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Correspondence

DEAR EDITOR,

I was interested in the recent appearance of problem **77.J** in the *Gazette*. It asked for a generalisation of the identity

$$\left[\sum_{i=1}^{n} i\right]^2 = \sum_{i=1}^{n} i^3.$$

The solution gave an interesting discussion of one method of deriving such identities.

Your readers may be interested in an article I wrote some years ago on this question. It contains more generalisations and describes a different method of obtaining them. The article appeared in the *Australian Senior Mathematics Journal*, Vol 2, No 1 in 1988. Readers who don't have access to this journal are welcome to write to me and I will gladly send them a copy of the article.

Yours sincerely,

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DEAR EDITOR,

'Who uses grads?' I regret I cannot now recall in sufficient detail the circumstances but back in the late sixties, when I was working for Fairey Surveys, I was shown a published map from the European continent which certainly showed grads. My colleague told me that a grad was a one hundredth part of a right angle. This reminds me of the (original?) definition of a metre as 10^{-7} of the meridianal distance on the geodesic from the equator to the pole. Does the grad then stem also from the French Revolution? Do French and other continental cartographers still use the grad, as primary or secondary angular measure, and do they indicate latitude and longitude in terms of grads?

In addition to *your* question, I would wish to ask (as I would also for temperatures in °Réaumear) 'and why do they bother?'

Yours sincerely,

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DEAR EDITOR,

I write in response to Robert Pargeter's letter 'Who uses grads?'. Now as far as I am concerned, the first question he should have asked is 'Who can remember what a grad is?'. I think I can recall that it is one hundredth part of a right angle (although I cannot find chapter and verse for this) and so was presumably an attempt to decimalise angular measure. As Robert implies, the attempt failed.

However, a much more successful attempt was the 'mil' – but thereby hangs a problem. If you look up 'mil' in either Chambers or Collins dictionaries you are told that it is the result of dividing a complete circle into 6400 parts, i.e. 0.05625 degrees. Now this may be a very useful unit in certain contexts, but why on earth should it be called a mil?

In fact in my experience, the mil as used by the military (trivial pun unavoidable) is one thousandth part of a radian i.e. 0.05729... degrees, and this had the useful property that it is the angle subtended by a (slightly curved) metre rule at 1000 metres.

Can any of your readers cast any light on the difference between these two mils, and in particular the background to the one defined in the dictionaries?

Yours sincerely,

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DEAR EDITOR,

Note 76.22 – Solid angles and the tetrahedron (*Math. Gaz.* **76** (November 1992) p. 397) reminded me that there is in fact an equivalent in three dimensions to the theorem that *the sum of the exterior angles of a convex plane polygon is 2π*. If the exterior angle in solid geometry is defined as 2π minus the sum of the face angles of the solid angle, then *the sum of the exterior angles of a convex polyhedron is 4π*.

This theorem was stated and proved in Note 1413, A theorem in solid geometry (*Math. Gaz.* **23,** (October 1939) p. 398) which was my own first contribution to the *Gazette*. The extension to four dimensions is interesting and is left to the reader.

Yours sincerely,

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Editor's note: There are two notes labelled 76.22 in the November 1992 *Gazette*. The one referred to above was correctly numbered but the sequence runs ... 76.20 76.21 76.22 76.20 76.21 76.22 76.23 ... This mistake was not mentioned in the 1992 index.

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DEAR EDITOR,

In issue 478 (March 1993), Athoen, King and Schilling have studied the game of snakes and ladders [1]. This game is much older than one might expect, so perhaps *Gazette* readers might like to know a bit about its history.

There is a 7th century Chinese 'Game of Promotion' (Shen-kuan t'u). Fawdry [2] says it is 'played on a board or plan representing an official career from the lowest to the highest grade, according to the imperial examination system. It is a kind of Snakes and Ladders, played with four dice: the object of each player being to secure promotion over the others.' Fawdry cites a Japanese work [3] for details – can any reader provide a copy and translation of this?

Bell and Cornelius, [4] discuss the same game as 'Promotion of the Mandarins' (Shing Kun t'o). They say it was played in the Ming Dynasty (1368-1616) with four or more players racing on a board with 98 spaces and throwing 6 dice to see how many equal faces appeared.

There are many Indian versions of the game. Bell and Cornelius [4] describe a Hindu version, called Moksha-patamu and describe numerous modern variants of the game.

The most extensive history of the game is by Andrew Topsfield of the Department of Eastern Art at the Ashmolean Museum, Oxford [5], but he is basically cataloguing the extant Indian boards. He says it is called Gyān caupad or Gyān chaupar in Hindi. He states that Moksha-patamu sounds like it is Telugu and that the latter name appeared in 1975 with no reference to a source and that Bell has repeated this. Game boards were drawn or painted on paper or cloth and hence have perished. The oldest extant version is believed to be an 84 square board of 1735, in the Museum of Indology, Jaipur. There were Hindu, Jain, Muslim and Tibetan versions representing a kind of Pilgrims Progress, finally arriving at God or Heaven or Nirvana. The number of squares varies from 72 to 360. He believes that the game has its origins in India, but he must have been unaware of the Chinese version. Topsfield also cites many earlier references and details – e.g. an Indian version of the game was described in 1916, but this was ignored by game historians until about 1975. The game has been called the Game of Karma. The earliest known English version has 100 squares in a spiral, registered in London by F. H. Ayres in October 1892 (example in the Bethnal Green Museum – see [6] for a reproduction).

The basic ideas in [1] were developed previously in [7], but their board had 10 snakes and 10 ladders with average number of moves equal to 47·98.

Some years ago I considered the average length of the game and developed ideas similar to [7] and [1]. After reading both papers and my own notes, I see a number of points which may encourage further work.

First, the game has two or three rules for finishing.

A One finishes by going exactly to the last square, or beyond it.

B One finishes by going exactly to the last square. If one throws too much, then one stands still.

C One finishes by going exactly to the last square. If one throws too much, one must count back from the last square. E.g., if there are 100 squares and one is at 98 and throws 6, then one counts: 99, 100, 99, 98, 97, 96 and winds up on 96. (I learned this from a neighbour's child but have not seen it elsewhere!)

Rule A is used in [7]: rule B is used in [1] – does the finishing rule make a significant difference?

A graph is given in [7] of the probability of finishing in n moves – this should be given in the study of any board.

What is the variance of the duration of the game? The simulations in [1] give durations varying between 7 and 242, though there is no longest game!

What is the average duration when there are several players and the game stops when one player finishes?

How does the average duration change with the number of snakes and ladders? For small numbers, I suspect it might depend primarily on the difference between these numbers, but clearly a large number of both snakes and ladders will cause a lot of cycling. To test this, one would take fixed numbers of snakes and ladders and make a number of random choices for their positions and compute the average duration in each case. There are many different boards, from at least five cultures, and it might be interesting to compare their average durations.

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Yours sincerely,

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