EFFECTS OF MANTLE ANELASTICITY ON THE CHANDLER WOBBLE

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ABSTRACT. On the basis of the perturbation principles of the normal mode and Love numbers, the theoretical Q of the Chandler wobble is derived by assuming that the wobble energy is totally dissipated within the mantle and by using Zschau's mantle rheology model. The results show that mantle anelasticity is likely to be the most important dissipative source of the Chandler wobble energy, and the theoretical Chandler Q is 71. Finally, the parameter α of the absorption band model is calculated, and the applicability of the model is discussed as well.

1 INTRODUCTION

The quality factor Q_w of the Chandler wobble (CW) not only represents the dissipative ratio of the wobble energy, but also provides an important basis for studying physical condition of the Earth's interior and establishing an united formula of Q on different time scale. For a long time past, the estimated values of Q_w have always been obtained by analyzing astronomical data. Owing to the fact that mechanisms of the damping and excitation have not been solved, that both of them hold each other brings the determination of Q_w a special difficulty. There is a wide discrepancy between the estimated Q_w (25—600).

What can cause the energy dissipation of the CW are (1) core viscosity, (2) electromagnetic coupling between core and mantle, (3) non—equilibrium pole tide, (4) mantle anelasticity. According to the present estimation both (1) and (2) can only explain a very small part of the wobble energy dissipation. Smith et al.(1981) analyzed the effects of mantle anelasticity on the CW eigenfrequency theoretically, and estimated the parameter of the absorption band model (Anderson et al. 1979) based on

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the observed Q_w . Okubo (1982) calculated the influences of anelasticity on the CW period. As a result of the imperfections in the determination of Q, the dissipative models of the elastic modulus they use are constant in the upper and lower mantle respectively. The complex modulus in Zschau's rheological model (Zschau et al. 1985) depends on depth and frequency. The purposes of this paper are to improve the theoretical estimation of the CW parameters, and to study the main dissipative source of the wobble energy.

2 DIRECT EFFECTS OF MANTLE ANELASTICITY

The effects of mantle anelasticity can be considered by introducing small, complex perturbations $\delta\kappa$ and $\delta\mu$ in the elastic moduli κ_o and μ_o, in which the real parts change the normal mode eigenfrequency and the imaginary parts give rise to dissipation of deformational energy. If $\delta\kappa$ and $\delta\mu$ are independent of longitude and latitude, then the first order perturbation $\delta\omega$ to the eigenfrequency ω_o of the elastic Earth is given by (Smith et al. 1981)

$$\frac{2\delta\omega}{\omega_o} = \int_o^a [(\frac{\delta\kappa}{\kappa_o})\kappa_o K + (\frac{\delta\mu}{\mu_o})\mu_o M]r^2 dr \qquad (1)$$

where K and M are scalar functions of radius, a is the mean radius of the Earth. We will assume that $\delta\kappa = 0$, only consider dispersion of μ_o in actual calculation.

Let the CW period of an elastic Earth which has an equilibrium pole tide be T_o, $T_o = 426.7$ sidereal days. The corresponding Q_w in this case is infinite. After taking the effects of anelasticity into account, from $\delta\omega$ we can obtain the change of the period δT, quality factor Q_w and damping time τ_w. Using $\delta\mu$ of the anelastic models 1066A–Zschau and PREM–Zschau respectively, which can be obtained by substituting stratified μ_o of models 1066A and PREM into Zschau's rheological model, we estimate the effects of anelasticity on the CW parameters. The results are shown in Table 1. It can be seen from Table 1 that compared with the seismic frequency band, a rigidity decrease of the mantle's interior at the CW frequency due to dispersion leads to the CW period to be lengthened by 9.0–10.0 days. It is about 2 per cent of the observed T_W. This agrees rather well with the result of 7−11 days estimated by Okubo. If the CW energy is totally dissipated within the mantle and the strain due to the pole tide is ignored, the theoretical Q_W is 68−77.

TABLE 1. Effects of mantle anelasticity on the CW parameters

	Model 1066A−Zschau	Model PREM−Zschau
$\delta\omega(10^{-5}\Omega)$	−5.37839+i1.85189	−4.86272+i1.63870
δT(day)	10.0	9.0
Q_W	68	77
τ_W (year)	25.2	28.5

3 INDIRECT EFFECT OF MANTLE ANELASTICITY

The variation in centrifugal potential associated with the wobble is

$$U = \sqrt{\frac{8\pi}{15}}\Omega^2 a^2 (m_1 Y_{21}^c + m_2 Y_{21}^s) = U_1 Y_{21}^c + U_2 Y_{21}^s \tag{2}$$

where m_1 and m_2 are the equatorial components of the position of the rotation axis, Y_{21}^c and Y_{21}^s are the real harmonics of degree 2, order 1. The equilibrium pole tide can be written as

$$H = \{\frac{\gamma}{g}(U_1 Y_{21}^c + U_2 Y_{21}^s) + \sum_{n=0}^{\infty}\frac{3}{2n+1}\frac{\rho_w}{\rho}\gamma'_n [H_{no} Y_{no} + \sum_{m=1}^{n}(H_{nm}^c Y_{nm}^c + H_{nm}^s Y_{nm}^s)] + c'\}\eta \tag{3}$$

where ρ and ρ_w are the mean density of the Earth and the ocean, $\gamma = 1 + k_2 - h_2$ and $\gamma'_n = 1 + k'_n - h'_n$ are the combinations of the Love numbers and loading Love numbers separately, c' is an arbitrary constant, η is the ocean function. If cross−coupling of different degree n and order m in the tidal height expansion is ignored, from (3) we have

$$\begin{bmatrix} H_{21}^c \\ H_{21}^s \end{bmatrix} = \frac{1}{g}\mathbf{W}\cdot\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{4}$$

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where the coefficient matrix **W**, which describes the response of the equilibrium ocean to the wobble, is

$$\mathbf{W} = \gamma[\mathbf{I} - \alpha_o \gamma' \mathbf{X}]^{-1} \cdot \mathbf{X} \tag{5}$$

Here $\alpha_o = 3\rho_w / (5\rho)$, $\gamma' = \gamma'_2$, **I** is the unit matrix, **X** is the matrix related to the ocean function. If the additional effects of the gravitational self–attraction and tidal loading of the ocean are neglected, Eq. (5) reduces to $\mathbf{W} = \gamma \mathbf{X}$.

Let $\delta\gamma$ and $\delta\gamma'$ be the perturbations in γ and γ' due to anelasticity. Using models PREM and PREM–Zschau yields $\delta\gamma = -0.0089 + i0.0026$ and $\delta\gamma' = -0.0163 - i0.0047$. The perturbation in **W** due to $\delta\gamma$ and $\delta\gamma'$ is given by

$$\delta\mathbf{W} = \frac{\delta\gamma}{\gamma}\mathbf{W} + [L\mathbf{I} + \frac{\alpha_o}{\gamma}(Tr\mathbf{W})\mathbf{W}]\delta\gamma' \tag{6}$$

Here Tr**W** is the trace of the matrix **W**, and $L = -\alpha_o \gamma |\mathbf{X}| / |\mathbf{D}|$ where $|\mathbf{X}|$ and $|\mathbf{D}|$ are the determinants of the matrices **X** and $(\mathbf{I} - \alpha_o \gamma' \mathbf{X})$.

We suppose that the equilibrium pole tide of the elastic Earth is $H = H_o exp(i\omega_o t)$. As to the anelastic Earth, it can be expressed as $H' = \Gamma H_o exp[i(\omega_o t + \beta)]$. By employing the ocean function coefficients (Balmino et al. 1973), we obtain $\Gamma = 0.986$ and $\beta = 0^\circ.2$.

Let $\mathbf{P} = \alpha_o(1 + k'_2)\mathbf{W}$ denote the matrix related to the perturbation in the inertia tensor caused by the equilibrium pole tide. Noting $k'_2 = k_2 - h_2$, we can also derive the perturbation in **P** due to $\delta\gamma$ and $\delta\gamma'$, i.e.

$$\delta\mathbf{P} = 2\frac{\delta\gamma}{\gamma}\mathbf{P} + [\alpha_o \gamma L\mathbf{I} + \frac{1}{\gamma^2}(Tr\mathbf{P})\mathbf{P}]\delta\gamma' \tag{7}$$

Then the perturbation $\delta(\triangle\omega)$ in the eigenfrequency can be approximately written as

$$\delta(\triangle\omega) = -\frac{1}{2}Tr(\delta\mathbf{P})\frac{\Omega^2 a^5}{3GA_M}\Omega \tag{8}$$

where G is the gravitational constant, and A_M is the equatorial moment of inertia of the mantle. The indirect effect of anelasticity on the CW period and Q are $\delta(\triangle T) = -0.9$ day and $\delta(Q_w) = 913$ separately. We have also performed a simple estimate in which the cross–coupling effect of different n and m is included. The result indicates that it is too small to affect our calculation.

Carton et al. (1986) showed that the effects of non–equilibrium part of the pole tide on the CW period and Q were 0.04–0.38 day and 1300–11000. We can conclude that the effects of the non–equilibrium pole tide on T_w and Q_w are much smaller than those of the anelastic perturbation of the equilibrium pole tide, and mantle anelasticity is likely to be the most important dissipative source of the CW energy.

The final estimate of the theoretical CW period, if we note that the indirect effect is to shorten the period, is $T_w = 434.8$ days. The quality factor corresponds to $Q_w = 71$ because both anelastic effects induce the wobble erengy dissipation. We believe this to be a better estimate of the CW parameters.

4 ESTIMATE OF PARAMETER α

If frequencies ω_1 and ω are located within the same absorption band, the relation between Q and frequency can be described as

$$Q(\omega) / Q_1(\omega_1) = (\omega / \omega_1)^{\alpha} \tag{9}$$

where ω_1 is a reference frequency in the seismic frequency band, we take $\omega_1 = 2\pi / 200\text{s}$. α is an undetermined constant. Using complex modulus evaluated based on Zschau's rheological model, we estimate many α values which correspond to these frequencies from $\omega = 10^{-3} / s$ to $10^{-15} / s$. The results show that all frequencies in the $\omega \geqslant 10^{-9} / s$ regime are basically located within the same absorption band, and in the band the variation of Q with frequency can be approximately described by numerically using a single parameter $\alpha = 0.21$. This is because the effect of viscosity is smaller in the high–frequency domain, and the response of the Earth to applied forces can be considered essentially as an elastic one. From $\omega < 10^{-9} / s$, dispersion increases obviously with decreasing frequency, there is a rapid change in α, and the absorption band model of a single parameter is no longer suitable in this case.

5 CONCLUSIONS

Mantle anelasticity makes the equilibrium pole tide decrease by 1.4 per cent and set up a small phase advance of $\beta = 0.^\circ 2$, its effect on the CW is much larger than that of the non–equilibrium part of the pole tide.

The anelastic effects lengthen the CW period by 8.1 days, the theoretical T_W and Q_W are 434.8 sidereal days and 71 respectively for an elliptical and rotating Earth which has an anelastic mantle, a fluid core and an equilibrium pole tide. This is in good agreement with most of the astronomical observation.

Mantle anelasticity is likely to be the most important dissipative source of the CW energy.

The variation of Q with frequency can be approximately expressed by using the absorption band model with $\alpha = 0.21$ in the regime from the seismic frequency band to $\omega \geqslant 10^{-9}/s$. In the case of $\omega < 10^{-9}/s$, the absorption band model of a single parameter is no longer suitable.

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