BUCKLING OF ELASTIC PLATES BY THE METHOD OF CONSTANT DEFLECTION LINES

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(Received 14 October 1969)

Communicated by B. Mond

1. Introduction

In a previous paper of this series – hereinafter to be referred to as [1] – the author introduced a new method for a large class of boundary value problems connected with the flexure analysis of elastic plates of arbitrary shape where the concept of 'Lines of Equal Deflection', i.e. lines which are obtained by intersecting the bent plate by planes parallel to the original plane of the plate, was introduced. The present paper extends this analysis to the buckling analysis of thin elastic plates with various forms of boundary conditions. It is shown that the proposed method appears to be a powerful tool for the investigation of those problems of elastic stability which could not be solved by conventional methods because of the difficulty of the mathematical treatment.

It is well known that the plate problems may be considered under three general classifications:

(i) bending problems, in which the plates are subjected to lateral loading only;

(ii) buckling problems, in which the plates are subjected to edge loading only;

(iii) combined loading problems, in which the plates are subjected to lateral loading and edge loading simultaneously.

In our previous analysis we have considered bending problems. We will now consider the remaining two types of problems. The thin plating structures used in aircraft are subjected to lateral loads from the pressure cabin or from the lift on the wings and to edge loading due to bending of fuselage and wings. The thin-walled structures used in ship construction are subjected to lateral pressure from the water and to edge loading due to bending of the hull. These and many other examples demonstrate the importance of plate problems subjected to a combination of lateral pressure and edge load.

2. Theory

From the point of view of the mathematical theory of elasticity, the technical theory of the buckling of thin plates is of an approximate character. We will assume here that the forces are applied in the plane of the plate and at its edges such that a plane stress system is induced in the plate. If in particular, large compressive stresses are developed in the plate, the plane state may become unstable and the plate may bend or buckle. We will further assume that the deflection $w(x, y)$ is always small; and that small bending of the plate does not affect the plane stresses set up by the forces applied at the edges. The second assumption is really conditioned on the first, for if the plate were to bend considerably, the stretch of the plate due to bending might modify the plane stress system appreciably.

Consider a laterally loaded elastic plate subjected to the action of a combination of compressive and shearing forces applied to its middle plane at the edges. The deflected form maintained by the plate in a state of neutral equilibrium may be described by a family of lines of equal deflection. Taking the $x \circ y$-plane as usual to be the middle plane of the plate and directing the z-axis perpendicular to that plane, we shall suppose that the family of lines of equal deflection $u(x, y) = \text{Const.}$ is known. Hence, intersections between the deflection surface $z = w(x, y)$ and the planes $z = \text{Const.}$ yield contours which after projection on to the $x \circ y$ plane are the level curves $u(x, y) = \text{Const.}$

Consider the equilibrium of an element of the plate bounded by any line of equal deflection. In our previous paper [1] it was assumed that the plate was bent by lateral loads alone and consequently the equilibrium equation, obtained by equating the total downward load acting on an element bounded by any line of equal deflection to the resultant upward contribution of the shear forces and the portion of the edge reaction which is due to the distribution along the edge of the twisting moment on the same contour $u(x, y) = \text{Const.}$, was found to be

$$\frac{d^3w}{du^3}\oint R\,ds + \frac{d^2w}{du^2}\oint F\,ds + \frac{dw}{du}\oint G\,ds - \int_{\Omega}\int q\,dx\,dy = 0 \tag{2.1}$$

where the contour integrals are taken around a closed path $u = \text{Const.}$ and the double integration over the area bounded by the closed contour $u = \text{Const.}$ R, F, G, etc. are the following expressions involving u and its partial derivatives:

$$\begin{aligned}
R &= -Dt^{\frac{3}{2}} \\
F &= -\frac{D}{t^{\frac{1}{2}}}[3u_{xx}u_x^2 + 3u_{yy}u_y^2 + u_{xx}u_y^2 + u_{yy}u_x^2 + 4u_{xy}u_xu_y] \\
G &= -\frac{D}{t^{\frac{3}{2}}}[u_{xxx}u_x^3 + u_{yyy}u_y^3 + (2-\mu)(u_{xxx}u_xu_y^2 + u_{yyy}u_x^2u_y \\
&\quad + u_{xyy}u_x^3 + u_{xxy}u_y^3) + (2\mu-1)(u_{xyy}u_xu_y^2 + u_{xxy}u_x^2u_y) \\
&\quad - 2(1-\mu)u_{xy}(u_xu_yu_{xx} - u_y^2u_{xy} - u_x^2u_{xy} + u_xu_yu_{yy}) \\
&\quad + (1-\mu)(u_{xx}-u_{yy})(u_{xx}u_y^2 - u_{yy}u_x^2)] \\
&\quad + \frac{2D(1-\mu)}{t^{\frac{5}{2}}}[u_{xy}(u_x^2-u_y^2) - u_xu_y(u_{xx}-u_{yy})]^2,
\end{aligned} \tag{2.2}$$

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where $t = u_x^2 + u_y^2$, D and μ are the flexural rigidity and Poisson's ratio of the plate respectively.

If, in addition to lateral loads, there are forces acting in the middle plane of the plate, stretching of this plane is produced and the corresponding stresses should be considered. Suppose that there are resultant forces N_x, N_y and N_{xy} per unit length in the middle plane of the plate acting on the sides of a small element $dxdy$ lying entirely inside the contour u = Const. in addition to a surface load and moments and shear forces (Fig. 1). The differential equation following the condition

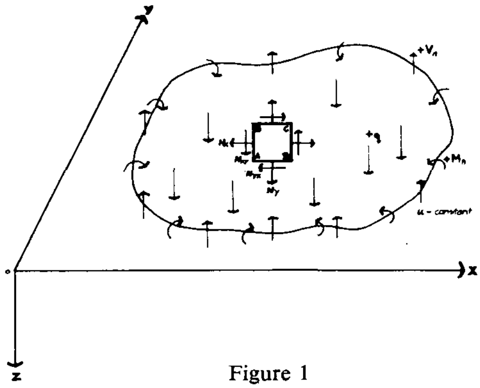

Figure 1

that there may be static equilibrium in a distorted configuration in which the normal displacement of a point of the middle surface of the plate is $w(x, y)$, can be derived from the equation (2.1) for bending by interpreting the quantity q properly. The quantity q is transverse force per unit area, which in general may vary along the surface of the plate and therefore is considered as a function of x and y. In the present case such a transverse force is also furnished by the vertical components of the plane stress system. The contributions of the inplane forces N_x, N_y and N_{xy} are readily seen from the Fig. 2 which is a plan view of the plate element $dxdy$. We

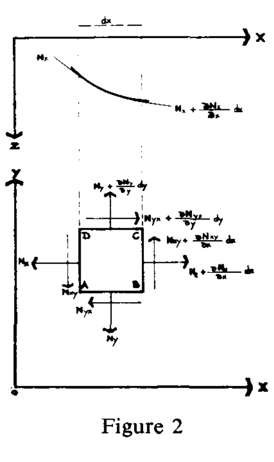

Figure 2

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must therefore find the resultant vertical contribution made by such forces. As shown in [2], the net downward contribution of the membrane forces is

$$\left[N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x\,\partial y} + N_y \frac{\partial^2 w}{\partial y^2}\right] dx\,dy$$

Therefore, it is to be expected that the governing equilibrium equation (2.1) will be altered as a consequence of such forces. Whereas in the previous case they had to support a downward load of intensity q per unit area and equation (2.1) resulted, they now have to support a downward load

$$q' = q + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x\,\partial y} + N_y \frac{\partial^2 w}{\partial y^2} = q + K \frac{d^2 w}{du^2} + L \frac{dw}{du}$$

per unit area and so the governing equation is

$$\begin{aligned} \frac{d^3 w}{du^3} \oint R\,ds + \frac{d^2 w}{du^2} \oint F\,ds + \frac{dw}{du} \oint G\,ds - \\ - \int_\Omega\!\!\int \left(q + K \frac{d^2 w}{du^2} + L \frac{dw}{du}\right) dx\,dy = 0 \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} K &= N_x u_x^2 + N_y u_y^2 + 2N_{xy} u_x u_y \\ L &= N_x u_{xx} + N_y u_{yy} + 2N_{xy} u_{xy} \end{aligned} \tag{2.4}$$

and as before, the contour integrals are taken around a closed path u = Const. and the double integrals over the area bounded by the closed contour u = Const.

While deriving the above expressions for K and L, we make use of the relations

$$\begin{aligned} \frac{\partial w}{\partial x} = \frac{dw}{du} \frac{\partial u}{\partial x} = \frac{dw}{du} u_x, \quad \frac{\partial w}{\partial y} = \frac{dw}{du} u_y, \\ \frac{\partial^2 w}{\partial x\,\partial y} = \frac{d^2 w}{du^2} u_x u_y + \frac{dw}{du} u_{xy}, \text{ etc.} \end{aligned} \tag{2.5}$$

The equation (2.3) should be used in determining the deflection surface of a plate if the forces N_x, N_y and N_{xy} are not small in comparison with the critical values of these forces. In the general case, these forces vary from one point in the plate to another and solution of the equation (2.3) is rather difficult. However, if this equation has a solution which is compatible with the boundary conditions of the problem considered, the equilibrium of the unbent configuration will be neutral.

3. Boundary conditions

Typical boundary conditions for a plate of arbitrary shape are here expressed in terms of the deflection w and its derivatives with respect to u. However, the

boundary conditions depend on the nature of fastening of the edge of the plate which, in general, will be a curved boundary with normal $\boldsymbol{n}$. If the boundary of the plate does not move in the direction perpendicular to the plane of the plate (a case corresponds to elastically supported edges), then clearly the contour of the plate belongs to the family of the lines of equal deflection and we may consider this contour without loss in generality as $u = 0$.
We thus get

$$\text{for } u = 0,\ w = 0 \text{ and } P\frac{d^2w}{du^2} + Q\frac{dw}{du} = \lambda\frac{dw}{du} \tag{3.1}$$

where λ is a Const. If $\lambda = 0$, we obtain simply supported edges, and for $\lambda = \infty$, we have clamping at the edges. The second condition in (3.1) is obtained from the consideration of the expression for M_n and $\partial w/\partial n$ as given in [1] viz:

$$M_n = P\frac{d^2w}{du^2} + Q\frac{dw}{du}, \tag{3.2}$$

$$\frac{\partial w}{\partial n} = \sqrt{t}\frac{dw}{du} \tag{3.3}$$

where

$$\begin{aligned} P &= -Dt, \\ Q &= -\frac{D}{t}\left[u_{xx}u_x^2 + u_{yy}u_y^2 + \mu u_{yy}u_x^2 + \mu u_{xx}u_y^2 + 2(1-\mu)u_{xy}u_xu_y\right]. \end{aligned} \tag{3.4}$$

Still one more condition is obtained at the centre, i.e., at the point of maximum deflection of the plate. The deflection of the plate at the centre in equilibrium position must be a finite quantity and therefore, the tangent plane at the centre must be horizontal.

4. Calculation of critical loads

A special class of problems is obtained from (2.3) by assuming $q \equiv 0$. In other words, there are assumed to be no lateral forces to cause bending. In addition, we always take homogeneous boundary conditions for w. For the sake of simplicity we also assume that the horizontal forces at the boundary are normal compressive forces which depend linearly on a factror of proportionality. Under these circumstances, it is clear that $w = 0$ is always a solution of (2.3), since w is assumed to satisfy homogeneous boundary conditions. This is also the unique solution for w when the applied compressive forces are small enough. However, there is always a critical value of the compressive forces at which the plane state becomes unstable and the plate bends, or buckles, in engineering terminology. Mathematically this means that a bifurcation of the solutions takes place for this critial value and solutions appear for which w is not identically zero. We will be here interested to

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investigate the lowest critical values for which a bifurcation takes place (or, for which bucking just begins).

It is to be noted that the discussion of the buckling problems of plates in the previous paragraph leads to a linear differential equation derived under the assumption that the deflections of the plate are small in comparison with its thickness. Therefore the solution of this differential equation applies only to the incipient state of buckling at which an infinitely small distortion of the plate is implied and consequently gives only the critical load at which the elastic equilibrium of the plate becomes unstable. It is obvious that the linear theory of plates no longer applies when the behaviour of the plate above the buckling load is to be investigated, since finite deflections of the order of magnitude of the plate thickness must be considered. This problem becomes a non-linear stress problem and requires a new basic theory.

5. Buckling of elliptic plates

As an illustration of the method, let us consider the case of a thin elliptic plate which is subjected to normal compressive forces $(-N)$ per unit length uniformly distributed around its edges (Fig. 3).

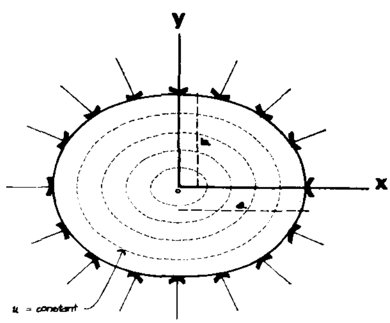

Figure 3

We will assume that a plane stress system is induced in the plate because the state of stress in a thin plate acted on by forces parallel to the mid-plane of the plate is approximately plane stress. The plane stress system will therefore be one of uniform compression throughout the elliptic plate. Our problem is to find the critical value of the compressive forces at which the plane state becomes unstable and the plate buckles. In the more general case in which normal forces and shearing forces are acting on the boundary of the elliptic plate, the same general equation (2.3) can be used for buckling analysis. However, if the forces are not uniformly distributed along the edges of the elliptic plate, there may be difficulty in solving the corresponding two-dimensional problem and determining N_x, N_y and N_{xy} as functions of x and y. But if the two dimensional problem is solved and the forces

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N_x, N_y and N_{xy} are determined then the problem of buckling of a thin elliptic plate can be fully analysed.

In our present case of an elliptic plate in a state of two-dimensional uniform compressive forces $(-N)$, we assume that the deflected surface of the plate is described by a family of lines of equal deflection $u(x, y) =$ const. which by symmetry consideration is taken to the form

$$u(x, y) = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \tag{5.1}$$

The differential equation of the deflection surface of the plate in this case is obtained from Eq. (2.3) by putting $q = 0$, i.e., by assuming that there is no lateral load and $N_x = N_y = -N$ and $N_{xy} = 0$ in the expressions for K and L. The buckling equation then becomes

$$\frac{d^3w}{du^3}\oint R\,ds + \frac{d^2w}{du^2}\oint F\,ds + \frac{dw}{du}\oint G\,ds + N\int_\Omega\int\left((u_x^2+u_y^2)\frac{d^2w}{du^2} + (u_{xx}+u_{yy})\frac{dw}{du}\right)d\Omega = 0 \tag{5.2}$$

Using Green's Theorem for the double integral and remembering that w and its derivatives with respect to u are constant on the line $u =$ Const., we finally obtain

$$\frac{d^3w}{du^3}\oint R\,ds + \frac{d^2w}{du^2}\oint F\,ds + \frac{dw}{du}\oint G\,ds + N\frac{dw}{du}\oint \sqrt{t}\,ds = 0 \tag{5.3}$$

Calculating the values of the expressions for R, F, G and t in (2.2), we obtain

$$\begin{aligned} R &= -\frac{8D}{p^3}, \\ F &= 4Dp\left[\frac{1-u}{a^2b^2} + 3\left(\frac{x^2}{a^6}+\frac{y^2}{b^6}\right)\right], \\ G &= \frac{2D(1-\mu)}{a^2b^2}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)p^5\left(\frac{x^2}{a^4}-\frac{y^2}{b^4}\right)(1-u), \\ t &= \frac{4}{p^2}, \end{aligned} \tag{5.4}$$

where

$$p^2 = \frac{1}{\dfrac{x^2}{a^4}+\dfrac{y^2}{b^4}} \tag{5.5}$$

Substituting the above expression into (5.3), we obtain the equation

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$$-8D\frac{d^3w}{du^3}\oint\frac{1}{p^3}\,ds+4D\frac{d^2w}{du^2}\oint p\left[\frac{1-u}{a^2b^2}+3\left(\frac{x^2}{a^6}+\frac{y^2}{b^6}\right)\right]ds$$

$$(5.6)\qquad +\frac{2D(1-\mu)}{a^2b^2}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\frac{dw}{du}\oint p^5\left(\frac{x^2}{a^4}-\frac{y^2}{b^4}\right)(1-u)ds$$

$$+2N\frac{dw}{du}\oint\frac{1}{p}\,ds=0$$

where the contour integrations are taken around the closed controur

$$u=1-\frac{x^2}{a^2}-\frac{y^2}{b^2}=\text{Const.}$$

and the double integration extends over the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1-u.$$

The values of these integrals are found to be

$$\oint\frac{1}{p^3}\,ds=\frac{\pi}{4}\frac{(1-u)^2}{a^3b^3}(3a^4+2a^2b^2+3b^4),$$

$$\oint p\,ds=2\pi ab,$$

$$(5.7)\qquad \oint p\left(\frac{x^2}{a^6}+\frac{y^2}{b^6}\right)ds=\pi ab(1-u)\left(\frac{1}{a^4}+\frac{1}{b^4}\right),$$

$$\oint p^5\left(\frac{x^2}{a^4}-\frac{y^2}{b^4}\right)ds=0,$$

$$\oint\frac{1}{p}\,ds=\pi ab(1-u)\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

with the help of (5.7), the differential equation (5.6) reduces finally to the form

$$(5.8)\qquad \frac{d^3w}{du^3}-\frac{2}{1-u}\frac{d^2w}{du^2}+\frac{\alpha^2}{4}\frac{1}{1-u}\frac{dw}{du}=0$$

where

$$(5.9)\qquad \alpha^2=\frac{4a^2b^2(a^2+b^2)}{3a^4+2a^2b^2+3b^4}\frac{N}{D}$$

It is convenient to introduce a new independent variable f by

$$(5.10)\qquad 1-u=\frac{f^2}{\alpha^2}$$

with respect to which Eq. (5.8) becomes

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$$f^2 \frac{d^3 w}{df^3} + f \frac{d^2 w}{df^2} + (f^2 - 1) \frac{dw}{df} = 0. \tag{5.11}$$

Denoting $dw/df = v$, we finally arrive at a Bessel's differential equation

$$f^2 \frac{d^2 v}{df^2} + f \frac{dv}{df} + (f^2 - 1)v = 0 \tag{5.12}$$

with the general solution

$$v = C_1 J_1(f) + C_2 Y_1(f) \tag{5.13}$$

where $J_1(f)$ and $Y_1(f)$ are Bessel functions of first order of the first and second kinds, respectively and C_1, C_2 are integration Constants. Boundary conditions at the edge $u = 0$ and at the center $u = 1$ must now be imposed. Let us consider the following two cases.

CASE I. *The edges of the plate are clamped.*

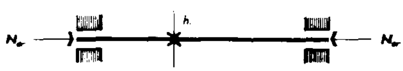

Figure 4

Suppose first that the plate is clamped (Fig. 4), so that we have the following conditions:

$$\begin{aligned} &\text{(i)} \quad w|_{u=0} = 0, \\ &\text{(ii)} \quad \left.\frac{dw}{du}\right|_{u=0} = 0, \\ &\text{(iii)} \quad \left.\sqrt{1-u}\,\frac{dw}{du}\right|_{u=1} = 0, \end{aligned} \tag{5.14}$$

further, clearly, $dw/du \neq \infty$ for $u = 1$.

The last condition is obtained by considering that the tangent plane to the deflection surface of the plate at the centre must be horizontal in order to satisfy the condition of symmetry. In terms of new variables v and f, these conditions reduce to

$$\begin{aligned} &\text{(i)} \quad w|_{f=\alpha} = 0 \\ &\text{(ii)} \quad v|_{f=\alpha} = 0 \\ &\text{(iii)} \quad v|_{f=0} = 0. \end{aligned} \tag{5.15}$$

Since the function $Y_1(f)$ becomes infinite at $f = 0$, the last condition in (5.15) requires that in dealing with a full plate, we must take $C_2 = 0$ in (5.13). Substituting now the second condition, we have aside from the trivial case obtained for $C_1 = 0$, non-vanishing solutions, given by

$$J_1(\alpha) = 0$$

or

(5.16) $$\sum_{n=0}^{\infty} (-1)^n \frac{\left(\frac{\alpha}{2}\right)^{1+2n}}{n!(1+n)!} = 0.$$

This transcendental equation has infinitely many roots,

(5.17) $$\alpha = 3.83,\ 7.02,\ 10.15, \cdots$$

the lowest non-zero one being $\alpha = 3.83$, which corresponds to the lowest critical buckling pressure obtained from (5.9)

(5.18) $$N_{cr} = \frac{3a^4+2a^2b^2+3b^4}{4a^2b^2(a^2+b^2)}(3.83)^2 D$$

Denoting $a/b = \rho > 1$, we represent this critical value by the formula

(5.19) $$N_{cr} = \gamma \frac{D}{b^2}$$

where

(5.20) $$\gamma = \frac{14.67(3\rho^4+2\rho^2+3)}{4(\rho^4+\rho^2)}.$$

The numerical values of the constant factor γ for various values of the ratio a/b are computed in Table I.

TABLE 1

$a/b = \rho$	1.0	1.1	1.2	1.3	1.4	1.5	2.0	3.0	4.0	5.0	∞
γ	14.67	13.46	12.63	11.98	11.66	11.38	10.82	10.76	10.83	10.88	11.00

CASE II. *The edges of the plate are simply supported.*

Suppose now that the plate is simply supported along its edges (Fig. 5). In

Figure 5

this case also, as in the previous case, we will consider that the lines of equal deflection form a family of similar and similarly situated concentric ellipses starting from the outer boundary as one of the lines. The differential equation (5.12) for the deflection surface of the plate therefore remains unchanged. Only the conditions (5.14) need to be altered. In this case, instead we have the following conditions:

$$(5.21)\qquad \begin{aligned} &(\text{i}) && w|_{u=0} = 0, \\ &(\text{ii}) && P\frac{d^2w}{du^2} + Q\frac{dw}{du}\bigg|_{u=0} = 0, \\ &(\text{iii}) && \sqrt{1-u}\,\frac{dw}{du}\bigg|_{u=1} = 0; \end{aligned}$$

further, clearly, dw/du is finite for $u = 1$.

The expressions (3.4.) for P and Q in this case reduce to

$$(5.22)\qquad \begin{aligned} P &= -\frac{4D}{p^2}, \\ Q &= 2Dp^2\left[\frac{\mu(1-u)}{a^2b^2} + \frac{x^2}{a^6} + \frac{y^2}{b^6}\right] \end{aligned}$$

As we see, the second condition in (5.21) may be satisfied in this particular case only approximately, because the functions P and Q appearing in this condition are not functions of u alone. We will therefore as in [1] satisfy this condition by taking the mean values of P and Q on the line u = Const. Consequently, we have for the second condition of (5.21) a modified condition

$$(5.23)\qquad (1-u)\frac{d^2w}{du^2} - \frac{1+u}{2}\frac{dw}{du}\bigg]_{u=0} = 0$$

Using the new variables, these conditions ultimately take the form

$$(5.24)\qquad \begin{aligned} &(\text{i}) && w|_{f=\alpha} = 0 \\ &(\text{ii}) && \frac{dv}{df} + \frac{\mu}{f}v\bigg]_{f=\alpha} = 0 \\ &(\text{iii}) && v|_{f=0} = 0 \end{aligned}$$

As in the previous case, the constant C_2 must be taken equal to zero. From the second condition, we obtain

$$(5.25)\qquad \frac{d}{df}J_1(f) + \frac{\mu}{f}J_1(f)\bigg]_{f=\alpha} = 0$$

Using the derivative formula for Bessel function $J_1(f)$

$$(5.26)\qquad \frac{dJ_1(f)}{df} = J_0(f) - \frac{J_1(f)}{f}$$

in which J_0 represents the Bessel function of zero order, we express the condition (5.25) to the form

$$(5.27)\qquad J_0(\alpha) - \frac{1-\mu}{\alpha}J_1(\alpha) = 0$$

Taking $\mu = 0.3$ and using tables of the function J_0 and J_1, we find the smallest root of the transcendental equation to be $\alpha = 2.05$. Then from Eq. (5.9)

$$N_{cr} = \frac{3a^4+2a^2b^2+3b^4}{4a^2b^2(a^2+b^2)}(2.05)^2 D$$
$$= \gamma_1 \frac{D}{b^2}$$

where

$$\gamma_1 = \frac{4.20(3\rho^4+2\rho^2+3)}{4(\rho^4+\rho^2)} \tag{5.29}$$

The numerical values of γ_1 for various values of ρ are computed in Table 2.

TABLE 2

ρ	1.0	1.1	1.2	1.3	1.4	1.5	2.0	3.0	4.0	5.0	∞
γ_1	4.20	3.85	3.62	3.43	3.34	3.26	3.10	3.08	3.10	3.11	3.15

6. Concluding remarks

We have thus obtained a direct method for the evaluation of the critical loads for plates having various forms of end and side constraints. To examine the effectiveness of the method, we have discussed a technically important problem. Since in the author's knowledge, there are no exact theoretical or experimental results for buckling of thin elliptic plates with which comparisons of our results can be made, we may as a basis for comparisons and also as confirmation of the proposed method consider that the case for elliptic plates reduces to that of circular plates in the limiting case when $a = b$ for which the exact solutions are known. It is interesting to note from the above two tables that for $a = b$, both cases I and II yield the exact result for the corresponding circular cases [2]. We may therefore, expect that for other cases (at least in the cases of uniform normal compressive inplane pressure), the method outlined above for buckling analysis may give us a fairly good result. It is to be mentioned that the buckling of a thin clamped elliptic plate compressed uniformly along the normal on its periphery has been attacked previously by S. Woinowsky-Krieger [3] by the use of the Rayleigh-Ritz method and by Y. Shibaoka [4] by the use of Mathieu Functions. The author believes that the solution obtained in this paper for buckling of clamped elliptic plates is an exact one.

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