Light Elements in the Universe
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Primordial nucleosynthesis in higher dimensional cosmology

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Abstract. We investigate nucleosynthesis and element formation in the early universe in the framework of higher dimensional cosmology. We find that temperature decays less rapidly in higher dimensional cosmology, which we believe may have nontrivial consequences *vis-a-vis* primordial physics.

Keywords. Cosmology: theory, cosmological parameters, early universe

1. The element formation

The work is primarily motivated by the consideration that both light element formation and higher dimensional cosmology are particularly relevant in the context of early universe.

From Einstein's field equations generalised to $(n+2)$dim. (Chatterjee & Bhui 1990) we get the following equations as

$$\frac{n(n+1)}{2}\frac{\dot{R}^2+k}{R^2}=\rho \tag{1.1}$$

$$-\frac{n\ddot{R}}{R}-\frac{n(n-1)}{2}\frac{\dot{R}^2+k}{R^2}=p \tag{1.2}$$

where ρ and p are the homogeneous mass density and pressure respectively. In the early universe one takes the radiation dominated case as our equation of state such that for a (n+2) dim. universe, $p=\frac{\rho}{n-1}$. One of the phenomenal successes favoring big bang cosmology is the almost correct prediction of the primeval nucleosynthesis, particularly the observed abundances of the light nuclei in the current universe.Now it can be shown via the field equations(1) and (2) that for the radiation dominated case we get

$$\rho=\frac{2n(n+1)}{(n+2)^2}(\frac{1}{t^2}) \tag{1.3}$$

Assuming absence of any dissipative mechanisms (for example, viscosity, friction etc.) and also that the laws of thermodynamics are valid in the early universe also with such huge temperature one gets from elementary thermodynamical considerations (Alvarez & Gavela 1983) that

$$\rho=\sigma T_{rad}^{n+2} \tag{1.4}$$

where σ is the higher dimensional Steffan's constant, hence it follows that

$$T_{rad}=\frac{2n(n+1}{(n+2)^2\sigma}t^{-\frac{2}{n+2}} \tag{1.5}$$

where T_{rad} is the temperature of radiation and 't' is the age of the universe. For the usual

4D spacetime it reduces to the well known relation $T_{kelvin} = 1.52 \times 10^{10} t^{-1/2}$ s. Equation (5) is the key equation for our attempt to investigate the effect of extra dimensions on the process of nucleosynthesis in early universe. We here study the situation when elementary particles have already materialized allowing us to take the low temperature approximation, $T << m_\mu c^2 = T_\mu$ for their distribution function. Here m_μ is the mass of a particular species. We here try to study the equilibrium condition for neutrinos with other species. As the reactions involving the neutrinos fall within the category of weak interactions and $T < T_\mu$ the cross section of a typical reaction is of the order of

$$A = f^2 h^{-4}(kT) \tag{1.6}$$

where f is the weak coupling constant. For simplicity it is further assumed that the constant A does not depend on the number of spatial dimensions. Moreover the number densities of the participating particles (say, muons) are, for (n+1) spatial dimensions, of the order of $(T/ch)^{n+1}$ and for reactions involving muons at low temperatures an exponential damping factor of $exp(-T/T_\mu)$ should also be considered. In the cosmological context one should also consider the rate of expansion of the background, which from equations (1) and (2) give

$$H^2 = \frac{\dot{R}^2}{R^2} \sim t^{-2} = T^{n+2} \tag{1.7}$$

Thus the ratio of the reaction rate to the expansion rate now becomes

$$\frac{Q}{H} \sim (\frac{T}{10^{10}K})^{\frac{n+4}{2}} exp(\frac{10^{12}K}{T}) \tag{1.8}$$

One recovers the familiar 4D form when $n = 2$. As the temperature falls below the critical level of $10^{10}K$, the exponential decays rapidly. One can at this stage call attention to a significant quantitative difference from the 4D case. From equations (5) and (8) it is tempting to suggest that as the temperature falls less rapidly in higher dimensional cosmology than the analogous 4D situation it takes relatively more time for the elementary particles to cool below the threshold temperature. More importantly the quotient $\frac{Q}{H}$ is more sensitive to temperature fluctuations in multidimensional universe. Thus Q is larger than H for $T > 10^{12}$ depending on the number of extra dimensions and in these temperature ranges the neutrinos would be in thermal equilibrium with the rest of the other species. As the temperature falls further below $10^{10}K$ both the terms in the rhs of equation(8) drop rapidly, which means that the reactions involving neutrinos run at a slower rate than compared to the expansion of the universe. This triggers the so called decoupling of the neutrinos from the rest of the other constituents of matter and as pointed earlier in the higher dimensional universe it takes relatively more time for this decoupling phase of the neutrinos to occur. However, the theoretical and observational consequences of this supposed time lag for the initiation of the decoupling era need to be worked out in more detail before any definite inferences could be drawn.

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