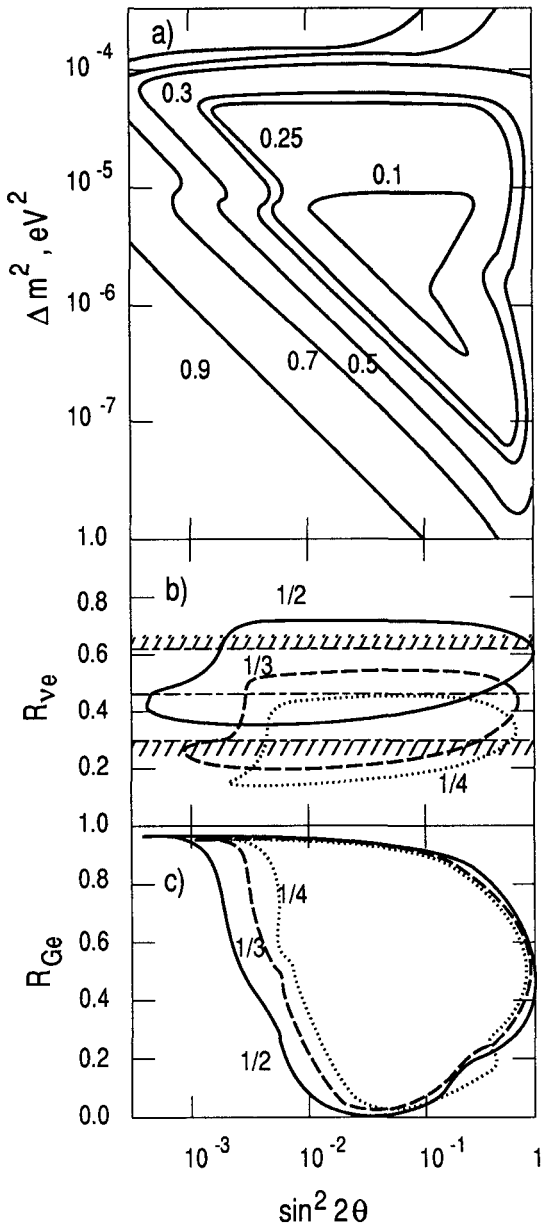
Figure 5.a). Lines of equal suppression (ISOSNU lines) of ^{37}Ar production rate in Cl-Ar experiment. Numbers on the curves are the suppression factors, R_{Ar}.

b). The suppression factor of number of $\nu e - \nu e$ events with the energies of final electrons $E_e > 9.5$ MeV as the function of $sin^2 2\theta$ at fixed suppression of Ar-production rate (numbers on the curves). The points of loops correspond to definite points of ISOSNU lines. The horizontal lines are the Kamiokande-II result with 1σ errors.

c). The same as in fig. b) for the Ge-production rate in Ga-Ge experiment.

4. Matter density perturbations and resonance conversion.

The influence of density perturbations on conversion depends on relation between the oscillation length, I_m, and the spatial scale of perturbation, I_{fI} (several model calculations were performed in [28, 29, 11]).

The smooth variations of density profile with typical scale $I_{fI} \cong R_0$ result in rather small change of suppression baths, which is controled by the energies $E_c \sim 1/\rho$ and $E_a \sim d\log\rho/dr$ (17,18). Such a change reduces mainly to snift of the edges of baths and is equivalent to renormalization of Δm^2 and $\sin^2 2\theta$. Futher on we will concentrate on the case $I_{fI} \cong I_m \ll R_0$.

4.1. CONDITIONS FOR MAXIMAL EFFECT OF DENSITY PERTURBATIONS

As the scales, I_{fI}, and amplitudes $A_\rho = \Delta\rho/\rho$ of possible density perturbations in the Sun are essentialy unknown and moreover A_ρ are considered to be small (no more than several percents), it is worthwhile to evaluate the minimal values of A_ρ and I_{fI} which can be observed with solar neutrino data. We begin with conditions under which the influence of ρ −perturbation on resonance conversion is maximal.

1). The perturbations should be placed in the resonance layer (at $\rho = \rho_R$ (E)). Indeed, density fluctuation, $\Delta\rho_{fI}$, can be considered as the perturbation of potential : $\Delta V = \sqrt{2} . G_F . \Delta\rho_{fI}/m_N$. The perturbation induces the transition between levels with probability P = 1 if ΔV is of order of level splitting, ΔH. The splitting is minimal in resonance (fig.1b) $\Delta H_R = \sqrt{2} G_F . \rho_R . \tan 2\theta/m_N$, and from the inequality $\Delta V > \Delta H$ one obtains $\Delta\rho_{fI} > \Delta\rho_R$ or $A_\rho = \Delta\rho_{fI}/\rho > \tan 2\theta$. The smaller $^R\theta$, the smaller A_ρ is sufficient for a large effect.

2). To avoid the averaging, which suppresses the perturbation effect , neutrino should be in definite level. Maximal effect takes place in the region of nonoscillatory transformation -- on the bottom of the bath.

3). The effect can be additionally enhanced if there are several density fluctuations, N_{fI}, on the way of neutrinos, and the parametric condition is fulfilled [30,31] :

$$l_{fI} \cong l_m(\rho) \tag{20}$$

In this case the effects of individual fluctuations add constructively. Now the same change of conversion probability can be obtained with smaller value of A_ρ : $A_\rho \sim 1/N_{fI}$. If the perturbations are distributed randomly in space, then a stochastic enhancement takes place and $A\rho \sim 1/\sqrt{N}_{fI}$.

4). The effect depends on the shape of density perturbation but in any cases perturbations should violate adiabaticity. Conditions 1). and 2). for a given ρ-fluctuations can be satisfied at definite neutrino energy only. Moreover they are compatible if the relation between the I_{fl} and the position of fluctuation inside the Sun, r_0 is fulfilled :

$$l_{fl} = \frac{2\pi m_N}{\rho(r_0)\sqrt{2}\,G_F tan\,2\theta}$$ (21)

The effect of density fluctuation looks like a peak on the bottom of the bath (under the considered conditions). The height of the peak equals

$$P^{max} \approx \left(\frac{A_\rho N_{fl}}{2tan\,2\theta}\right)^2$$

and the width is iversely proportional to number of structures $\Delta E \sim tan\,2\theta/N_{fl}$. If the condition (21) is not fulfilled, then two peaks appear corresponding to the resonance at $E = E_R$ and to parametric condition at $E = E_P$ (parametric resonance). If neutrino state contains the admixtures of both levels (which is realized on the nonadiabatic edge), then the ρ-perturbation results in that the enhancement of P will be altered with energy by suppression.

4.2. EFFECTS OF ρ-PERTURBATION. NUMERICAL RESULTS

To illustrate the results discussed above Krastev and myself have calculated the probabilities of resonant conversion (averaged over oscillations in final state) for

$$\rho(r) = \rho^{ssm}(r)\cdot\left[1+A_\rho\theta(r-r_0)\cdot\theta(r_1-r)\cdot sin\,2\pi r/l_{fl}\right]$$

where ρ^{ssm} is the profile of standard solar model, $\theta(r)$ is the step-function (the perturbations are in the interval $r \div r_1$), A_ρ = const. In fig. 6 the effects on the bottom of the bath (a-c) and on the nonadiabatic edge (d) are shown.

4.3. DENSITY VARIATIONS AND SOLAR NEUTRINO FLUX

Let us adopt that variations of boron neutrinos can be observed if they take place in the energy region $\Delta E/E \geq 0.2$ and if the amplitude of variations is $\Delta P/P \geq 0.3$. For beryllium neutrinos line the region of variations can be much smaller $\Delta E/E > 0.02$ and even very narrow peaks of fig. 6 can induce an appreciable effect. Using the results of sect. 4.2,3. the parameters of smallest density fluctuations ($I_{fl}A_\rho$) are

evaluated, which give an observable variations : $(\rho 10^{-3}R_\odot, 3.10^{-3})$ at r_0 = $0.2R_\odot$, $(10^{-2}R_\odot, 0.03)$ at $r_0 = 0.5R_\odot$ $(0.1R_\odot, 0.1)$ at $r_0 = 0.7R_\odot$.
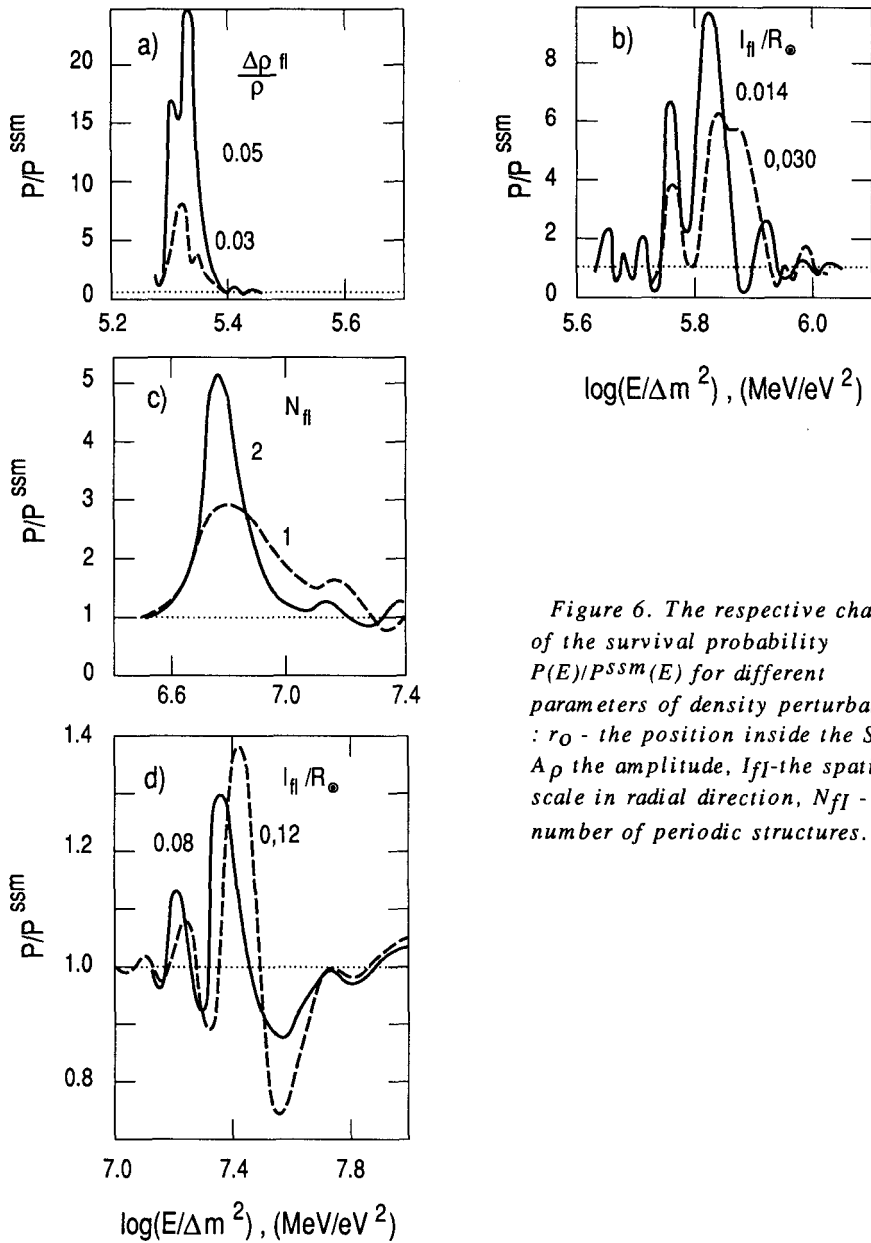

Figure 6. The respective change of the survival probability $P(E)/P^{ssm}(E)$ for different parameters of density perturbations : r_0 - the position inside the Sun, $A\rho$ the amplitude, I_{fl}-the spatial scale in radial direction, N_{fl} - the number of periodic structures.

a). r_0 = $0.18R_\odot$, $\sin^2 2\theta$ = 0.01, N_{fl} = 1, I_{fl} = $0;015R_\odot$.
b). r_0 = $0.30R_\odot$, $\sin^2 2\theta$ = 0.01, N_{fl} = 1, A_ρ = 0.05.
c). r_0 = $0.51R_\odot$, $\sin^2 2\theta$ = 0.25, A_ρ = 0.2, I_{fl} = $0.035R_\odot$
d). r_0 = $0.74R_\odot$, $\sin^2 2\theta$ = 0.25, A_ρ = 0.2, N_{fl} = 2.

The most strong respective change of P (E) occurs on the bottom of the baths, where in absence of ρ-perturbation the suppression of ν -flux is strong. Then a scenario can be considered in which without conversion the ν -flux is large, so that $Q^0_{Ar} = (13 - 15)$ SNU but due to flavor conversion and ρ−variations the Q_{Ar} change from 1 to 4-5 SNU [31].

Density variations may be related to the magnetic fields [32], to the instabilities induced by rotation or nuclear energy release. Semiannual variations of signals on the Earth may be stipulated by nonsphericity of the Sun and to the annual change of Earth position. Another possibility is the ρ-variations due to g-mode of oscillations of the Sun; the parametric and/or stochastic enhancement of perturbation effect may take place. The change of the oscillation activity (power of vibrations) with time can induce the corresponding change of ν -signals.

5. Conclusion

1). Resonant neutrino conversion -- the change of flavor of neutrino state according to matter density variations -- is the analog of well known and established phenomena from different fields of physics.

2). For conversion the massiveness and mixing of neutrinos are needed only. Conversion takes place inside the Sun if $\Delta m^2 = (10^{-8} - 10^{-4})$ eV^2 and $\sin^2 2\theta > 10^{-4}$. Spin-flavor precession demands sufficiently large magnetic moment of neutrino : $> 10^{-11}\mu_B$.

3). Resonant conversion allows to explain the suppressions of signals in Cl-Ar and Kamiokande-II experiments. Moreover it can explain the possible variations of signals via magnetic field variations (in the spin-flavor case) or via density profile variations with $\Delta\rho/\rho > 3.10^{-3}$ and $I_{fI} > 10^{-3}R_\odot$ (in flavor as well as spin-flavor cases). Inverseiy, the search for time variations of solar neutrino signals is sensitive to $\Delta\rho/\rho > 3.10^{-3}$ and $I_{fI} > 10^{-3}R_\odot$

4). Resonant conversion gives the unique method of measuring the neutrino masses and mixing in very plausible region of magnitudes, which is not achieved by usual experiments. If the conversion effects will not be found then large region of Δm^2 and $\sin^2 2\theta$ as well as $\mu_\nu > 10^{-11}\mu_B$ can be excluded.

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