

ERRATUM

Rotating planar gravity currents at moderate Rossby numbers: fully resolved simulations and shallow-water modelling – ERRATUM

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First, we wish to clarify that in equations (3.14)–(3.15) of Salinas *et al.* (2019) all variables are dimensional except for the angular velocity ω and the coefficient k . After (3.15), we switch to dimensionless variables for the streamwise direction x , the front location x_N , the Ekman layer δ_E , the local height h (scaled by the initial height h_0) and time t (scaled by $1/\Omega$, with Ω the angular velocity of the rotating system).

Therefore, equations (3.18)–(3.19) must be corrected to

$$\mathcal{C}^2 \frac{d}{dt} [(2 + \omega)\omega x_N^4] = 2kE^{1/2}\omega x_N^2, \quad (1)$$

$$\frac{d\hat{\omega}}{dt} = - \left[2k \left(\frac{3}{2} \right)^{1/3} x_0^{-2/3} \mathcal{C}^{-2/3} E^{1/2} \right] \frac{\hat{\omega}^{5/3}}{1 + \hat{\omega}} = -K_{su} \frac{\hat{\omega}^{5/3}}{1 + \hat{\omega}}, \quad (2)$$

where $\hat{\omega} = -\omega$, x_0 is the initial location of the front in the streamwise direction and K_{su} is the spin-up constant of the model without mixing (see expression in brackets in (2)). Recall that \mathcal{C} and E are the Coriolis and Ekman numbers, respectively.

Second, note the change of K_{su} . The corrected value of K_{su} introduces corrections in the last column of table 2 (see below). Also, the corrected value of K_{su} motivates the replacement of figure 14 of the original paper with figure 1 below. As for K_{su}^m – the spin-up constant of the model with turbulent mixing – it should be stressed that even if its expression remains unchanged (equation (3.21)), the value of K_{su}^m is modified due to the change of K_{su} . The corrected value of K_{su} and K_{su}^m introduces corrections in the last column of table 1.

Third, point (3) in § 5 of the original paper should also be changed. Overall, the corrected (3.19) yields fair agreement between the spin-up SW no-mixing predictions and the DNS results, in particular for large Sc . Conversely, the corrected mixing model (at least in its present form) overestimates the mean drift velocity of the slow expanding front for the range of parameters of this investigation.

The good performance of the spin-up model without mixing in the present flow merits attention, but a conclusive understanding of this effect is beyond the scope of this erratum. A plausible explanation is that the local Richardson number Ri at the interface is not small during the spin-up, and hence the mixing and momentum

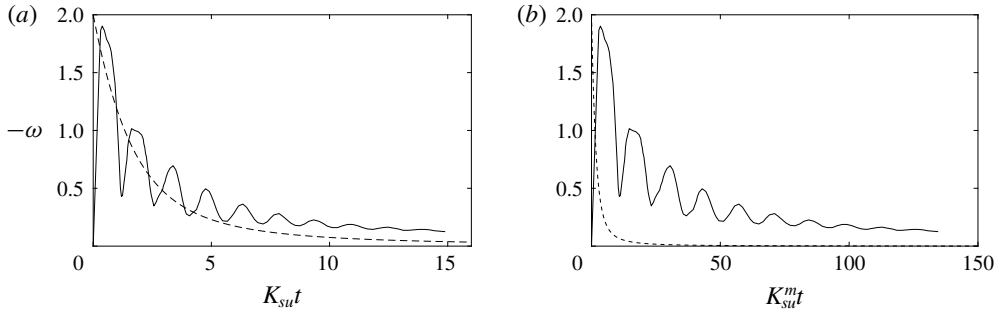


FIGURE 1. Spin-up $-\omega = -\tilde{v}/(C\tilde{x})$ as a function of (a) $K_{su}t$ and (b) $K_{su}^m t$. Parameter t is scaled with $1/\Omega$ (here $t = C\tilde{t}$): SW theoretical model (dashed line) (a) without mixing and corrected K_{su} and (b) with mixing and corrected K_{su}^m ; DNS, maximum angular velocity for case S1-C15-N (solid line).

	Approach	Reference	C	Sc	BC	$d\bar{x}_F/d\tilde{t}$
Increasing C	DNS	S1-C10-F	0.1	1	FS	1.6×10^{-3}
	DNS	S1-C15-F	0.15	1	FS	1.0×10^{-3}
	DNS	S1-C25-F	0.25	1	FS	7.0×10^{-4}
	SW	(no mixing)	0.1		FS	2.4×10^{-2}
	SW	(no mixing)	0.15		FS	1.7×10^{-2}
	SW	(no mixing)	0.25		FS	1.1×10^{-2}
	SW	(mixing)	0.1		FS	9.1×10^{-2}
	SW	(mixing)	0.15		FS	6.2×10^{-2}
	SW	(mixing)	0.25		FS	3.9×10^{-2}
Increasing Sc (without wall friction)	DNS	S1-C15-F	0.15	1	FS	1.0×10^{-3}
	DNS	S5-C15-F	0.15	5	FS	3.0×10^{-3}
	DNS	SI-C15-F	0.15	∞	FS	9.3×10^{-3}
	SW	(no mixing)	0.15		FS	1.7×10^{-2}
	SW	(mixing)	0.15		FS	6.2×10^{-2}
Increasing Sc (with wall friction)	DNS	S1-C15-N	0.15	1	NS	7.6×10^{-3}
	DNS	S5-C15-N	0.15	5	NS	1.2×10^{-2}
	DNS	SI-C15-N	0.15	∞	NS	1.7×10^{-2}
	SW	(no mixing)	0.15		NS	3.5×10^{-2}
	SW	(mixing)	0.15		NS	9.9×10^{-2}

TABLE 1. Mean ‘drift’ velocity $d\bar{x}_F/d\tilde{t}$ of the slow expanding front (computed for $\tilde{t} \geq 100$) corresponding to the slope of the dashed lines in figures 2 and 9. DNS refers to the fully resolved simulations (see table 1 of the original paper) and SW refers to the corrected shallow-water theoretical model without mixing (2) and with mixing (3.21). FS and NS refer to free slip and no slip for DNS, and to $k = 1/2$ and $3/2$ (see § 3.3 for definition) for SW, respectively. Note that the DNS value (run SI-C15-N) corrects a misprint in the original paper.

transfer (drag) are small, making the classical Ekman layer a good approximation. We estimate Ri as follows:

$$Ri = \frac{\frac{g}{\rho_0} \left| \frac{\partial \rho}{\partial z} \right|}{\left(\frac{\partial u}{\partial z} \right)^2} \approx \frac{g'/\delta_\rho}{(V/\delta_V)^2}, \quad (3)$$

where ρ and u are the local density and velocity, respectively, z is the vertical coordinate, ρ_0 is the density of the ambient fluid, g' is the reduced gravity, δ_ρ (δ_V) is the characteristic thickness of the density (momentum) transition layer at the interface between the current and the ambient and V is the characteristic speed difference across δ_V . We approximate δ_ρ , δ_V and V as

$$\delta_\rho \approx \frac{h_0}{\sqrt{ReSc}}, \quad \delta_V \approx h_0\sqrt{E}, \quad V \approx \Omega h_0 x_0 \hat{\omega} x_N, \quad (4a-c)$$

where Re is the Reynolds number, Sc is the Schmidt number and x_0 , $\hat{\omega}$ and x_N are dimensionless. Note that the scaling law for δ_ρ was confirmed in the case of non-rotating gravity currents, in the same range of Re and Sc as here, by the DNS of Bonometti & Balachandar (2008); see, for example, their figure 17. The local Richardson number thus reads (dimensional variables)

$$Ri \approx E\sqrt{ReSc} \frac{g'h_0}{(\hat{\omega}x_N)^2} \frac{1}{(\Omega h_0 x_0)^2}, \quad (5)$$

which can be written using the definition of C as

$$Ri \approx E\sqrt{ReSc} \frac{C^{-2}}{(\hat{\omega}x_N)^2} \frac{1}{x_0^2}, \quad (6)$$

where all variables are now dimensionless. Finally, we use (3.13) and (3.17) to eliminate E and x_N , respectively, and obtain our estimate of Ri as

$$Ri \approx \left(\frac{3}{2}x_0^4\right)^{-2/3} Re^{-1/2} Sc^{1/2} C^{-5/3} \hat{\omega}^{-4/3}. \quad (7)$$

During spin-up the estimated Ri increases because $\hat{\omega}$ decreases, and for the values of Re , Sc and C of our DNS the typical Ri is larger than 0.25. We also note that a sharp density transition layer enhances stability, and indeed the spin-up model without mixing shows better agreement for DNS with large Sc .

REFERENCES

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