OVERSTABLE CONVECTION IN A NON-UNIFORM MAGNETIC FIELD

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ABSTRACT. The linear spectrum of a non-homogeneous, compressible and thermally conducting planar plasma with a vertical gravitational field and a sheared horizontal magnetic field is studied. It is shown that the spectrum has two continuous parts. The Alfvén continuum of linear ideal MHD is unaffected by radiative conduction, but the slow continuum is removed and replaced by a new continuous part which has been called isothermic continuum. Purely exponentially growing and overstable normal modes have been determined numerically and the distribution of their eigenfrequencies in the complex plane has been studied. The eigenfrequencies lie on specific curves in the complex plane which are partially determined by the isothermic continuum.

1. INTRODUCTION

Overstable magnetoconvection in a superadiabatic zone has been invoked as a possible driving mechanism for the rapid oscillations observed in Ap-stars and for waves and oscillations in sunspot regions. Shibahashi (1983) and Cox (1984) used a local analysis to obtain expressions for the complex frequencies of overstable motions. Hermans and Goossens (1986, in press), however, pointed out that a local analysis yields an incomplete picture of the adiabatic spectrum of an non-homogeneous plasma slab. In particular, the continuous parts of the spectrum, which are due to the stratification of the density, the pressure and the magnetic field, are lost. Therefore, it was anticipated that a local analysis might give an oversimplified picture of overstability in a stratified and thermally conducting plasma.

The present paper gives a preliminary report of analytical and numerical investigations of the linear spectrum of a non-homogeneous, thermally conducting and planar plasma with a vertical gravitational

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field and a horizontal sheared magnetic field. The numerical investigation uses a code which combines the Galerkin method with a finite element discretization. The code has been developed originally by W. Kerner and coworkers to study the resistive Alfvén spectrum and has now been adapted to include non-adiabatic effects, of which radiative conduction is the most important, in the energy equation and gravitational acceleration in the equation of motion.

2. ANALYTICAL DISCUSSION

Let us first recall that the linear and adiabatic motions of a non-homogeneous plasma slab with a horizontal magnetic field and a vertical gravitational field are governed by two first-order ordinary differential equations, which have regular singularities for values of z where $\sigma^2 = \sigma^2{}_A(z)$ or $\sigma^2{}_c(z)$ with $\sigma^2{}_A$ and $\sigma^2{}_c$ the squares of the Alfvén and cusp frequency respectively.

$$\sigma_A^2 = \frac{(\vec{k}.\vec{B}_o)^2}{\mu\rho_o} , \quad \sigma_c^2 = \frac{\gamma p_o}{\gamma p_o + B_o^2/\mu}\,\sigma_A^2 ,$$

with ρ_o, p_o and $\vec{B}_o$ the equilibrium density, pressure and magnetic field respectively. μ is the magnetic permeability, $\vec{k}$ is the horizontal wave vector and γ is the ratio of specific heats. These singularities give rise to the Alfvén and cusp continuum in the spectrum of linear ideal MHD. When radiative conductivity is included in the energy equation, the linear motions are governed by a system of ordinary differential equations of fourth order, that can be written as

$$D(z)\,\frac{d\xi_z}{dz} = A_1(z)\xi_z + A_2(z)P' + A_3(z)T' ,$$

$$D(z)\,\frac{dP'}{dz} = B_1(z)\xi_z + B_2(z)P' + B_3(z)T' ,$$

$$\kappa D(z)\,\frac{d^2T'}{dz^2} = C_1(z)\xi_z + C_2(z)P' + C_3(z)T' .$$

ξ_z is the z-component of the Lagrangian displacement, P' and T' are the perturbation of the total pressure and the temperature respectively, and, κ is the coefficient of radiative conductivity. D(z) is given by

$$D(z) = \rho_o(p_o + B_o^2/\mu)(\sigma^2 - \sigma_i^2)(\sigma^2 - \sigma_A^2) .$$

Let us note that the regular singularities now occur for values of z where $\sigma^2 = \sigma^2{}_A(z)$ or $\sigma^2 = \sigma^2{}_i(z)$ with $\sigma^2{}_i$ the square of the isothermic frequency,

$$\sigma_i^2 = p_o\sigma_A^2/(p_o + B_o^2/\mu) < \sigma_c^2 .$$

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Again, these regular singularities correspond to continuous parts in the spectrum. The Alfvén continuum is not affected by the radiative conductivity. The cusp continuum, however, is removed and replaced by the isothermic continuum.

We note also that the spatial eigenfunctions of the eigenmodes that correspond to a frequency in the Alfvén continuum or in the isothermic continuum, are in general non-square integrable. The spatial structure of these singular normal modes will be discussed elsewhere.

3. NUMERICAL RESULTS

The linear spectrum of a horizontal, polytropic layer (n = 1.5) with a sheared horizontal magnetic field,

$$\vec{B}_o = B_o\{\cos(\frac{\pi}{2} z)\vec{1}_x - \sin(\frac{\pi}{2} z)\vec{1}_y\} ,$$

is calculated for weak magnetic fields. The horizontal wavevector $\vec{k}$ is chosen parallell to the x-axis so that the continua reach the origin. As the magnetic field is weak, the Alfvén, cusp and isothermic continuum overlap. The fast magneto-acoustic modes are damped in time by the conductivity and are not of interest here. There exists a branch of discrete, exponentially damped modes which are modified by conductivity and magnetic field. This branch is not considered here.

The remaining part of the slow-gravity part of the spectrum can be divided into five well separated branches (See Fig. 1). Branches I and

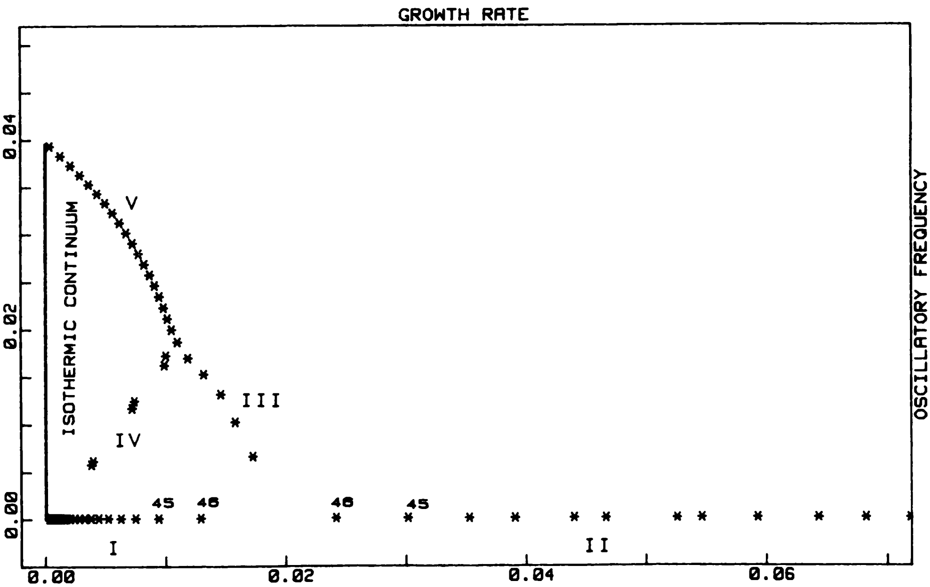

Figure 1: The spectrum of overstable modes. Roman figures indicate the different branches of the spectrum. Arabic figures indicate the number of nodes of the corresponding spatial eigenfunction.

II consist of exponentially growing eigenmodes and have the same finite number of eigenmodes. There is a one to one correspondence between the eigenmodes of branches I and II in the sense that to an eigenmode of branch I with a given number of nodes in the spatial eigenfunction ξ_z there corresponds an eigenmode of branch II with the same number of nodes in ξ_z. Further, the number of nodes increases with increasing eigenfrequencies for eigenmodes on branch I and with decreasing eigenfrequencies for eigenmodes on branch II. As the magnetic field strength and/or the radiative conductivity are increased, the eigenfrequencies are shifted towards the right on branch I and towards the left on branch II and the eigenfrequencies of the corresponding eigenmodes with the largest number of nodes in ξ_z merge, become complex and produce two overstable eigenmodes. Eventually, for strong enough magnetic fields and large enough radiative conductivity, all unstable eigenmodes can have become overstable.

Branch III consists of overstable modes that emerge when two unstable eigenmodes coalescence as described above. The oscillatory part of the eigenfrequency of an overstable mode lies in the isothermic continuum. For overstable eigenmodes on branch III it was found that, for a given magnetic field strength and radiative conductivity, the oscillatory part of the eigenfrequency increases and the growth rate decreases as the number of the nodes of ξ_z increases. For overstable eigenmodes with a growth rate smaller than a critical value which we could not determine numerically, branch III splits up into branches IV and V. So, branches IV and V have eigenmodes with relatively small growth rates and eigenfrequencies close to the isothermic continuum. The spatial behaviour of the eigenfunctions of eigenmodes on branches IV and V is very characteristic as the eigenfunctions are almost exclusively confined to the vicinity of the layer where the oscillatory part of the eigenfrequency equals the local isothermic frequency, and, this behaviour becomes more pronounced as the growth rate decreases.

4. CONCLUSION

The overstable normal modes of a polytropic, conductive plasma slab with a vertical gravitational field and a sheared horizontal magnetic field are related to the isothermic continuum. The oscillatory part of the eigenfrequencies of the overstable modes lie inside the isothermic continuum, while the spatial behaviour of the smallest growing overstable modes is determined by the equilibrium profile of the square of the isothermic frequency. The dependency of the overstable modes on the isothermic continuum should be an argument to be carefull with the use of a local analysis in non-homogeneous equilibria.

5. REFERENCES

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