Can. J. Math., Vol. XXV, No. 5, 1973, pp. 1000–1001

METACYCLIC INVARIANTS OF KNOTS AND LINKS: CORRIGENDUM

STEVEN W. REYNER

Theorem 1 of [**1**] is incomplete due to the fact that there are representations into Γ_p which are neither onto nor cyclic. For example, if G is the group $\pi_1(S^3 - 4_1)$, the group of the four knot, and $\Delta(t)$ is its Alexander polynomial, then $\Delta(\mathrm{t}) = 1 - 3t + t^2$. Since 5 divides $\Delta(2^2)$, Theorem 1 of [**1**] states that there is a representation of G onto Γ_5. However, there is no representation onto Γ_5; there is a representation onto the subgroup, the dihedral group of order 10.

We use the notation of [**1**] throughout. To correctly state the theorem, we need the following definition: a representation ρ of G into Γ_p is non-cyclic if $\rho(G)$ is not cyclic. The corrected theorem is now:

THEOREM 1. *G can be non-cyclicly represented into Γ_p if and only if the odd prime p divides $\Delta(q^{b_1}, \ldots, q^{b_u})$ for some b. The number of inequivalent representations is equal to*

$$\sum_b \frac{p^{d(b)} - 1}{p - 1},$$

where $d(b)$ is the largest integer d such that p divides $E_d(q^{b_1}, \ldots, q^{b_u})$. Furthermore, the index of the image of G in Γ_p is the greatest common divisor of $b_1, \ldots, b_u$, and $p - 1$.

Proof. Fox [**1**] actually proved the corrected theorem except for the last (added) property. If $\rho : x_j \to \omega^{a_j}\xi^{b_{e(j)}}$ represents G non-cyclicly into $\Gamma_p = |\omega, \xi : \omega^p, \xi^{p-1}, \xi\omega\xi^{-1}\omega^{-q}|$, then for some distinct integers i and j, $(q - 1)a_i : q^{b_{e(i)}} - 1$ and $(q - 1)a_j : q^{b_{e(j)}} - 1$ are not equal and

$$[\omega^{a_j}\xi^{b_{e(j)}}, \omega^{a_i}\xi^{b_{e(i)}}] = \omega^{a_i(1-q^{b_{e(j)}})-a_j(1-q^{b_{e(i)}})}$$

is a power of ω distinct from the identity. Consequently:

(1) $\rho(G)$ is generated by ω and $\xi^{b_{e(k)}}$, for $k = 1, \ldots, u$;

(2) the index of $\rho(G)$ in Γ_p equals the index of the group generated by $\xi^{b_{e(k)}}$, $k = 1, \ldots, u$ in the cyclic group generated by ξ; and

(3) this index is just the greatest common divisor of $b_{e(1)}, \ldots, b_{e(u)}$, and $p - 1$, as claimed.

Received May 26, 1972 and in revised form, September 20, 1972.

Although some representations may be of different index than others, each subgroup of G determined by a non-cyclic representation into Γ_p will have index p in G.

REFERENCE

1. R. H. Fox, *Metacyclic invariants of knots and links*, Can. J. of Math. *22* (1970), 193–201.

State University of New York College,
Oswego, New York