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SCHRÖDINGER OPERATORS AND THE KATO SQUARE ROOT PROBLEM

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The general theme of this thesis is the harmonic analysis of Schrödinger operators and its applications. We focus on two distinct but related open problems in this field.

The first problem is the construction of potential-dependent averaging operators and is primarily considered in the first part of this thesis. Here, a Hardy–Littlewood-type maximal operator adapted to the Schrödinger operator $\mathcal{L} := -\Delta + |x|^2$ and acting on $L^2(\mathbb{R}^n)$ is constructed. This is achieved through the use of the Gaussian grid Δ_0^γ constructed in [5] with the Ornstein–Uhlenbeck operator in mind. At the scale of this grid, the maximal operator resembles the classical Hardy–Littlewood operator. At a larger scale, the constituent averaging operators of the maximal function are decomposed over the cubes from Δ_0^γ and weighted appropriately. Through this maximal function, a new class of weights, A_p^+, is defined with the property that for any $w \in A_p^+$ the heat maximal operator associated with $\mathcal{L}$ is bounded from $L^p(w)$ to itself. This class contains any other known class that possesses this property and contains weights of exponential growth. In particular, it is strictly larger than A_p. Some of this work has appeared in [3].

The second problem that we consider is the Kato square root problem for divergence-form elliptic operators with potential $V : \mathbb{R}^n \to \mathbb{C}$. This is the equivalence statement

$$\|(L + V)^{1/2}u\|_{L^2(\mathbb{R}^n)} \simeq \|\nabla u\|_{L^2(\mathbb{R}^n)} + \|V^{1/2}u\|_{L^2(\mathbb{R}^n)},$$

where $L + V := -\mathrm{div}(A\nabla) + V$ and the perturbation A is an L^∞ complex matrix-valued function satisfying an ellipticity condition. One possible path to a solution for this problem is by proving square function estimates for perturbations of associated nonhomogeneous Dirac-type operators. At present, there is no general method to obtain such square function estimates other than for potentials bounded both from

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above and below (see [1]). We develop such a method by adapting the homogeneous framework introduced by Axelsson *et al.* in their seminal paper [2]. Two distinct approaches are considered when adapting this framework. The second such approach yields a satisfying solution to the potential-dependent Kato problem for a large class of potentials. This class includes any potential V with range contained in some sector of angle $\omega_V \in [0, \pi/2)$ and for which $|V|$ belongs either to the reverse Hölder class RH_2 in any dimension or $L^{n/2}(\mathbb{R}^n)$ for $n > 4$. Some of this work has appeared in [4].

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