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ERRATUM

PERIODS OF DRINFELD MODULES AND LOCAL SHTUKAS WITH COMPLEX MULTIPLICATION – ERRATUM

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B. Errata

B.1. First Error

In [2, formulas (1.13) and (1.12) and Definition 5.21] it is claimed that $v(\omega_\psi)$ and $v_\psi(u)$ are integers. However, in general they only lie in the rational numbers $\mathbb{Q}$, because the valuation v is normalized to be an isomorphism $v\colon Q_v^\times/A_v^\times \xrightarrow{\sim} \mathbb{Z}$, but the arguments of v in both formulas lie in Q_v^{alg} instead of Q_v.

This error is harmless, as the integrality of $v_\psi(u)$ and $v(\omega_\psi)$ is nowhere used.

B.2. Second Error

In [2, formula (1.13) and Definition 4.10] there is an *error in the definition* of $v(\omega_\psi)$.

As in most of [2], we fix a finite separable semi-simple Q-algebra E. That is, E is a product of finite separable field extensions of Q. We fix a finite place v of Q and consider the decomposition of the separable Q_v-algebra $E_v := E \otimes_Q Q_v = E_{v,1} \times \cdots \times E_{v,s}$ into a product of finite field extensions $E_{v,i}$ of Q_v as after [2, Definition 4.1]. We fix a finite Galois extension $K \subset Q^{\mathrm{alg}}$ of Q and we let $L := K_v \subset Q_v^{\mathrm{alg}}$ be the closure of K. It is a finite Galois extension of Q_v. We fix a $\psi \in H_E$. The canonical extension $\psi \otimes \mathrm{id}_{Q_v}\colon E_v \to L$ will be denoted again by ψ and factors through the quotient $E_{v,i(\psi)}$ of E_v; see [2, Definition 4.5].

Let $\underline{\hat{M}}$ be a local shtuka over $R := \mathcal{O}_L$ with complex multiplication by $\mathcal{O}_{E_v}$ as in [2, Definition 4.3]. It may arise from a good model $\underline{\mathcal{M}}$ of an A-motive over R as in [2,

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Example 3.2]. We consider the one-dimensional L-vector space

$$\begin{aligned}\mathrm{H}^{\psi}(\underline{\hat{M}},L) \;:=\; & \big\{\omega\in \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},L)\colon [a]^*\omega=\psi(a)\cdot\omega \;\;\forall a\in\mathcal{O}_{E_v}\big\}\\ & \xrightarrow{\sim} \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},L)/([a]^*-\psi(a)\colon a\in\mathcal{O}_{E_v})\cdot \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},L), \end{aligned} \tag{B.1}$$

where the isomorphism comes from [2, Proposition 4.9] using the fact that E is separable over Q.

In [2, formula (1.13) and Definition 4.10] there is an *error in the definition* of $v(\omega_\psi)$ for $L[\![y_{i(\psi)}-\psi(y_{i(\psi)})]\!]$-generators ω_ψ of $\mathrm{H}^{\psi}(\underline{\hat{M}},L[\![y_{i(\psi)}-\psi(y_{i(\psi)})]\!])$. Namely, there, as a reference integral structure on the L-vector space

$$\mathrm{H}^{\psi}(\underline{\hat{M}},L)=\mathrm{H}^{\psi}(\underline{\hat{M}},L[\![y_{i(\psi)}-\psi(y_{i(\psi)})]\!])/\big(y_{i(\psi)}-\psi(y_{i(\psi)})\big),$$

the R-module

$$\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R):=\big\{\omega\in \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R)\colon [a]^*\omega=\psi(a)\cdot\omega \;\;\forall a\in\mathcal{O}_{E_v}\big\}$$

was used (which was denoted without the $\sim$ on $\widetilde{\mathrm{H}}$) and in [2, Formula (1.13) and Definition 4.10]. Then $v(\omega_\psi)$ was defined to be

$$v^{\sim}(\omega_\psi):=v(\tilde{x})\in\mathbb{Q}, \tag{B.2}$$

where $\tilde{x}\in L^\times$ is such that $\tilde{x}^{-1}\big(\omega_\psi \bmod y_{i(\psi)}-\psi(y_{i(\psi)})\big)$ is an R-generator of $\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R)$. (To clarify the error we write $v^{\sim}(\omega_\psi)$ instead of $v(\omega_\psi)$ in this erratum.)

However, in the rest of [2] the R-submodule

$$\mathrm{H}^{\psi}(\underline{\hat{M}},R):=\mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R)/([a]^*-\psi(a)\colon a\in\mathcal{O}_{E_v})\cdot \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R)\subset \mathrm{H}^{\psi}(\underline{\hat{M}},L)$$

is used as a reference integral structure on $\mathrm{H}^{\psi}(\underline{\hat{M}},L)$. (See Lemma B.1 for why the latter is an inclusion and how the two integral structures can be compared.) Correspondingly, the following definition for $v(\omega_\psi)$ is used in [2]

$$v(\omega_\psi):=v(x)\in\mathbb{Q}, \tag{B.3}$$

where $x\in L^\times$ is such that $x^{-1}\big(\omega_\psi \bmod y_{i(\psi)}-\psi(y_{i(\psi)})\big)$ is an R-generator of $\mathrm{H}^{\psi}(\underline{\hat{M}},R)$. Indeed, in [2, Section 5.12] the generator $\omega^\circ_\psi:=1$ of $\mathrm{H}^{\psi}(\underline{\hat{M}},R)$ is used, which might not lie in $\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R)$. Afterwards, any other generator ω_ψ is compared to the generator ω°_ψ. This error occurs in [2, Theorems 1.3 and 5.24 and Corollaries 5.22 and 5.25]. In terms of the valuation $v^{\sim}(\omega_\psi)$ from formula (B.2), all these theorems and corollaries have to be reformulated, as explained later. However, with the definition of $v(\omega_\psi)$ in formula (B.3), all these theorems and corollaries are correct.

Note that if $\underline{\hat{M}}=\underline{\hat{M}}_v(\underline{\mathcal{M}})$ arises from a good model $\underline{\mathcal{M}}$ of an A-motive over R as in [2, Example 3.2] then $\mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R)=\mathrm{H}^1_{\mathrm{dR}}(\underline{\mathcal{M}},R):=\sigma^*\mathcal{M}\otimes_{A_R,\,\gamma\otimes\mathrm{id}_R}R$, and hence

$$\begin{aligned}\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R)&=\widetilde{\mathrm{H}}^{\psi}(\underline{\mathcal{M}},R):=\big\{\omega\in \mathrm{H}^1_{\mathrm{dR}}(\underline{\mathcal{M}},R)\colon [a]^*\omega=\psi(a)\cdot\omega\;\;\forall a\in\mathcal{O}_E\big\},\\ \mathrm{H}^{\psi}(\underline{\hat{M}},R)&=\mathrm{H}^{\psi}(\underline{\mathcal{M}},R):=\mathrm{H}^1_{\mathrm{dR}}(\underline{\mathcal{M}},R)/([a]^*-\psi(a)\colon a\in\mathcal{O}_E)\cdot \mathrm{H}^1_{\mathrm{dR}}(\underline{\mathcal{M}},R)\end{aligned}$$

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$$\text{inside} \qquad \mathrm{H}^{\psi}(\underline{\hat{M}},L) = \mathrm{H}^{\psi}(\underline{\mathcal{M}},L) = \widetilde{\mathrm{H}}^{\psi}(\underline{\mathcal{M}},R) \otimes_R L = \mathrm{H}^{\psi}(\underline{\mathcal{M}},R) \otimes_R L.$$

We next show how the two integral structures can be compared.

Lemma B.1. *The integral structures $\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R)$ and $\mathrm{H}^{\psi}(\underline{\hat{M}},R)$ are free R-modules of rank one and contained in the L-vector space $\mathrm{H}^{\psi}(\underline{\hat{M}},L)$. The natural R-morphism*

$$\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R) \hookrightarrow \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R) \twoheadrightarrow \mathrm{H}^{\psi}(\underline{\hat{M}},R)$$

is injective with cokernel isomorphic to $R/R\cdot\psi(\mathfrak{D}_{E_v/Q_v})$, where $\mathfrak{D}_{E_v/Q_v}$ is the different of $E_v = \prod_{i=1}^{s} E_{v,i}$ over Q_v.

Proof. The morphism fits into the diagram

$$\begin{array}{ccccc} \widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R) & \hookrightarrow & \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R) & \twoheadrightarrow & \mathrm{H}^{\psi}(\underline{\hat{M}},R) \\ \big\downarrow & & \big\downarrow & & \big\downarrow ? \\ & \nearrow & \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},L) & \searrow & \\ \mathrm{H}^{\psi}(\underline{\hat{M}},L) & & \xrightarrow{\quad\sim\quad} & & \mathrm{H}^{\psi}(\underline{\hat{M}},L), \end{array} \tag{B.4}$$

in which the lower isomorphism was described in formula (B.1), the lower triangle is the tensor product of the upper row with L and the injectivity of the right vertical arrow still has to be proved. Note that the argument will not use the specific situation of de Rham cohomology of local shtukas. It will only use the isomorphism (B.1) coming from [2, Proposition 4.9] and the freeness of the R-module $\mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R)$ over $\mathcal{O}_{E,v} \otimes_{A_v} R$ (see later).

The Q_v-algebra E_v acts on $\mathrm{H}^{\psi}(\underline{\hat{M}},L)$ through the character $\psi\colon E_v \twoheadrightarrow E_{v,i(\psi)} \hookrightarrow L$. By [3, §III.6, Proposition 12], there exists an element $y \in \mathcal{O}_{E_{v,i(\psi)}}$ such that $\mathcal{O}_{E_{v,i(\psi)}} = A_v[y] = A_v[Y]/(m)$, where $m \in A_v[Y]$ is the minimal polynomial of y over A_v. The image $\gamma(m)$ under the map $\gamma\colon A_v[Y] \hookrightarrow R[Y]$ has $\psi(y)$ as a zero and correspondingly factors as

$$\gamma(m) = \big(Y - \psi(y)\big)\cdot g(Y)$$

for a monic polynomial $g(Y) \in R[Y]$. The derivative $m' := \frac{dm}{dY} \in A_v[Y]$ satisfies

$$\psi\big(m'(y)\big) = \gamma(m)'\big(\psi(y)\big) = g\big(\psi(y)\big). \tag{B.5}$$

Recall that $A_{v,R}$ is the v-adic completion of A_R. By [2, Proposition 4.8] we can decompose $\underline{\hat{M}} = \bigoplus_{i=1}^{s} \underline{\hat{M}}_i$ into local shtukas $\underline{\hat{M}}_i$ over R with complex multiplication by $\mathcal{O}_{E_{v,i}}$. In particular,

$$\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R) := \big\{\omega \in \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}}_{i(\psi)},R)\colon [a]^*\omega = \psi(a)\cdot\omega \;\; \forall a \in \mathcal{O}_{E_{v,i}}\big\} \qquad \text{and}$$

$$\mathrm{H}^{\psi}(\underline{\hat{M}},R) := \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}}_{i(\psi)},R)/([a]^* - \psi(a)\colon a \in \mathcal{O}_{E_{v,i}})\cdot \mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}}_{i(\psi)},R)$$

can be computed from

$$\mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}}_{i(\psi)},R) := \sigma^*\hat{M}_{i(\psi)} \otimes_{A_{v,R},\gamma\otimes\mathrm{id}_R} R$$

instead of $\mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R)$. Moreover, by the same proposition $\underline{\hat{M}}$ is free over $\mathcal{O}_{E_v,R} := \mathcal{O}_{E_v} \otimes_{A_v} R[\![z]\!] = \mathcal{O}_{E_v} \widehat{\otimes}_{\mathbb{F}_v} R$ of rank one, and we may choose a generator of $\hat{M}$. This generator

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provides an isomorphism

$$\mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}}_{i(\psi)},R) \cong (\mathcal{O}_{E_{v,i(\psi)}} \widehat{\otimes}_{\mathbb{F}_q} R) \underset{A_{v,R},\gamma\otimes\mathrm{id}_R}{\otimes} R = \mathcal{O}_{E_{v,i(\psi)}} \underset{A_v,\gamma}{\otimes} R = R[Y]/(\gamma(m)).$$

Since $[a]^* - \psi(a) = [a]^* - \gamma(a)$ for $a \in A_v$ already annihilates $\mathrm{H}^1_{\mathrm{dR}}(\underline{\hat{M}},R)$, this yields the upper vertical isomorphisms in the following diagram:

$$\begin{array}{ccc}
\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R) & \hookrightarrow & \mathrm{H}^{\psi}(\underline{\hat{M}},R) \\
\downarrow \cong & & \downarrow \cong \\
\{f \in \mathcal{O}_{E_{v,i(\psi)}} \otimes_{A_v,\gamma} R \colon (y\otimes 1 - 1\otimes\psi(y))\cdot f = 0\} & \hookrightarrow & (\mathcal{O}_{E_{v,i(\psi)}} \otimes_{A_v,\gamma} R)/(y\otimes 1 - 1\otimes\psi(y)) \\
\| & & \| \\
\{f \in R[Y]/(\gamma(m)) \colon (Y-\psi(y))\cdot f = 0\} & \hookrightarrow & R[Y]/\big(\gamma(m), Y-\psi(y)\big) \\
\| & & \| \\
g(Y)\cdot R[Y]/(\gamma(m)) & \hookrightarrow & R[Y]/(Y-\psi(y)).
\end{array}$$

The injectivity of the horizontal maps follows from diagram (B.4). The lower left equality holds because $R[Y]$ has no $(Y-\psi(y))$-torsion. Next, $\mathrm{H}^{\psi}(\underline{\hat{M}},R) \cong R[Y]/(Y-\psi(y)) \cong R$ is free, and hence contained in $\mathrm{H}^{\psi}(\underline{\hat{M}},R)\otimes_R L = \mathrm{H}^{\psi}(\underline{\hat{M}},L)$. Finally, the image of the lower horizontal map is the ideal

$$R\cdot g\big(\psi(y)\big) = R\cdot\psi\big(m'(y)\big) = R\cdot\psi(\mathfrak{D}_{E_{v,i(\psi)}/Q_v}) = R\cdot\psi(\mathfrak{D}_{E_v/Q_v})$$

(see [3, §III.4, Proposition 10 and §III.6, Corollary 2]. □

Corollary B.2. *For an L-generator ω_ψ of $\mathrm{H}^{\psi}(\underline{\hat{M}},L)$, the two valuations in formulas (B.2) and (B.3) satisfy*

$$v(\omega_\psi) - v^{\sim}(\omega_\psi) = v(\mathfrak{D}_{\psi(E_v)/Q_v}) = v\big(\psi(\mathfrak{D}_{E_v/Q_v})\big) = v\big(\psi(\mathfrak{D}_{E/Q})\big).$$

Proof. Let $x,\tilde{x} \in L^\times$ be elements such that $x^{-1}\omega_\psi$ is an R-generator of $\mathrm{H}^{\psi}(\underline{\hat{M}},R)$ and $\tilde{x}^{-1}\omega_\psi$ is an R-generator of $\widetilde{\mathrm{H}}^{\psi}(\underline{\hat{M}},R)$. Then $x/\tilde{x}$ is an R-generator of $\psi(\mathfrak{D}_{E_v/Q_v})$ by Lemma B.1, and the corollary follows. □

Now let $\underline{M}$ be an A-motive over a finite Galois extension $K \subset Q^{\mathrm{alg}}$ of Q with complex multiplication by a finite separable semi-simple Q-algebra E. Assume that $\psi(E) \subset K$ for all $\psi \in H_E$. Fix a $\psi \in H_E$ and let ω_ψ be a generator of the $K[\![y_\psi - \psi(y_\psi)]\!]$-module $\mathrm{H}^{\psi}(\underline{M}^{\eta}, K[\![y_\psi - \psi(y_\psi)]\!])$. For every $\eta \in H_K$, let $\underline{M}^{\eta}$ and $\omega^{\eta}_{\psi} \in \mathrm{H}^{\eta\psi}(\underline{M}^{\eta}, K[\![y_{\eta\psi} - \eta\psi(y_{\eta\psi})]\!])$ be obtained by extension of scalars via η. With the corollary and the computation

$$\begin{aligned}\sum_{\eta\in H_K} v(\omega^{\eta}_{\psi}) - v^{\sim}(\omega^{\eta}_{\psi}) &= \sum_{\eta\in H_K} v(\eta\psi(\mathfrak{D}_{E/Q})) = v\Big(\prod_{\eta\in H_K}\eta\psi(\mathfrak{D}_{E/Q})\Big) = v\big(N_{K/Q}(\mathfrak{D}_{\psi(E)/Q})\big) \\ &= v\Big(N_{\psi(E)/Q}\big(N_{K/\psi(E)}(\mathfrak{D}_{\psi(E)/Q})\big)\Big) = [K:\psi(E)]\cdot v(\mathfrak{d}_{\psi(\mathcal{O}_E)/A}),\end{aligned}$$

we obtain a reformulation of [2, Theorems 1.3 and 5.24 and Corollaries 5.22 and 5.25] in terms of $v^{\sim}(\omega_\psi)$, which is even more analogous to [1, Theorem II.1.1(i)].

Theorem 1.3′. *Let ω_ψ be a generator of the $K[\![y_\psi-\psi(y_\psi)]\!]$-module $\mathrm{H}^\psi(\underline{M},K[\![y_\psi-\psi(y_\psi)]\!])$. For every $\eta\in H_K$, let $\underline{M}^\eta$ and $\omega_\psi^\eta\in\mathrm{H}^{\eta\psi}(\underline{M}^\eta,K[\![y_{\eta\psi}-\eta\psi(y_{\eta\psi})]\!])$ be obtained by extension of scalars via η, and choose an E-generator $u_\eta\in\mathrm{H}_{1,\mathrm{Betti}}(\underline{M}^\eta,Q)$. Then for every place $v\neq\infty$ of C, we have*

$$\tfrac{1}{\#H_K}\sum_{\eta\in H_K}v(\textstyle\int_{u_\eta}\omega_\psi^\eta)=Z_v(a^0_{E,\psi,\Phi},1)-\mu_{\mathrm{Art},v}(a^0_{E,\psi,\Phi})+\tfrac{1}{\#H_K}\sum_{\eta\in H_K}\big(v^{\sim}(\omega_\psi^\eta)+v_{\eta\psi}(u_\eta)\big).\qquad\square$$

Corollary 5.22′. *Let $\varphi,\psi\in H_{E_v}$ with $i(\varphi)=i(\psi)=:i$ and assume that $E_{v,i}$ is separable over Q_v. Let $u\in\mathrm{H}_{1,v}(\underline{\hat{M}}_{E_v,\varphi},Q_v)$ be a generator as an E_v-module and let ω_ψ be an $L[\![y_i-\psi(y_i)]\!]$-generator of $\mathrm{H}^\psi(\underline{\hat{M}}_{E_v,\varphi},L[\![y_i-\psi(y_i)]\!])$. Then $\int_u\omega_\psi:=u\otimes\mathrm{id}_{\mathbb{C}_v(\!(z-\zeta)\!)}\big(h^{-1}_{v,\mathrm{dR}}(\omega_\psi)\big)$ has valuation*

$$v\big(\textstyle\int_u\omega_\psi\big)=Z_v(a_{E_v,\psi,\varphi},1)-\mu_{\mathrm{Art},v}(a_{E_v,\psi,\varphi})+v^{\sim}(\omega_\psi)+v_\psi(u),$$

where $v^{\sim}(\omega_\psi)$ and $v_\psi(u)$ are defined in forumla (B.2) and [2, Definition 5.21] respectively. $\square$

Theorem 5.24′. *Let $\underline{\hat{M}}$ be a local shtuka over R with complex multiplication by the ring of integers $\mathcal{O}_{E_v}$ in a commutative, semi-simple, separable Q_v-algebra E_v with local CM-type Φ, and assume that $\psi(E_v)\subset L$ for all $\psi\in H_{E_v}$ and that L is separable over Q_v. Let $u\in\mathrm{H}_{1,v}(\underline{\hat{M}},Q_v)$ be an E_v-generator and let ω_ψ be an $L[\![y_{i(\psi)}-\psi(y_{i(\psi)})]\!]$-generator of $\mathrm{H}^\psi(\underline{\hat{M}},L[\![y_{i(\psi)}-\psi(y_{i(\psi)})]\!])$. Then the period $\int_u\omega_\psi:=u\otimes\mathrm{id}_{\mathbb{C}_v(\!(z-\zeta)\!)}\big(h^{-1}_{v,\mathrm{dR}}(\omega_\psi)\big)$ has valuation*

$$v\big(\textstyle\int_u\omega_\psi\big)=Z_v(a_{E_v,\psi,\Phi},1)-\mu_{\mathrm{Art},v}(a_{E_v,\psi,\Phi})+v^{\sim}(\omega_\psi)+v_\psi(u),$$

where $v^{\sim}(\omega_\psi)$ and $v_\psi(u)$ are defined in formula (B.2) and [2, Definition 5.21] respectively. $\square$

Corollary 5.25′. *Keep the situation of Theorem 5.24′. For every $\eta\in H_L$, note that $i(\eta\psi)=i(\psi)$, let $\underline{\hat{M}}^\eta$ and $\omega_\psi^\eta\in\mathrm{H}^{\eta\psi}(\underline{\hat{M}}^\eta,L[\![y_{i(\psi)}-\eta\psi(y_{i(\psi)})]\!])$ be obtained by extension of scalars via η and choose an E_v-generator $u_\eta\in\mathrm{H}_{1,v}(\underline{\hat{M}}^\eta,Q_v)$. Then*

$$\tfrac{1}{\#H_L}\sum_{\eta\in H_L}v(\textstyle\int_{u_\eta}\omega_\psi^\eta)=Z_v(a^0_{E_v,\psi,\Phi},1)-\mu_{\mathrm{Art},v}(a^0_{E_v,\psi,\Phi})+\tfrac{1}{\#H_L}\sum_{\eta\in H_L}\big(v^{\sim}(\omega_\psi^\eta)+v_{\eta\psi}(u_\eta)\big).\qquad\square$$

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