

# Lorentz violation and gravity

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**Abstract.** In the last decade, a variety of high-precision experiments have searched for miniscule violations of Lorentz symmetry. These searches are largely motivated by the possibility of uncovering experimental signatures from a fundamental unified theory. Experimental results are reported in the framework called the Standard-Model Extension (SME), which describes general Lorentz violation for each particle species in terms of its coefficients for Lorentz violation. Recently, the role of gravitational experiments in probing the SME has been explored in the literature. In this talk, I will summarize theoretical and experimental aspects of these works. I will also discuss recent lunar laser ranging and atom interferometer experiments, which place stringent constraints on gravity coefficients for Lorentz violation.

**Keywords.** gravitation, relativity

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## 1. Introduction

General Relativity (GR) encompasses all known gravitational phenomena and remains the best known fundamental theory of gravity. No convincing deviations from GR have been detected thus far (Will 2005, Battat *et al.* 2007, Müller *et al.* 2008, and Chung *et al.* 2009). Nonetheless, there remains widespread interest in continuing ever more stringent tests of GR in order to find possible deviations. This is motivated by the idea that small deviations from GR might be the signature of an as yet unknown unified fundamental theory that successfully meshes GR with the Standard Model of particle physics.

The principle of local Lorentz symmetry is a solid foundation of GR. However, the literature abounds with theoretical scenarios in which this symmetry principle might be broken. One promising possibility is that miniscule deviations from perfect local Lorentz symmetry might be observable in high-precision experiments and observations (for reviews see CPT 2008, Bluhm 2006, and Amelino-Camelia *et al.* 2005). The Standard-Model Extension (SME) is a general theoretical framework for tests of Lorentz symmetry, in both gravitational and nongravitational scenarios (Kostelecký & Potting 1995, Colladay & Kostelecký 1997, Colladay & Kostelecký 1998, and Kostelecký 2004). The SME is an effective field theory that includes all possible Lorentz-violating terms in the action while also incorporating the known physics of the Standard Model and GR. The Lorentz-violating terms are constructed from gravitational and matter fields and coefficients for Lorentz violation, which control the degree of Lorentz-symmetry breaking.

A wide variety of experiments with matter have set constraints on many of the coefficients for Lorentz violation in the SME (for a summary and references see the data tables in Kostelecký & Russell 2009). Recently, the gravitational sector and matter-gravity couplings in the SME have been studied (Bailey & Kostelecký 2006, Bailey 2009, and Kostelecký & Tasson 2009). It turns out that some novel effects can occur that are controlled by certain matter sector coefficients in the SME, which are unobservable in the absence of gravity. In addition, experimental work constraining SME coefficients in the pure-gravity sector has begun, including gravimeter experiments (Müller *et al.* 2008)

and lunar laser ranging (Battat *et al.* 2007). The theoretical and experimental aspects of the signals for Lorentz violation in gravitational experiments will be discussed in this talk. For convenience, natural units ( $\hbar = c = 1$ ) are used.

## 2. Theory

In Kostelecký (2004), the SME with both gravitational and nongravitational couplings was presented in the general context of a Riemann-Cartan spacetime. This extended earlier work on the Minkowski spacetime limit. We focus here on the special cases of gravity couplings in the matter sector of the SME and also on the pure-gravity sector.

In the matter sector of the SME, the matter-gravity couplings expected to dominate in many experimental scenarios for ordinary matter (protons, neutrons, and electrons) can be described in terms of Dirac spinor fields for spin-1/2 fermions. The lagrangian density for this limit takes the form

$$\mathcal{L}_m = \frac{1}{2}iee^\mu{}_a\bar{\psi}(\gamma^a - c_{\nu\lambda}e^{\lambda a}e^\nu{}_b\gamma^b + \dots)\overleftrightarrow{D}_\mu\psi - e\bar{\psi}(m + a_\mu e^\mu{}_a\gamma^a + \dots)\psi + \dots, \quad (2.1)$$

where the ellipses represent additional terms in the SME, neglected here for brevity. Following standard methods, the spinor fields  $\psi$  and the gamma matrices  $\gamma^a$  are incorporated into the tangent space of the spacetime at each point using the vierbein  $e_\mu{}^a$ . The symbol  $e$  represents the determinant of the vierbein. The coefficients for Lorentz violation appearing in equation (2.1) are  $c_{\mu\nu}$  and  $a_\mu$ , which vanish in the limit of perfect local Lorentz symmetry for the matter fields. Note also that the covariant derivative appearing in this lagrangian is both the spacetime covariant derivative and the  $U(1)$  covariant derivative, and so contains additional couplings to gravity through the spacetime connection.

In the pure-gravity sector of the SME, and in the Riemann-spacetime limit, the relevant lagrangian is written as

$$\mathcal{L}_g = \frac{1}{2\kappa}e[(1-u)R + s^{\mu\nu}R_{\mu\nu}^T + t^{\kappa\lambda\mu\nu}C_{\kappa\lambda\mu\nu}] + \mathcal{L}'. \quad (2.2)$$

In this expression,  $R$  is the Ricci scalar,  $R_{\mu\nu}^T$  is the trace-free Ricci tensor,  $C_{\kappa\lambda\mu\nu}$  is the Weyl conformal tensor, and  $\kappa = 8\pi G$ , where  $G$  is Newton's gravitational constant. The leading Lorentz-violating gravitational couplings are controlled by the 20 coefficients for Lorentz violation  $u$ ,  $s^{\mu\nu}$ , and  $t^{\kappa\lambda\mu\nu}$ . The matter sector and possible dynamical terms governing the 20 coefficients are contained in the additional term denoted  $\mathcal{L}'$ . In the limit of perfect local Lorentz symmetry for gravity these coefficients vanish.

In all sectors, the action in the SME effective field theory maintains general coordinate invariance. However, under local Lorentz transformations and diffeomorphisms of the localized matter and gravitational fields, or what are called *particle transformations*, the action is not invariant. When Lorentz violation is introduced in the context of a general Riemann-Cartan geometry, some interesting geometric constraints arise. For example, introducing the coefficients for Lorentz violation in the matter and gravity sectors as nondynamical or prescribed functions generally conflicts with the Bianchi identities. If instead the coefficients arise through a dynamical process, as occurs in a spontaneous Lorentz-symmetry breaking scenario, conflicts with the geometry are avoided (Kostelecký 2004).

The approach of Kostelecký & Tasson (2009) and Bailey & Kostelecký (2006) is to treat the coefficients for Lorentz violation as arising from spontaneous Lorentz-symmetry breaking. Although specific models with dynamical vector and tensor fields can reproduce the terms in the lagrangians (2.1) and (2.2), it is a challenging task to study the

gravitational effects in a generic, model-independent way. However, the weak-field or linearized gravity regime offers some simplifications to the analysis. It is possible, under certain mild assumptions on the dynamics of the coefficients for Lorentz violation, to extract effective linearized equations. In this case the effects of Lorentz violation on gravity and matter involve only the vacuum expectation values of the coefficients for Lorentz violation (denoted  $\bar{a}_\mu$ ,  $\bar{s}^{\mu\nu}$ , etc.).

By including matter-gravity couplings, the  $\bar{a}_\mu$  coefficients, which have remained largely elusive in nongravitational tests, become accessible to experiments. In contrast, stringent constraints on the  $c_{\mu\nu}$  coefficients already exist (see the data tables in Kostelecký & Russell 2009). As shown in Kostelecký & Tasson (2009), the dominant Lorentz-violating effects on matter fields arise from an effective vector potential  $\tilde{a}_\mu$  that takes the form

$$\tilde{a}_\mu = \frac{1}{2}\alpha h_{\mu\nu}\bar{a}^\nu - \frac{1}{4}\alpha\bar{a}_\mu h^\nu{}_\nu, \tag{2.3}$$

where  $h_{\mu\nu}$  are the metric fluctuations around a Minkowski background and  $\alpha$  is a constant.

In the pure-gravity sector, the weak-field gravity analysis in Bailey & Kostelecký (2006) reveals that the leading Lorentz-violating effects are controlled by the nine independent coefficients in  $\bar{s}^{\mu\nu}$ . In the post-newtonian limit, attempting to match the pure-gravity sector of the SME to the standard Parametrized Post-Newtonian (PPN) formalism involves constraining  $\bar{s}^{\mu\nu}$  to an isotropic form in a special coordinate system with one independent coefficient  $\bar{s}^{00}$ . This reveals that there is a partially overlapping relationship between the two approaches (see figure 1 in Bailey & Kostelecký 2006). Thus, the pure-gravity sector of the SME describes effects outside of the PPN while, conversely, the PPN describes effects outside of the pure-gravity sector of the SME. The relationship between the PPN and other sectors of the SME is not known at present.

Many of the dominant effects in the pure-gravity sector of the SME can be studied via an effective post-newtonian lagrangian for a system of point masses given by

$$\begin{aligned} L = & \frac{1}{2} \sum_a m_a \bar{v}_a^2 + \frac{1}{2} \sum_{ab} \frac{Gm_a m_b}{r_{ab}} (1 + \frac{3}{2}\bar{s}^{00} + \frac{1}{2}\bar{s}^{jk}\hat{r}_{ab}^j\hat{r}_{ab}^k) \\ & - \frac{1}{2} \sum_{ab} \frac{Gm_a m_b}{r_{ab}} (3\bar{s}^{0j}v_a^j + \bar{s}^{0j}\hat{r}_{ab}^j v_a^k \hat{r}_{ab}^k) + \dots, \end{aligned} \tag{2.4}$$

where  $v_a$  is the velocity of mass  $m_a$  and  $r_{ab}$  is the relative euclidean distance between two masses. In (2.4), the nine coefficients for Lorentz violation  $\bar{s}^{\mu\nu}$  are projected into their space and time components  $\bar{s}^{00} = \bar{s}^{jj}$ ,  $\bar{s}^{jk}$ , and  $\bar{s}^{0j}$ . For other applications, such as the time-delay effect, it is necessary to directly use the post-newtonian metric which was obtained in Bailey & Kostelecký (2006).

### 3. Matter-gravity tests

The chief effects from the coefficients  $\bar{a}_\mu$  can be described as an effective modification to the gravitational force between two test bodies. Supposing there is a source body  $S$ , such as the Earth, and a test body  $T$  near the surface, an addition to the usual vertical force arises that takes the form

$$\tilde{F}_z = -2g(\alpha\bar{a}_t^T + \alpha\bar{a}_t^S m^T/m^S), \tag{3.1}$$

in a laboratory reference frame. In this expression  $m^T$  and  $m^S$  are the masses of the test body and source, respectively, and  $g$  is the local gravitational acceleration. The force modification in (3.1) depends on the time components of the  $\bar{a}_\mu$  coefficients for

the test body and source, denoted  $\bar{a}_t^T$  and  $\bar{a}_t^S$ . For ordinary matter, these coefficients are ultimately related to the 12  $\bar{a}_\mu$  coefficients for the electron, neutron, and proton (denoted  $\bar{a}_\mu^e$ ,  $\bar{a}_\mu^n$ , and  $\bar{a}_\mu^p$ ).

Two types of effects arise from the signal in (3.1). When the coefficients  $\bar{a}_\mu$  take on a flavor dependence, violations of the Weak Equivalence Principle (WEP) occur, as two test bodies of different compositions experience different accelerations. The standard reference frame for reporting measurements of coefficients for Lorentz violation is the Sun-centered celestial equatorial reference frame or SCF for short (Kostelecký & Mewes 2002). When relating the laboratory frame coefficients in (3.1) to the SCF, sidereal day and yearly time dependence from the Earth's rotation and revolution is introduced. Therefore, an additional effect occurs where the signal for Lorentz violation depends on the time of day and season. These effects have direct experimental consequences for both Earth-bound and space-based tests of WEP (Nobili *et al.* 2003), as well as ordinary single-flavor gravimeter-type tests. Estimated sensitivities for specific tests and a single constraint on one of the 12 coefficients  $\bar{a}_\mu$  for ordinary matter, implied by existing analysis (Schlamminger *et al.* 2008), are described in more detail in Kostelecký & Tasson (2009).

#### 4. Pure-gravity sector tests

Within the pure-gravity sector of the SME, the primary effects on orbital dynamics, due to the nine coefficients  $\bar{s}^{\mu\nu}$ , can be obtained from the point-mass lagrangian in equation (2.4). From the post-newtonian metric, modifications to the classic solar-system tests of GR can be determined, such as the gyroscope experiment and the time-delay effect.

One particularly sensitive test of orbital dynamics in the solar system is lunar laser ranging. Highly-sensitive laser pulse timing is achievable using the reflectors on the lunar surface. The dominant Lorentz-violating corrections to the Earth-Moon coordinate acceleration can be calculated from equation (2.4). In the ideal case, a full analysis would incorporate the effects from the pure-gravity sector of the minimal SME, as well as the standard dynamics of the Earth-Moon system, into the orbital determination program. However, it can be shown that the dominant effects can be described as oscillations in the Earth-Moon distance. The frequencies of these oscillations involve the mean orbital and anomalistic frequencies of the lunar orbit, and the mean orbital frequency of the Earth-Moon system (see Table 2 in Bailey & Kostelecký 2006). Using past data spanning over three decades, Battat *et al.* (2007) placed constraints on 6 combinations of the  $\bar{s}^{\mu\nu}$  coefficients at levels of  $10^{-7}$  to  $10^{-10}$ . Substantial improvement of these sensitivities may be achievable in the future with APOLLO (Murphy *et al.* 2008). Also of potential interest are Earth-satellite tests, since satellites of differing orientation can pick up sensitivities to distinct coefficients.

For Earth-laboratory experiments, a modified local gravitational acceleration arises due to the  $\bar{s}^{\mu\nu}$  coefficients, similar to that which occurred in (3.1). Again, due to the Earth's rotation and revolution relative to the SCF, this acceleration acquires a time variation that can be searched for in appropriate experiments, such as gravimeter tests. Such an experiment was performed by Müller *et al.* (2008) and, more recently, these results were combined with lunar laser ranging analysis to yield new constraints on 8 out of the 9 independent  $\bar{s}^{\mu\nu}$  coefficients (Chung *et al.* 2009).

The classic time delay effect of GR becomes modified in the presence of Lorentz violation. The correction to the light travel time for a photon passing near a mass  $M$  can be obtained by studying light propagation with the post-newtonian metric of the pure-gravity sector of the minimal SME. If the signal is transmitted from an observer at position  $r_e$ , reflected from a planet or spacecraft at position  $r_p$ , the time delay of light

can be written in a special coordinate system as

$$\Delta T_g \approx 4GM \left[ (1 + \bar{s}^{TT}) \ln \left( \frac{r_e + r_p + R}{r_e + r_p - R} \right) + \bar{s}^{JK} \hat{b}^J \hat{b}^K \right]. \quad (4.1)$$

In this expression,  $R$  is the distance between the observer and the planet or spacecraft and  $\hat{b}$  is the impact parameter unit vector. The coefficients for Lorentz violation are expressed in the SCF, as denoted by capital letters. This result holds for near-conjunction times when the photon passes near the mass  $M$ . As discussed in Bailey (2009), time-delay tests can be useful to constrain the isotropic  $\bar{s}^{TT}$  coefficient, among others. It would be of definite interest to perform data analysis searching for evidence of nonzero  $\bar{s}^{\mu\nu}$  coefficients in sensitive time-delay experiments such as Cassini (Bertotti *et al.* 2003), BepiColombo (Iess & Asmar 2007), and SAGAS (Wolf *et al.* 2009).

In addition to effects discussed above, many standard results of GR receive modifications in the presence of Lorentz violation. This includes classic tests such as the perihelion shifts of the inner planets, the gravitational redshift, and the classic gyroscope experiment. In addition, tests beyond the solar system, such as with binary pulsars (Kramer 2009 and Stairs 2009), can also probe SME coefficients. In the case of binary pulsars, the dominant effects arise from changes in the orbital elements of the pulsar-companion oscillating ellipse. For more details the reader is referred to Bailey (2009) and Bailey & Kostelecký (2006).

## 5. Acknowledgments

Q.G. Bailey wishes to thank the International Astronomical Union for the travel grant that was provided to attend the symposium.

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