

# A NEW TECHNIQUE FOR THE ANALYTICAL DETERMINATION OF A FUNDAMENTAL SYSTEM OF POSITIONS AND PROPER MOTIONS

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## ABSTRACT

A survey of the various tasks involved in the construction of the Fifth Fundamental Catalogue (FK5) is given. One of these tasks is the determination of the FK5 system which will be performed with the aid of new analytical methods developed at Heidelberg. The basic characteristics of these methods are described, and information on the most significant errors in the FK4 system is given.

## 1. INTRODUCTION

The basic reference coordinate system on the sky is defined by the positions and proper motions of the stars given in the Fourth Fundamental Catalogue (FK4) which was compiled by Fricke and Kopff (1963). Soon after the completion of the FK4 new observations of FK4 stars revealed considerable errors in the FK4. Especially the right ascensions south of about  $\delta = -50^\circ$  showed large systematic errors as was demonstrated by Anguita (1974). After the IAU (1974) had urged the compilation of a Fifth Fundamental Catalogue (FK5) work started at the Astronomisches Rechen-Institut, Heidelberg. Various reports on problems and progress in the work on the FK5 were given by Fricke (1974, 1978, 1980a, 1982a) and by Fricke and Gliese (1979).

The improvement of a fundamental catalogue can be subdivided into several subtasks which shall be shortly described below. One of these subtasks consists in the elimination of regional errors in the FK4 system. This will be performed with new analytical methods developed at Heidelberg. The determination of the systematic errors is now in progress and the transformation describing the transition from the FK4 to the FK5 system will be available by the end of 1984. The whole FK5 is expected to be completed in 1986.

## 2. TASKS INVOLVED IN THE CONSTRUCTION OF A FUNDAMENTAL CATALOGUE

The most important aim of a fundamental catalogue is to represent an inertial system. It is realized by the mean positions and proper motions of stars with the best history of observations and by specifying the precessional quantities used in the derivation of the proper motions. Except for galactic rotation the positions of the fundamental stars should define space fixed coordinate axes at any instant of time and there should be no distortion in the lines of constant right ascension and declination. A fundamental catalogue yields the basic reference coordinate system for the differential or photographic determination of star positions, and the fundamental proper motions provide the basis for studies of the kinematics of the galaxy. It is therefore obvious that errors in a fundamental catalogue will have far reaching consequences in many fields of astronomical research, some of which will be discussed in Section 9. The reference system in international use since 1963 is the FK4 (Fricke, Kopff, 1963). It is represented by the mean positions and proper motions of 1535 stars brighter than about apparent magnitude 7.5 and based on Newcombs expressions for general precession. The compilation of the FK5 was not only aimed at the elimination of various errors in the FK4 but also envisaged the inclusion of new bright and faint stars in order to provide a close-meshed net of coordinates on the sky. In particular, the following subtasks are involved in the improvement of the FK4:

- (a) Improvement of the expressions for general precession,
- (b) Derivation of corrections to the zero points in right ascension and declination (equinox and equator),
- (c) Elimination of regional errors of the FK4 system,
- (d) Determination of individual corrections to each FK4 position and proper motion,
- (e) Selection and inclusion of additional bright and faint stars as new fundamental stars.

## 3. COMPLETED TASKS

The tasks (a) and (b) listed in Section 2 are already completed. On the basis of 512 FK4 and FK4 Sup stars Fricke (1977) derived the correction  $\Delta p_1 = +1''.10$  per century to Newcomb's value for the lunisolar precession. In combination with the new values for the planetary masses adopted by the IAU (1976) at the XVI General Assembly in Grenoble improved expressions for the precession quantities were derived by Lieske et al. (1977).

The fundamental reference frame is dynamically defined by the earth's equator as principal plane, and the zero point is chosen to be the

intersection of this plane with the ecliptic. For the determination of the earth's equator, which is the reference plane for measuring the declinations, observations of stars in both culminations are sufficient, in principal. The practical determination of the equinox on the other hand depends ultimately on observations of the Sun. Since it is very difficult to relate observations of stars to those of the Sun, the dynamical equinox will in general deviate from its practical realization in a fundamental catalogue. In a series of papers Fricke (1979, 1980b, 1981) has reported on the determination of the equinox. On the basis of observations of the Sun, planets and minor planets, from observations of lunar occultations and from studies of the FK4 proper motions Fricke (1982b) derived the following correction to the FK4 equinox:

$$E(T) = +0.035 + 0.085 (T - 19.50),$$

where T is counted in centuries. This means that all FK4 right ascensions have to be increased by  $0.035$  at 1950 and that all FK4 proper motions need a correction of  $+0.085$  per century. With these corrections applied the catalogue equinox should coincide with the dynamical equinox as far as possible.

No significant correction to the FK4 equator was found by Fricke.

#### 4. BASIC MATERIAL FOR IMPROVING THE FK4

In this Section a short description of the procedure used for the selection of appropriate star catalogues for the improvement of the FK4 shall be given. A catalogue of star positions can contribute to the FK5 primarily only, if a sufficiently large number of FK4 stars has been observed so that the systematic relation "catalogue minus FK4" (Cat - FK4) can be determined. If this is not the case the catalogue can be used at most in a second approximation or with a lower weight. Depending on the method of observing and the program of observations a catalogue can contribute to the following tasks:

- (a) Improvement of the FK4 system. This requires that the observed positions have been determined by absolute or quasi-absolute methods.
- (b) Determination of individual corrections to the FK4 positions and proper motions, selection and inclusion of additional bright fundamental stars. In this case differential observations, for instance with respect to the FK4, can additionally be used. We will, however, determine the systematic relation Cat - FK4 also for the catalogues based on differential observations and remove the systematic differences (which should be small) from the catalogue positions.
- (c) Extension of the FK4 to fainter magnitudes ( $m_v \leq 9.0$ ). In order to ensure that the system of the faint stars will be the same as that of the bright stars the observations must have been performed by using objective screens and impersonal micrometers.

For the systematic and individual improvement of the FK4 one has to analyse mainly such observations which were not yet included in the FK4. For the selection and inclusion of new bright and faint fundamental stars it is, however, necessary to know their history of observation. Since the effects of latitude variation and magnitude equation (use of screens) were not known before only 1900 no observations of the past century can be used for fundamental catalogues. For FK5 we have considered about 270 catalogues with absolute, quasi-absolute and differential observations, obtained at about 60 observatories in 25 countries. These catalogues contain nearly 700 000 star positions resulting from about 3.5 millions single observations performed with meridian circles, transit instruments, vertical circles, and astrolabes.

Three so-called master catalogues (MC) have been constructed containing the reservoir of all candidate stars for the inclusion in the FK5. MCI contains the stars of the IRS list (AGK3R and SRS) and the NPZT stars. These stars are the candidates for the extension of the FK4 to fainter magnitudes. Each catalogue with observations of faint stars is identified with MCI so that the history of observation for the faint stars is obtained. MCII consists of the FK4, FK4 Sup, and N30 stars. Its purpose is twofold. First, the FK4 stars in a catalogue can be identified; they are needed for deriving the systematic relation  $\text{Cat} - \text{FK4}$  and for the individual improvement of the FK4. Second, the FK4 Sup and N30 stars in a catalogue are found which are the candidates for the new bright fundamental stars. MCIII is a subset of the N30 catalogue (Morgan, 1952) transformed to the FK4 system; its construction is described by Bien et al. (1978). The purpose of this catalogue is to provide the systematic relation between the FK4 and those catalogues of observation which contain none or no sufficient number of FK4 stars for performing a direct comparison  $\text{Cat} - \text{FK4}$ . If there are enough N30 stars available in a catalogue, we determine the systematic relation  $\text{Cat} - \text{MCIII}$  which is used for transforming such catalogues to the FK4 system. Systematic relations  $\text{Cat} - \text{MCIII}$  will, however, never be used for the derivation of the FK5 system.

##### 5. METHODS FOR DETERMINING SYSTEMATIC DIFFERENCES $\text{CAT} - \text{FK4}$ AS APPLIED IN THE WORK ON THE FK5

The basic data for improving the FK4 are the differences  $\Delta_i$ ,  $i = 1, \dots, N$  between the  $N$  observed FK4 positions in a catalogue and the positions computed on the basis of the data given in the FK4:

$$\Delta_i = \begin{cases} \Delta\alpha_i \cos \delta_i = (\alpha_i(\text{Cat}) - \alpha_i(\text{FK4})) \cos \delta_i, \\ \Delta\delta_i = \delta_i(\text{Cat}) - \delta_i(\text{FK4}). \end{cases}$$

Each  $\Delta_i$  is computed for the mean epoch of observation of the star and referred to the equinox and equator B1950. With the factor  $\cos \delta_i$  in  $\Delta\alpha_i$  the differences are reduced to the equator and therefore of comparable size. The differences  $\Delta_i$  are in general splitted up in two

components, a systematic part  $\Delta_{i,s}$ , and a random part  $\epsilon_i$ :

$$\Delta_i = \Delta_{i,s} + \epsilon_i.$$

The systematic parts  $\Delta_{i,s}$  per star define a smooth function  $\Delta(\alpha, \delta, m)$  (where  $\Delta = \Delta\alpha \cos \delta$  or  $\Delta = \Delta\delta$ ) on that part of the sphere which is covered by observations. This function represents the systematic relation Cat-FK4 between a catalogue of observation and the FK4. An extensive study of methods for determining the systematic relations Cat-FK4 was performed by Bien et al. (1978). As a result of this study we apply for the determination of Cat-FK4 the so-called Analytical Method which was developed by Brosche (1966) and extended by Schwan (1977). We apply in parallel as a check a Numerical Method which is a computerized version of classical techniques. A detailed description of both methods is given by Bien et al. (see Sections 3.2 and 5) so that a short summary of the main characteristics is sufficient here.

The basic properties of the numerical method are the following:

(a) The systematic part is assumed to be the sum of three components depending on  $\delta, \alpha$ , and the apparent magnitude  $m$  so that one has per star

$$\Delta_i = \Delta_{i,s}(\delta_i) + \Delta_{i,s}(\alpha_i) + \Delta_{i,s}(m_i) + \epsilon_i$$

yielding the systematic relation Cat-FK4

$$\Delta(\alpha, \delta, m) = \Delta_\delta + \Delta_\alpha + \Delta_m$$

where  $\Delta_\delta, \Delta_\alpha, \Delta_m$  are functions of  $\delta, \alpha$ , and  $m$  exclusively;

(b) The systematic parts  $\Delta_\delta, \Delta_\alpha, \Delta_m$  are determined by computing weighted average values in intervals of  $\delta, \alpha$ , and  $m$ ;

(c) These average values are computed in a set of grid points of  $\alpha, \delta$ , and  $m$  yielding the systematic relation Cat-FK4 primarily in tabular form;

(d) The systematic part for arbitrary values of  $\alpha, \delta, m$  is obtained by interpolation;

(e) The choice of intervals in  $\alpha, \delta$ , and  $m$  determines the degree of smoothing. No criterion, for the choice of intervals is applied in the numerical method, but the same set of intervals is used for all comparisons.

In the analytical method the systematic relation Cat-FK4 is represented by a series development using normalized products  $Y_j$  of Hermite polynomials  $H_p$ , Legendre polynomials  $L_n$ , and Fourier terms  $F_{m\lambda}$ . Each FK4 star in a catalogue of observation yields an equation of condition of the form

$$\Delta_i = \sum_j a_j Y_j(\alpha_i, x(\delta_i), y(m_i)) + \varepsilon_i$$

for the unknown coefficients  $a_j$  where

$$Y_j = R_{pnm\ell} H_p(y(m)) L_n(x(\delta)) F_{m\ell}(\alpha).$$

$x(\delta)$  and  $y(m)$  are transformed declinations and magnitudes which are determined by the distribution of the FK4 stars in a catalogue on the sky. With these transformations the  $Y_j$  form a normalized, complete and orthogonal system of functions. In the series development only functions  $Y_j$  with coefficients  $a_j$  significantly different from zero are included. The F-test and a significance level of 5% is used in the search for significant functions.

It should be mentioned that the spherical harmonics in the earlier work by Brosche (1966) and Schwan (1977) have now been replaced by the products of Legendre polynomials and Fourier terms. In combination with the use of the transformed declinations  $x(\delta)$  this type of functions is much more suited for modelling the systematic differences, especially if a catalogue covers only a small zone of declination or a polar cap. For more details reference is made to Bien et al. (1978).

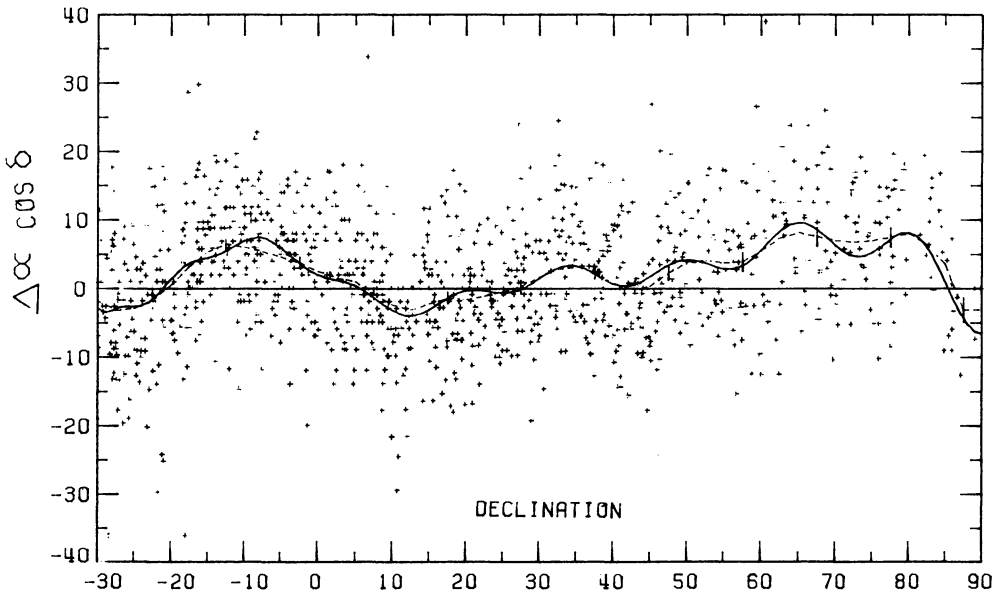


Fig. 1. Wash 5/50—FK4;  $\Delta\alpha \cos \delta [0.001]$  versus the declination. Crosses: total differences Cat—FK4 for the mean epoch of observation (1967.4). Systematic part  $\Delta\alpha \cos \delta$  according to the analytical method (full line) and numerical method (dashed line).

The main characteristics of the analytical method are:

- (a) the systematic part is essentially defined by a single number, the significance level,
- (b) a general dependence of the systematic part on  $\alpha$ ,  $\delta$ , and  $m$  simultaneously is taken into account,
- (c) the systematic relation is defined for any values of  $\alpha$ ,  $\delta$ , and  $m$  within the catalogue limits, no interpolation is necessary.

A graphical representation of the  $\delta$ -dependent part  $\Delta\alpha_{\delta} \cos \delta$  resulting from the numerical and analytical method is shown in Fig.1 for the comparison Wash 5/50 (Hughes & Scott, 1983) with the FK4. Each cross is the total difference  $\Delta\alpha \cos \delta$  (Wash 5/50 - FK4) plotted versus the declination. The full line is the result of the analytical method and has been obtained by evaluating those significant functions depending on the declination exclusively. The dashed line is the result of the numerical method yielding primarily mean values of the  $\Delta\alpha \cos \delta$  at grid points in declination which are multiples of  $2^{\circ}5$ . For some of these mean values estimates of their mean errors are indicated by error bars. Both results agree within the mean errors. The dispersion of the residuals is  $0^{\text{S}}008$  which is a rather small value for an absolute catalogue.

## 6. THE ADVANTAGES OF DETERMINING SYSTEMATIC DIFFERENCES

The determination of the systematic differences Cat - FK4 is a very laborious and time consuming task. One may therefore ask whether it is not legitimate to derive fundamental positions and proper motions just by averaging all suited catalogue positions without separating the work in the derivation of the system and of individual corrections. Some remarks on the most important reasons for the use of systematic differences should therefore be made.

- (a) The systematic relations Cat - FK4 are of general importance for the reduction of all observations given in a catalogue to the FK4 system.
- (b) With the aid of the systematic relations an absolute or quasi-absolute catalogue contributes to the mean system on the whole area where the systematic relations are determined. In order to demonstrate the essential importance of the systematic relations for the determination of a fundamental system (with emphasis on the word "system") consider the following hypothetical example. Suppose there are two absolute or quasi-absolute catalogues Cat<sub>1</sub>, Cat<sub>2</sub> with observations of FK4 stars in the same part of the sky and suppose in addition that both catalogues have no common FK4 stars, so that no mean position can be computed per FK4 star. With the aid of the systematic relations Cat<sub>1</sub> - FK4 and Cat<sub>2</sub> - FK4, however, a mean systematic correction can be derived on the whole part of the sky covered by observations.

(c) The systematic relations provide information on the behaviour of instrumental systems thus yielding one criterion for the

(d) Assignment of weights to catalogues as a whole.

(e) Residuals per star are provided which are used in addition to differential observations for the individual improvement of the FK4 positions.

## 7. ANALYTICAL DETERMINATION OF A SYSTEM OF MEAN POSITIONS AND PROPER MOTIONS

A new analytical method for the determination of mean positions and proper motions of stars was developed by Schwan (1983) with the aim to derive the FK5 system directly from all functions representing the systematic relations Cat-FK4 for the catalogues under consideration. Omitting the mathematical details this method shall be briefly described here.

Suppose one has  $K$  absolute or quasi-absolute catalogues  $\text{Cat}_k$ ,  $k=1, \dots, K$  with FK4 stars and mean epochs  $T_k$ . With the aid of the analytical method described in Section 5 one can derive for each of these catalogues the systematic relation  $\Delta_k = \text{Cat}_k - \text{FK4}$  yielding a representation of the form

$$\Delta_k(\alpha, \delta, m) = \text{Cat}_k - \text{FK4} = \sum_j a_{jk} Y_{jk}(x_k, y_k, \alpha).$$

The systematic relations of the  $K$  catalogues define systematic corrections to the mean FK4 positions and proper motions in the zone of declination,  $Z_0$ , which is common to all catalogues. Before one can derive these corrections two modifications of the systematic relations have to be performed. As was mentioned in Section 5 variables  $x_k, y_k$  are used which are transformed declinations and apparent magnitudes. These transformations depend on the data given in the catalogue  $\text{Cat}_k$  so that in general the transformed declination  $x_k(\delta)$  for a catalogue  $\text{Cat}_k$  is different from the value  $x_\ell(\delta)$  for a catalogue  $\text{Cat}_\ell$  (and correspondingly for  $y_k$  and  $y_\ell$ ). In the zone  $Z_0$  the same new variables  $x_0, y_0$  for all catalogues can be chosen and substituted in the previous equation so that one has

$$\Delta_k = \sum_j a_{jk} Z_{jk}(x_0(\delta), y_0(m), \alpha), \quad k=1, \dots, K.$$

The second modification consists in the reduction of all catalogues to the same zero point in right ascension and in the elimination of so-called "catalogue errors" in declination. For more details reference is made to Gliese (1963) and Kopff et al. (1964). These corrections are functions of the declination which are approximated by polynomials in  $x_0$  and eliminated from the systematic relations.

The systematic relations  $\Delta_k = \text{Cat}_k - \text{FK4}$  for each of the  $K$  catalogues yields on the common declination  $Z_0$  the following equation of condition



$$\Delta_k = \xi_0 + \eta(T_k - T_0) + \epsilon_k, \quad k=1, \dots, K,$$

for a systematic correction  $\xi_0$  to the mean FK4 positions at the mean epoch  $T_0$  of all catalogues, and for a systematic correction  $\eta$  to the proper motions. In the classical procedure the quantities  $\Delta_k, \xi_0, \eta$  (and  $\epsilon_k$ ) are numbers determined in a set of grid points of  $\alpha, \delta$  and  $m$ . In the analytical approach each  $\Delta_k$  is a known function of  $\alpha, x_0, y_0$  which is derived with the aid of the analytical method described in Section 5. The corrections  $\xi_0, \eta$  which we want to determine are now represented by series developments using again normalized products  $Y_j$  of Hermite polynomials, Legendre polynomials and Fourier terms (compare Section 5)

$$Y_j = R_{pnml} H_p(y_0) L_n(x_0) F_{ml}(\alpha) .$$

The equations of condition take the form

$$\sum_j a_{jk} Z_{jk} = \sum_r b_r Y_r + (T_k - T_0) \sum_s c_s Y_s + \epsilon_k, \quad k=1, \dots, K$$

and the coefficients  $b_r, c_s$  are determined by the method of least squares:

$$\sum_k w_k \int \rho \epsilon_k^2 \text{ is a minimum.}$$

$w_k$  is the weight assigned to a catalogue and  $\rho$  is the weighting function of the Hermite polynomials.

The systematic corrections to the mean positions and proper motions are given by the functions

$$\xi_0(\alpha, \delta, m) = \sum_r b_r Y_r(x_0, y_0, \alpha) ,$$

$$\eta(\alpha, \delta, m) = \sum_s c_s Y_s(x_0, y_0, \alpha) .$$

Significant functions only are included in the development. Estimates of the mean errors of the coefficients  $b_r, c_s$  and of the derived quantities  $\xi_0, \eta$  can be computed.

## 8. RESULTS

The analytical method for deriving a system of mean positions and proper motions was tested in a determination based on observations made with the Washington six-inch transit circle from about 1910 to 1970. The method has proved to work successfully; the results for the Washington observations were published and discussed by Schwan (1983).

Furthermore, on the basis of about 25 new absolute and quasi-absolute catalogues and by employing the mean FK4 positions an improved system of positions and proper motions has been derived. Since the investigations are of experimental nature only, no detailed results can be given here. The most significant deficiencies of the FK4 system,

however, have become very clear and shall be briefly mentioned.

The largest systematic errors occur in the proper motions in right ascension, especially in the declination zone south of  $\delta = -60^\circ$  where corrections of  $-0^{\text{S}}.060$  per century corresponding to nearly one second of arc per century have been found. North of  $\delta = -60^\circ$  the systematic corrections to the FK4 proper motions in general do not exceed  $\pm 0^{\text{S}}.020/\text{cy}$ .

The systematic correction  $\Delta\mu' = \mu'(\text{FK5}) - \mu'(\text{FK4})$  to the FK4 proper motions in declination are much smaller compared to those in right ascension. The maximum deviations occur at  $\delta = -80^\circ$  and  $\delta = -70^\circ$  where the values  $\Delta\mu' = -0^{\text{S}}.30/\text{cy}$  and  $+0^{\text{S}}.30/\text{cy}$ , respectively, have been found. For other declinations the corrections  $\Delta\mu'$  are in general much smaller than  $\pm 0^{\text{S}}.20$  per century.

## 9. CONCLUSIONS

There will be large systematic differences between the systems of proper motions of the FK4 and FK5 resulting from the following changes:

(a) Introduction of improved expressions for precession yielding the following corrections to the FK4 proper motions:

$$\Delta\mu = \mu(\text{FK5}) - \mu(\text{FK4}) = -\Delta m - \Delta n \sin\alpha \tan\delta,$$

$$\Delta\mu' = \mu'(\text{FK5}) - \mu'(\text{FK4}) = -\Delta n \cos\alpha,$$

where  $\Delta m$ ,  $\Delta n$  are the corrections to the rates of general precession. Based on the determinations by Fricke (1977b) and Lieske et al. (1977) numerical values for  $\Delta m$ ,  $\Delta n$  are given by Aoki et al. (1983), and by Lederle and Schwan (1984).

(b) Elimination of the fictitious motion of the equinox involving an increase of all FK4 proper motions by  $0^{\text{S}}.085$  per century,

(c) Elimination of regional systematic errors (see Section 8).

These corrections will have considerable effects on investigations based on the study of stellar proper motions, e.g. on the determination of the galactic rotation or the solar motion, on the determination of the distance of the Hyades cluster with the aid of the convergence point method, or on the distance of cepheids derived by using statistical parallaxes. The influence of systematic errors in proper motion systems on statistical parallaxes was discussed by Fricke (1966). On the basis of preliminary corrections to the FK4 proper motion system (see Section 8) Fricke (1983) found the following effects on the statistical parallaxes of stars: if  $r_{\text{FK5}} = 100 \text{ pc}$ ,  $500 \text{ pc}$ ,  $800 \text{ pc}$  are the statistical parallaxes derived from the preliminary FK5 proper motions for three assemblies of stars which are uniformly distributed along the galactic equator then  $r_{\text{FK4}} = 103 \text{ pc}$ ,  $575 \text{ pc}$ ,  $1010 \text{ pc}$  are the corresponding distances determined

from the FK4 proper motions. This example clearly demonstrates the far reaching consequences involved in the systematic improvement of fundamental proper motions.

#### ACKNOWLEDGEMENTS

The present paper is mainly a report on progress in the construction of the FK5 carried out under the supervision of Prof.Dr. W.Fricke. Many of my colleagues at the Astronomisches Rechen-Institut Heidelberg are fully involved in this task. Without the joint efforts of this group of people a work as the compilation of a fundamental catalogue would never be possible.

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#### Discussion:

- GUINOT:** Can you use the analytical method with zone catalogues?
- SCHWAN:** We no longer use spherical harmonics but rather products of Legendre polynomials and Fourier terms. This system is defined on a cylindrical area. This eliminates problems with zonal as well as with polar catalogues.
- GUINOT:** Did you use or do you intend to use astrolabe catalogues?
- SCHWAN:** We have included astrolabe catalogues, for instance, the general catalogue of astrolabes and also the Chinese astrolabe catalogue.
- LIESKE:** Have you noticed any significant discrepancies between the tabular and the analytical methods?
- SCHWAN:** Not within our limits. We have tested both methods using the Wa50 catalogue as an example and there were no significant differences.