characteristic subgroups. Each chapter contains several sets of exercises and at the end some 'Worked-Out Problems' and supplementary exercises. No answers are provided. There is a substantial bibliography and a competent index. Proofs throughout are adequately full.

My first impression was that the book might suit a first course in group theory for those with fairly limited background knowledge, provided they were competent at coping with sophisticated notation and in reading proofs. There are various things in the book that I like very much, such as diagrams that use 'string art' to show how the element 5 generates $\mathbb{Z}_{12}$ and how cycle notation works, and the diagrams illustrating the three isomorphism theorems are also very helpful, but I think opportunities have been missed to illustrate dihedral groups, and transformations in 3D in a more ambitious way. Unfortunately there are rather a lot of other problems. As can be inferred from the section titles already quoted, the English used is not always idiomatic. Definite and indefinite articles are often omitted; more worryingly, the logical structure of assertions and questions is sometimes misleading. For instance, on page 89 we read 'In other words, the integer a is a unit modulo n, meaning that $ab \equiv 1 \pmod n$ for some integer b', when what is meant is, presumably, 'In other words, the integer a is a unit modulo n if (and only if) $ab \equiv 1 \pmod n$ for some integer b'. There are other similar examples, while quite a few of the exercise questions contain typos or ambiguities. The order of the material is not always helpful (examples should always come immediately after definitions, and there should be more of them) and is sometimes simply wrong; there are unhelpful elisions between relations and sets or functions; and there is unexplained notation. Further, the author introduces a large number of concepts at an early stage, so that, for example, we meet semigroups and monoids before groups (in a section described as 'optional', but the results in it are used later on, as is some of the vocabulary) and torsion is defined as early as Chapter 4, although it is not used later. This seems to me a recipe for information overload. Add a significant number of typos, and the impression is of a book which could have been a lot more useful with more careful proofreading and subediting. Learners with the limited background knowledge that the first chapters envisage will find the going tougher than need be, although the book might serve as a useful reference resource for the more experienced.

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A course in complex analysis by Saeed Zakeri, pp. 428, £50 (hard), ISBN 978-0-691-20758-2, Princeton University Press (2021)

This book is a second-level course in complex analysis for beginning graduate students. It presupposes some background knowledge in analysis and topology, but not much in the way of functional analysis or measure theory. Throughout, the author emphasises geometrical and topological aspects of the subject but does not neglect more computational function theoretic concerns. The book is superbly produced on good quality paper with wide margins and excellent use of colour in the many helpful diagrams. Each of the 13 chapters culminates in a very satisfactory highlight, without being overwhelming in detail or complexity.

The first 7 chapters establish the groundwork and cover the rudiments of complex analysis (essentially a rapid review of an introductory course), the general homology form

https://doi.org/10.1017/mag.2023.134 Published online by Cambridge University Press

of Cauchy's theorem, Möbius maps (including their dynamics), normality, conformal maps, and harmonic functions. The remaining 6 chapters look at zeroes of holomorphic functions (and the factorisation theorems of Weierstrass and Hadamard), interpolation and approximation, extension theorems, the great theorems on the ranges of holomorphic functions, and the uniformisation theorem. There are worked examples throughout the text and a magnificent collection of 368 interesting and well-structured exercises at the ends of chapters, some giving alternative proofs of theorems.

Although the book opens with a rather ominous quotation from Thurston about reading mathematics textbooks, "Even one page a day can be quite fast", the author works hard to assist the reader. The writing is clear and unfussy with a strong narrative thrust and constant concern for the needs of an independent reader meeting the material for the first time. But experts will also find much to admire in the author's exemplary organisation of results and choice of proofs: the latter range from the classical "pole-pushing" proof of Runge's theorem to four contrasting approaches to the theorems of Bloch, Schottky, Montel and Picard, complete with a roadmap of interdependencies, and Zalcman's remarkable rescaling argument from the 1970s.

The contextualisation of results is flawless; thus, contrasts with the behaviour of real-valued functions are constantly drawn out, and the marginal photographs and historical remarks about the named contributors to the subject add much to the exposition. Unusual features include the theorems on the boundary behaviour of maps in the Riemann mapping theorem, a generalised Schwarz reflection principle for analytic arcs, a very full account of conformal metrics (including Ahlfor's far-reaching generalisation of Pick's version of Schwarz's lemma), and a detailed treatment of holomorphic branched coverings. I particularly liked the incremental generalisation of results: for example, arriving at Painlevé's vast generalisation of Riemann's theorem on removable singularities which guarantees holomorphic extensions for bounded holomorphic functions on $U - K$ where K is not just a point (as in Riemann) but a compact set with Hausdorff measure 0. And some things which are often skated over are given a full treatment, such as the use of Runge's theorem in constructing sequences of functions with surprising limiting behaviour.

I have reviewed books in the *Gazette* for nearly 40 years and none has given me greater pleasure than this one. Beautifully produced, beautifully written, on an incomparably beautiful area of mathematics, this is an inspirational book that I shall gratefully return to again and again.

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3000 years of analysis by Thomas Sonar, pp. xx+706, £109.99 (hard), ISBN 978-3-030-58221-0, also available as an e-book, Birkhäuser (Springer Nature Switzerland AG) 2021

You will gather from the title of this book that 'analysis' is interpreted in a broad sense; the author quotes from *Encyclopedia Britannica*: 'a branch of mathematics that deals with continuous change and with certain general types of processes that have emerged from the study of continuous processes ...'. This implies that Zeno's paradoxes of motion and any matters connected with indivisibility or discreteness of time or space or numbers (the 'continuum') come within the scope of the book. In fact the scope is much wider than this: it aims to give a summary of the main political events happening alongside scientific developments and to give quite detailed biographies of the main characters in the story, including reproductions of paintings or photographs or, in the case of classical figures,

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