Ergod. Th. & Dynam. Sys. (1983), **3**, 161–166
Printed in Great Britain

BOOK REVIEWS

TOPICS IN ERGODIC THEORY
(*Cambridge Tracts in Mathematics, no. 75*)

By WILLIAM PARRY: pp. 110. Cambridge University Press: Cambridge, 1981.

This is a collection of the author's favourite topics. It does not pretend to be comprehensive, but the author compensates for this by describing where these topics fit in the general theory. It would make a good text for an introductory graduate course, possibly supplemented with additional material from such books as [**3**] or [**4**].

Parry's main themes are the uses of algebraic and functional analytic tools in ergodic theory. These methods are traditional sources of inspiration, and still serve well to introduce students to the subject. These students should have a firm grasp of basic functional analysis. For example, the proof of the first stated theorem in the book uses the Hahn–Banach, Riesz representation, and Hahn decomposition theorems.

The author also includes the initial steps of the more modern parts of the subject, such as the convergence of conditional information and the Rohlin–Sinai characterization of Kolmogorov automorphisms.

The book contains five chapters and a useful appendix on spectral multiplicity of unitary operators that distils those parts of the theory of interest to ergodic theorists. There is a good selection of problems, ranging from routine to provokingly cryptic (Exercise 9, p. 17).

After an introduction to motivate the origins and development of ergodic ideas, the author uses irrational rotations and their generalizations to examine specific transformations in Chapter 1. The chapter concludes with complete proofs of the usual ergodic theorems, including Wiener's L^p dominated ergodic theorem. Chapter 2 contains the Shannon–McMillan–Breiman theorem on convergence of conditional information and related material on the associated martingales. Various notions of mixing are discussed in Chapter 3. Entropy as an invariant and related topics are covered in Chapter 4. The last and most interesting chapter returns to some specific results about perturbing transformations and flows to eliminate eigenfunctions. This is perhaps the only book on ergodic theory with a proof using Bessel functions! This chapter also contains other stimulating examples to complement the previous theory, including Furstenberg's minimal non-ergodic homeomorphism of the 2-torus.

A student who masters this book will be well prepared to study more advanced research. Such a student would undoubtedly want to know more about the Bernoulli theory, and could learn this from reading first [**3**] and then [**2**], where the fundamental Rohlin tower technique, which Parry does not need or mention, is heavily exploited. Another good direction would be smooth dynamics, where the recent [1] would be useful.

Parry has managed to compress a stimulating cross-section of ergodic theory into a book of relatively modest size. Those who study it will be amply rewarded.

REFERENCES

[1] Michael Irwin. *Smooth Dynamical Systems.* Academic Press: New York, 1980.
[2] Donald S. Ornstein. *Ergodic Theory, Randomness, and Dynamical Systems.* Yale Univ. Press: New Haven, 1974.
[3] Paul Shields. *The Theory of Bernoulli Shifts.* Univ. of Chicago Press: Chicago, 1973.
[4] Ya. G. Sinai. *Introduction to Ergodic Theory.* Princeton Univ. Press: Princeton, 1977.

D. LIND

RECURRENCE IN ERGODIC THEORY AND COMBINATORIAL NUMBER THEORY

By H. FURSTENBERG: pp. 199. Princeton University Press: Princeton, New Yersey, 1981

Ergodic theorists will be aware of the author's long-standing interest in the application of dynamics and probability theory to the solution of number theoretical problems. Already in his Annals study 'Stationary processes and prediction theory' [1] and in his 'skew products' paper [2] there appears a proof of Weyl's theorem on the uniform distribution of polynomial functions of a natural number, and in his 'Disjointness' paper [4] he obtains fascinating results concerning semi-groups of endomorphisms of a torus.

Here we find the most convincing testimony for the power of dynamical techniques in combinatorial number theory. The book under review is devoted to ergodic theoretic and topological dynamical proofs of the following well known theorems:

VAN DER WAERDEN. (V.d.W)
If $\mathbb{Z}$ is partitioned into sets $B_1, B_2, \ldots, B_q$, then one of the B_i contains arithmetic progressions of arbitrary length.

HINDMAN. (H)
If $\mathbb{N}$ is partitioned into $B_1, \ldots, B_q$, then one of the B_j has the property that there exist $x_1 \leq x_2 \leq \cdots$ such that all the elements x_i together with $x_{i_1} + \cdots + x_{i_k}$, $(i_1 < i_2 \cdots < i_k)$, belong to B_j.

RADO. (R)
This is more complicated to state, but it gives necessary and sufficient conditions on a system of equations

$$\sum a_{ij} x_j = 0$$

to have a solution in a single element of an arbitrary finite partition of $\mathbb{N}$.

https://doi.org/10.1017/S0143385700001875 Published online by Cambridge University Press