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Non-complemented Spaces of Operators, Vector Measures, and c_o

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Abstract. The Banach spaces $L(X, Y)$, $K(X, Y)$, $L_{w^*}(X^*, Y)$, and $K_{w^*}(X^*, Y)$ are studied to determine when they contain the classical Banach spaces c_o or ℓ_∞. The complementation of the Banach space $K(X, Y)$ in $L(X, Y)$ is discussed as well as what impact this complementation has on the embedding of c_o or ℓ_∞ in $K(X, Y)$ or $L(X, Y)$. Results of Kalton, Feder, and Emmanuele concerning the complementation of $K(X, Y)$ in $L(X, Y)$ are generalized. Results concerning the complementation of the Banach space $K_{w^*}(X^*, Y)$ in $L_{w^*}(X^*, Y)$ are also explored as well as how that complementation affects the embedding of c_o or ℓ_∞ in $K_{w^*}(X^*, Y)$ or $L_{w^*}(X^*, Y)$. The ℓ_p spaces for $1 = p < \infty$ are studied to determine when the space of compact operators from one ℓ_p space to another contains c_o. The paper contains a new result which classifies these spaces of operators. A new result using vector measures is given to provide more efficient proofs of theorems by Kalton, Feder, Emmanuele, Emmanuele and John, and Bator and Lewis.

1 Introduction

If each of X and Y is a real, infinite dimensional Banach space, $L(X, Y)$ is the space of all continuous linear transformations (operators) $T: X \to Y$, and $\mathcal{J}$ is a proper operator ideal, then is $\mathcal{J}$ complemented in $L(X, Y)$? This question has long been of interest to functional analysts. Particular attention has been paid to the case when $\mathcal{J} = K(X, Y) :=$ the space of compact operators from X to Y. See Emmanuele and John [6] for an historical perspective and a guide to the extensive literature on this topic.

Since Kalton presented his results at the Gregynog Colloquium in 1972 and published these results for the broader mathematical community [11], his results and techniques have been the primary tools used by researchers in this area. The sharpest *complementation* results for $K(X, Y)$ are as follows. Kalton [11] showed that if ℓ_1 is complemented in X, then $K(X, Y)$ is not complemented in $L(X, Y)$. Appealing to results in [11], Feder [7] showed that $K(X, Y)$ is not complemented if there is a non-compact operator $T: X \to Y$ which has an unconditional compact expansion. Feder [8] subsequently showed that if $c_o \hookrightarrow Y$, then $K(X, Y)$ is not complemented in $L(X, Y)$. Noting that Kalton's hypothesis, as well as both hypotheses of Feder, implied that $c_o \hookrightarrow K(X, Y)$, Emmanuele [5] and John [10] showed that if $c_o \hookrightarrow K(X, Y)$, then $K(X, Y)$ is not complemented in $L(X, Y)$.

In the next section we use vector valued measures to give simple arguments showing that c_o frequently embeds in $K(X, Y)$ and to unify and extend results in [5,7,8,10,

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11]. In the concluding section we investigate specifically when c_o embeds as a complemented subspace of $K(\ell_p, \ell_q)$. Our notation and terminology are standard. See [2, 3] for undefined terms.

2 Vector Measures and Spaces of Operators

For the convenience of the reader, we begin with a brief discussion demonstrating that frequently c_o embeds in $K(X, X)$ and $\ell_\infty \hookrightarrow L(X, X)$. Kalton [11, p. 267] observed that $\ell_\infty \hookrightarrow L(\ell_2, \ell_2)$, and it is not difficult to check that $c_o \hookrightarrow K(\ell_2, \ell_2)$. Of course, ℓ_2 has an unconditional (Schauder) basis.

More generally, suppose that X is an infinite dimensional complemented subspace of Y and X has an unconditional compact expansion of the identity (*i.e.*, $T_n \in K(X, X)$ such that $\sum_{n=1}^\infty T_n(x)$ converges to x unconditionally for each $x \in X$ [6, 11]). Since X is infinite dimensional and $\sum T_n$ is not norm convergent, we may assume that $\|T_n\| \not\to 0$. Let $\mathcal{F}$ be the finite-cofinite algebra of subsets of $\mathbf{N}$, and let $P \colon Y \to X$ be a projection. Define $\mu \colon \mathcal{F} \to K(X, X)$ by

$$\mu(A) = \begin{cases} \sum_{n \in A} T_n \circ P & \text{if } A \text{ is finite,} \\ \sum_{n \notin A} T_n \circ P & \text{if } \mathbf{N} \setminus A \text{ is finite.} \end{cases}$$

It is not difficult to see that μ is finitely additive. Further, since $\sum T_n(x)$ converges unconditionally to x, μ is bounded and $\mu(\{n\}) \not\to 0$. (Thus, μ is not strongly additive.) An application of the Diestel–Faires theorem [3, p. 20] shows that $c_o \hookrightarrow K(Y, Y)$. An appeal to [11] and explicitly [12] shows that $\ell_\infty \hookrightarrow L(Y, Y)$.

More generally, it is known that if X is infinite-dimensional and $c_o \hookrightarrow L(X, Y)$, then $\ell_\infty \hookrightarrow L(X, Y)$ (see [12]). The conditions permitting ℓ_∞ to embed isomorphically into $K(X, Y)$ are quite specific: Kalton [11] showed that $\ell_\infty \hookrightarrow K(X, Y)$ *if and only if* $\ell_\infty \hookrightarrow X^*$ *or* $\ell_\infty \hookrightarrow Y$.

The first theorem in this section is a vector measure generalization of results in [11]. (It is not difficult to see that there are countably many functionals separating the points of $L(X, Y)$ if X is separable and Y is the dual of a separable space.) Let $\mathcal{P}$ be the σ-algebra consisting of all subsets of $\mathbf{N}$.

Theorem 2.1 *If $\mu \colon \mathcal{P} \to X$ is a bounded, finitely additive vector measure with $\mu(\{n\}) = 0$ for each $n \in \mathbf{N}$ and there are countably many points in X^* which separate the points in the range of μ, then there exists an infinite set $M \subseteq \mathbf{N}$ so that $\mu(A) = 0$ for all $A \subseteq M$.*

Proof Since $\mathbf{R} \setminus \mathbf{Q}$ is uncountable and $\mathbf{Q}$ is dense in $\mathbf{R}$, we partition $\mathbf{N}$ into uncountably many infinite sets $(U_\alpha)_{\alpha \in \Delta}$ so that $U_\alpha \cap U_\beta$ is finite if $\alpha \neq \beta$. Note that $\mu(\bigcup_{i \in F} U_i) = \sum_{i \in F} \mu(U_i)$ for all finite subsets F of Δ. We assert that there exists $\alpha \in \Delta$ so that $\mu(B) = 0$ for all $B \subseteq U_\alpha$. Suppose not, and for each $\alpha \in \Delta$ choose $B_\alpha \subseteq U_\alpha$ so that $\mu(B_\alpha) \neq 0$. Since there are countably many points in X^* which separate $\{\mu(A) : A \in \mathcal{P}\}$, we may assume that there is an $x^* \in X^*$ so that $\|x^*\| = 1$ and $\{\alpha : x^*\mu(B_\alpha)\} \neq 0$ is uncountable. Without loss of generality, suppose $p > 0$ and $W = \{\alpha \in \Delta : x^*\mu(B_\alpha) > p!\}$ is uncountable. If F is a finite subset of W, then $\operatorname{card}(F) \cdot p \leq \|\mu \bigcup_{i \in F} B_i)\|$, and we easily contradict the boundedness of μ. ■

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In the sequel, let (e_n) denote the canonical unit vector basis of c_o and (e_n^*) denote the canonical unit vector basis of ℓ_1.

The following argument immediately yields an improvement of [11, Lemma 3].

Corollary 2.2 *Suppose that (x_n) is a normalized unconditional basic sequence whose closed linear span is complemented in X and $S\colon [x_n : n \geq 1] \to Y$ is an operator so that no subsequence of $(S(x_n))$ converges. Then $K(X,Y)$ is not complemented in $L(X,Y)$.*

Proof Let $Q\colon X \to [x_n]$ be a projection, let $J\colon Y \to \ell_\infty$ be an operator such that J is an isometry on $[T(x_n) : n \geq 1]$, and let (P_A), $A \subset \mathbf{N}$ be the family of projections associated with the unconditional basis (x_n). Define $\mu\colon \mathcal{P} \to L(X,Y)$ by $\mu(A) = SP_AQ$. If A is finite, $\mu(A)$ is compact. Suppose by way of contradiction that $P\colon L(X,Y) \to K(X,Y)$ is a projection. Now μ and $P\mu$ are bounded and finitely additive, and $\mu(\{n\}) - P\mu(\{n\}) = 0$ for every $n \in \mathbf{N}$. Let M be an infinite set such that $J\mu(M)_{|_{[x_n]}} = JP\mu(M)_{|_{[x_n]}}$. But SP_MQ and JSP_MQ are not compact. Thus, we have a contradiction. ■

As a specific application of Corollary 2.2 note that *if ℓ_1 embeds complementably in X and Y is infinite dimensional, then $K(X,Y)$ is not complemented in $L(X,Y)$.*

Corollary 2.3 *If $c_o \hookrightarrow Y$ and X is infinite dimensional, then $K(X,Y)$ is not complemented in $L(X,Y)$.*

Proof Let $L\colon c_o \to Y$ be an isomorphism, and let (P_M) be the family of projections associated with the seminormalized and unconditional basic sequence $(y_n) = (L(e_n))$. Choose a normalized w^*-null sequence (x_n^*) in X^* [2, Chapter XII], and let $J\colon Y \to \ell_\infty$ be an operator so that $J_{|_{[y_n]}}$ is an isometry. Define

$$S\colon X \to [y_i : i \geq 1] \subseteq Y$$

by $S(x) = \sum x_n^*(x)L(e_n)$. If X_o is any separable subspace of X that norms $(x_n)_{n=1}^\infty$, then $JP_MS_{|_{X_o}}$ is compact if and only if M is finite. Suppose that $K(X,Y)$ is complemented in $L(X,Y)$, and let $P\colon L(X,Y) \to K(X,Y)$ be a projection. Define $\mu\colon \mathcal{P} \to L(X_o, \ell_\infty)$ by $\mu(A) = JP_AS - JPP_AS$, and apply Theorem 2.1 to find an infinite set M so that $JP_MS = JPP_MS$ on X_o. Since $JP_MS_{|_{X_o}}$ is not compact, we have a contradiction. ■

Analogous to Corollary 2.2 and the italicized statement following it, the proof of Corollary 2.3 immediately produces the following improvement of results in [8].

Corollary 2.4 *If Y contains a seminormalized unconditional basic sequence (y_i), (P_M) is the family of projections associated with (y_i), $S\colon X_o \to [y_i : i \geq 1]$ is an operator, and X_o is a separable subspace of X so that $P_MS_{|_{X_o}}$ is not compact for any infinite subset M, then $K(X,Y)$ is not complemented in $L(X,Y)$.*

Remark Essentially the only difference in the proof of Corollary 2.2 and Corollary 2.3 (Corollary 2.4) involves whether $S \circ P_M$ or $P_M \circ S$ is used in defining the operator-valued measure to which Theorem 2.1 is applied.

Moreover, Theorem 2.1 has applications to other operator ideals. For example, suppose $\ell_1 \overset{c}{\hookrightarrow} X$ and (y_n) is a bounded sequence in Y which has no weakly convergent subsequence. Defining an operator $S\colon X \to Y$ and an operator-valued measure

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μ precisely as in Corollary 2.2 produces the next result. The weakly compact operators from X to Y are denoted by $W(X,Y)$.

Corollary 2.5 ([1, Theorem 3]) *If $\ell_1 \overset{c}{\hookrightarrow} X$ and $W(X,Y) \neq L(X,Y)$, then $W(X,Y)$ is not complemented in $L(X,Y)$.*

Corollary 2.6 ([5, 10]) *If $c_o \hookrightarrow K(X,Y)$, then $K(X,Y)$ is not complemented in $L(X,Y)$.*

Proof Suppose that $K(X,Y) \overset{c}{\hookrightarrow} L(X,Y)$. By Corollary 2.3 or Corollary 2.4, $c_o \not\hookrightarrow Y$. Suppose that (T_n) is a sequence in $K(X,Y)$ which is equivalent to (e_n). Then $\sum T_n(x)$ is weakly absolutely summable and consequently unconditionally convergent for all $x \in X$. Define $\mu\colon \mathcal{P} \to L(X,Y)$ by $\mu(A)(x) = \sum_{n\in A} T_n(x)$, and let $\nu = P \circ \mu$. Since $\nu(\{n\}) \not\to 0$, the Diestel–Faires theorem ensures that $\ell_\infty \hookrightarrow K(X,Y)$. Therefore, $\ell_\infty \hookrightarrow X^*$ (equivalently, $\ell_1 \overset{c}{\hookrightarrow} X$) or $\ell_\infty \hookrightarrow Y$. Corollaries 2.2 and 2.3 provide the contradiction that finishes the proof. ■

An operator $T\colon X \to Y$ is said to have an unconditional compact expansion if there exists a sequence (T_n) in $K(X,Y)$ so that $\sum_{n=1}^{\infty} T_n(x)$ converges unconditonally to $T(x)$ for all $x \in X$. As noted in the introduction, Feder [7] showed the following.

$(*)$ The existence of a non-compact operator T with an unconditional compact expansion implies that $K(X,Y)$ is not complemented in $L(X,Y)$.

Emmanuele observed that the existence of such a non-compact T ensures that $c_o \hookrightarrow K(X,Y)$. Specifically, if $\mathcal{F}$ denotes the finite-cofinite algebra of subsets of $\mathbf{N}$ and μ is defined by

$$\mu(A) = \begin{cases} \sum_{n\in A} T_n & \text{if } A \text{ is finite,} \\ -\sum_{n\notin A} T_n & \text{if } \mathbf{N}\setminus A \text{ is finite,} \end{cases}$$

then μ is finitely additive, the unconditional convergence of $\sum_{n=1}^{\infty} T_n(x)$ ensures that μ is bounded, and the non-compactness of T ensures that $\sum_{n=1}^{\infty} T_n$ is not Cauchy and that μ is not strongly additive. Another application of the Diestel–Faires theorem promises that $c_o \hookrightarrow K(X,Y)$.

While Corollary 2.6 certainly subsumes $(*)$, Feder's result has applications where c_o is not mentioned explicitly. The next result, a complement to Kalton [11, Lemma 3], follows directly from $(*)$ and the proof of Corollary 2.2.

Corollary 2.7 *If $1 \le p < \infty$, ℓ_p is complemented in X, and there exists a non-compact operator $T\colon \ell_p \to Y$, then $K(X,Y)$ is not complemented in $L(X,Y)$.*

3 $L(\ell_p, \ell_q)$ and c_o

As noted earlier in this paper, the list of infinite-dimensional Banach spaces X for which $c_o \hookrightarrow K(X,X)$ and $\ell_\infty \hookrightarrow L(X,X)$ is extensive. Furthermore, the preceding section suggests that criteria assisting one in determining the presence of c_o in spaces of operators would be beneficial. Emmanuele provided a useful tool for identifying copies, even complemented copies, of c_o in spaces of operators [5].

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Theorem 3.1 *Let X and Y be Banach spaces satisfying the following assumption: there exists a Banach space G with an unconditional basis (g_n) and biorthogonal coefficients (g_n^*) and two operators $R\colon G \to Y$ and $S\colon G^* \to X^*$ such that $(R(g_i))$ and $(S(g_i^*))$ are normalized basic sequences. Then $c_o \hookrightarrow K(X,Y)$.*

Moreover, if $(R(g_i))$ and $(S(g_i^*))$ are basic and Y (or X^*) has the Gelfand–Phillips property, then $K(X,Y)$ contains a complemented copy of c_o.

As an application of this result, Emmanuele observed that if $\ell_1 \hookrightarrow X$ and $\ell_p \hookrightarrow Y$ for some $p \geq 2$, then $c_o \hookrightarrow K(X,Y)$ and, of course, $K(X,Y)$ is not complemented in $L(X,Y)$.

We extend Emmanuele's observation in this section. The statement of a generalization of Theorem 3.1 and additional definitions will be helpful in our study.

A bounded subset A of X is called a limited subset of X if every w^*-null sequence in X^* tends to zero uniformly on A, and X has the Gelfand–Phillips property if every limited subset of X is relatively compact. Separable Banach spaces have the Gelfand–Phillips property ([14], [2, p. 116]).

The space of all $w^* - w$ continuous operators $T\colon X^* \to Y$ (resp. all compact and $w^* - w$ continuous operators) is denoted by $L_{w^*}(X^*,Y)$ (resp. $K_{w^*}(X^*,Y)$). Ruess [13] contains a discussion of $L_{w^*}(X^*,Y)$ and $K_{w^*}(X^*,Y)$, as well as applications of the following well-known isometries:

$$L_{w^*}(X^*,Y) \cong L_{w^*}(Y^*,X); \quad K_{w^*}(X^*,Y) \cong K_{w^*}(Y^*,X), (T \mapsto T^*)$$
$$W(X,Y) \cong L_{w^*}(X^{**},Y); \quad K(X,Y) \cong K_{w^*}(X^{**},Y), (T \mapsto T^{**}).$$

See also Drewnowski [4] for an extension of results in [11] to the space $K_{w^*}(X^*,Y)$. Theorem 3.1 is extended in [9].

Theorem 3.2 *Let X and Y be Banach spaces satisfying the following assumption: there exists a Banach space G with an unconditional basis (g_n) and biorthogonal coefficients (g_n^*) and two operators $R\colon G \to Y$ and $S\colon G^* \to X$ such that $(R(g_i))$ and $(S(g_i^*))$ are seminormalized sequences and either $(R(g_i))$ or $(S(g_i^*))$ is a basic sequence. Then $c_o \hookrightarrow K_{w^*}(X^*,Y)$ (indeed, in any subspace H of $L_{w^*}(X^*,Y)$ that contains $X \otimes_\lambda Y$).*

Moreover, if $(R(g_i))$ and $(S(g_i^*))$ are basic and Y (or X) has the Gelfand–Phillips property, then $K_{w^*}(X^*,Y)$ contains a complemented copy of c_o.

If $1 < p < \infty$, then we say p' is conjugate to p if $\frac{1}{p} + \frac{1}{p'} = 1$, *i.e.*, $(\ell_p)^* \cong \ell_{p'}$.

Theorem 3.3 *Suppose $1 < p < \infty$, p' is conjugate to p, and $S\colon \ell_p \longrightarrow X$ is a non-compact operator. For $p' \leq p \leq q$ or $p \leq p' \leq q$, if $R\colon \ell_q \to Y$ is a non-compact operator, then $c_o \hookrightarrow K_{w^*}(X^*,Y)$. Furthermore, if X or Y is Gelfand–Phillips (separability is sufficient), then $c_o \overset{c}{\hookrightarrow} K_{w^*}(X^*,Y)$. However, if $p < q < p'$, then there exist spaces X and Y and appropriate operators S and R so that $c_o \not\hookrightarrow K_{w^*}(X^*,Y)$.*

Proof *Case 1:* $p' \leq p \leq q$. Since $S\colon \ell_p \to X$ is a non-compact operator, we can find a $\delta > 0$ and a sequence (x_n) in B_{ℓ_p} such that $\|S(x_n) - S(x_m)\| > \delta$ if $n \neq m$. Since ℓ_p is reflexive, B_{ℓ_p} is weakly compact. Thus, without loss of generality we may assume $(a_n) = (x_n - x_{n+1})$ is weakly null.

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Observe that $\|S(a_n)\| > \delta$ for all $n \in \mathbf{N}$. Thus, $(a_n) \not\to 0$. Hence (a_n) is weakly null and seminormalized. By the Bessaga–Pelycznski selection principle (a_n) contains a subsequence (a_{n_k}) which is equivalent to a block basic sequence (h_n) of (e_n^p).

Note that ℓ_p is perfectly homogeneous for all $1 \leq p < \infty$, so we may assume (a_n) is equivalent to (e_n^p). Thus, (a_n) is basic. Since $p' \leq p$, there is a natural injection J from $\ell_{p'}$ into ℓ_p which sends $(e_n^{p'})$ to (a_n). Note that the Bessaga–Pełczyński selection principle also applies to the sequence $(S(a_n))$. Hence, we have (a_n) equivalent to $(J((e_n^{p'})))$, and without loss of generality $(S(J((e_n^{p'})))) = (S(a_n))$ is a seminormalized basic sequence in X.

Similarly, one can find a weakly null, seminormalized sequence (b_n) equivalent to (e_n^q) in ℓ_q so that $(R(b_n))$ is a seminormalized basic sequence in Y. Since $p \leq q$, there is a natural injection U from ℓ_p into ℓ_q which sends (e_n^p) to (b_n). Hence, we have (b_n) equivalent to $(U((e_n^p)))$, and without loss of generality $(R(U((e_n^p)))) = (R(b_n))$ is a weakly null, seminormalized basic sequence in Y. (The Bessaga–Pełczyński selection principle applies to the sequence $(R(b_n))$.) Therefore, by Theorem 3.2, $c_o \hookrightarrow K_{w^*}(X^*, Y)$.

Case 2: $p \leq p' \leq q$. Repeat the argument for Case 1.

Case 3: $p < q < p'$. Since $p < q < p'$, every operator from ℓ_q to ℓ_p is compact and every operator from $\ell_{p'}$ to ℓ_q is compact, *i.e.*, $K_{w^*}((\ell_p)^*, \ell_q) = K(\ell_{p'}, \ell_q) = L(\ell_{p'}, \ell_q)$. In fact, this space of compact operators is reflexive. Thus c_o cannot embed in $K_{w^*}((\ell_p)^*, \ell_q)$. In this case, let $X = \ell_p$, $Y = \ell_q$, and let $S\colon \ell_p \to \ell_p$ and $R\colon \ell_q \to \ell_q$ be identity operators. ∎

Corollary 3.4 *If $\ell_1 \hookrightarrow X$ and there exists a $p \geq 2$ with a non-compact operator $A : \ell_p \to Y$, then $c_o \hookrightarrow K(X, Y)$.*

Proof Since $\ell_1 \hookrightarrow X$, $L_1 \hookrightarrow X^*$ [2, Notes and Remarks, Chapter X]. The Rademacher functions span a copy of ℓ_2 in L_1, and thus $\ell_2 \hookrightarrow X^*$. The perfect homogeneity of the unit vector basis of ℓ_p [15] and the non-compactness of the operator A produces a non-compact operator $B\colon \ell_2 \to Y$ (as in the proof of Case 1 of Theorem 3.3). Theorem 3.3 guarantees that $c_o \hookrightarrow K_{w^*}(X^{**}, Y)$. The isometry $K_{w^*}(X^{**}, Y) \cong K(X, Y)$ finishes the argument. ∎

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