THE ELECTRIC FIELDS IN NONNEUTRAL BEAM MODELS OF PULSAR MAGNETOSPHERES

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The electric fields in the polar cap region of the magnetosphere of a pairless orthogonal rotator are being calculated using a modified Poisson's equation which takes explicit account of the effect of the displacement current on the potential Φ. This is the first step in a program to determine how return currents are formed in these models and the resulting global current flow patterns. The potential is being calculated from $0.1\,R_L$ to $3\,R_L$, where R_L is the light cylinder radius, using known results for Φ at $0.1\,R_L$ with an inner last closed magnetic field line corresponding to that of a point dipole. Considerable difficulty is being encountered because of the mixed nature of the equation and the irregular boundaries. Possible methods of overcoming these difficulties are indicated.

The nonneutral beam model of pulsar magnetospheres has been reviewed by Arons (1979) and its latest developments discussed by Arons (1981). The analyses of these models to date have been local in detail although a general picture of the global current flow is envisioned (Arons 1981). In almost all cases these models require a return current to maintain a quasisteady state in the corotating frame. To examine how these return currents are formed requires a knowledge of the electric and magnetic fields in the whole magnetosphere out to several R_L. As a first step in this direction we are calculating Φ and the resulting electric fields from 0.1 to 3 R_L for the case of the pairless orthogonal rotator. Although this case does not require a return current in a global sense to some degree of approximation (Scharlemann et al. 1978), the formation of a return current instead of a sheath with a trapped particle population is a very real possibility and all of the forces required to form the current are present. Moreover, only half of the magnetosphere needs to be considered with $\Phi = 0$ everywhere on the equatorial plane. We assume a steady-state and use the results of Scharlemann et al. (1978) as a boundary condition at 0.1 R_L. Examination of the length scales involved from the surface of the star out to 3 R_L shows that a completely numerical approach would be quite impractical.

W. Sieber and R. Wielebinski (eds.), Pulsars, 29–30.

The modified Poisson's equation used with spherical coordinates referenced to the rotation axis is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right)$$

$$+\frac{1}{r^2\sin^2\theta}\left(1-\frac{\Omega^2r^2}{c^2}\sin^2\theta\right)\frac{\partial^2\Phi}{\partial\phi^2}$$

$$=-4\pi\eta\left(1-\frac{\Omega^2r^2}{c^2}\sin^2\theta\right)-\frac{2\underline{\Omega}\cdot\underline{B}}{c}\ , \qquad (1)$$

where η is the real charge density, Ω is the rotation rate and $\underline{B}$ is the magnetic vector. The last term on the right-hand side should lead to a significant fraction of the full potential drop of the polar cap near the light cylinder ($r\sin\theta = c/\Omega$). Additional boundary conditions are $\Phi = 0$ on the closed magnetosphere and $\partial\Phi/\partial r = 0$ at 3 R_L. Equation (1) is elliptic to $r\sin\theta = R_L$ and hyperbolic beyond. This mixed nature of the equations and the irregular boundaries pose serious computational problems.

We are investigating overcoming these problems as follows. Equation (1) reduces to an ordinary differential equation on the light cylinder which is solved for Φ on that surface. The elliptic part of the problem is then solved by successive overrelaxation (SOR) which works well in this irregular region. The hyperbolic part of the problem is reduced to a series of two-dimensional elliptic problems by Fourier analysis in Φ which are being solved by SOR. Resynthesis of these results should give the solution, but we are presently having oscillation problems using this method. It may be necessary and preferable to use the Yale Sparse Matrix Method for the hyperbolic part of the problem which does not require optimizing relaxation parameters as in SOR. With this problem overcome, the electric fields will be calculated from Φ by standard differencing techniques.

This research was supported by the U.S. National Science Foundation.

REFERENCES

Arons, J.: 1979, Space Sci. Rev. 24, p. 437.
Arons, J.: 1981, these proceedings.
Scharlemann, E.T., Arons, J., and Fawley, W.M.: 1978, Astrophys. J. 222, p. 297.