

# The effect of secular resonances on the long-term orbital evolution of uncontrolled near-Earth objects

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**Abstract.** A set of 29 low order secular resonance relations in the near-Earth satellite dynamics has been obtained and their influence on dynamic evolution of objects has been investigated on time intervals of 100 years. The analysis of the results shows that secular resonances are very common phenomena in the near-Earth orbital space. Sharp resonances are concentrated in the orbital space areas where the semi-major axes  $a \geq 20000$  km and the inclinations  $i \geq 50^\circ$ . In some areas objects have several secular resonances. The superposition of several secular resonances or orbital and secular resonances are the source of randomness in the motion of objects.

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To study the effect of secular resonances on uncontrolled near-Earth objects we use a numerical-analytical approach consisting in (Bordovitsyna *et al.* 2012):

- identifying the secular resonances based on an analytical method on condition that the third body orbit is an ellipse with revolving nodal and apsidal lines;
- numerical modeling the long-term orbital evolution using the software package Numerical Model of Motion of Satellite Systems (Bordovitsyna *et al.* 2009) realized in a parallel programming environment at the Tomsk State University computing cluster;
- MEGNO-analysis of dynamic evolution of objects (Cincotta *et al.* 2003).

A complete set of 29 low order resonance relations has been presented in Bordovitsyna *et al.* (2014). This set may be considered as a small modification of the resonance relation list obtained by Cook (1962) and by Hughes (1980, 1981).

The long-term orbital evolution of uncontrolled satellites of systems GLONASS, GPS and BEIDOU IGSO have been simulated numerically using the mentioned software package. It was shown that the superposition of several sharp secular resonances or proximity of the object's mean motion to the tesseral resonance in the presence of a sharp secular resonance are the source of randomness in the motion of objects. This effect appears after the eccentricity increases up to 0.6. See details in (Bordovitsyna *et al.* 2014).

To study the distribution of secular lunisolar resonances in the near-Earth orbital space and their effect on the uncontrolled object dynamics, an extensive numerical experiment has been carried out. Numerical values of 29 secular resonance relations have been calculated for the following collections of eccentricity ( $e$ ), inclination ( $i$ ) and semi-major axis ( $a$ ):

$$e = 0.01 \text{ and } 0.1 - 0.9 \text{ with step } 0.1; \quad i = 0^\circ - 90^\circ \text{ with step } 5^\circ; \\ a = 8000 \text{ km} - 55000 \text{ km with step } 1 \text{ km.}$$

The influence of secular resonances on the dynamic evolution of 200 model near-Earth space objects has been investigated on the time interval of 100 years. Objects which go through (1) one or several secular resonances, (2) tesseral and secular resonances simultaneously and (3) have no resonances have been considered. The analysis of the

obtained results allows the following preliminary conclusions (Bordovitsyna & Tomilova 2014 a, b).

– Secular resonances are very common phenomena in the near-Earth orbital space, while sharp resonances are concentrated in the orbital space areas, where  $a \geq 20000$  km and  $i \geq 50^\circ$ .

– Secular resonances where the value of resonance relation  $\leq 10^{-8}$  rad/s affect the orbital evolution of near-Earth objects distinctly.

– Secular resonances with the mean motion of the Sun are more significant than the similar resonances with the mean motion of the Moon, and this has good agreement with Breiter (1999).

– Secular lunisolar resonances produce growth of  $e$  (for any  $i$ ). This conclusion has good agreement with other authors results (e.g., Chao & Gick 2004; Rossi 2008). Even a single sharp resonance can transform a circular orbit into a high-eccentric one.

– The superposition of several sharp secular resonances or proximity of the object's mean motion to the tesseral resonance in the presence of a sharp secular resonance are the source of randomness in the motion of objects.

– Objects from the region of the orbital space with semi-major axes 40000 km and higher and inclinations more than  $55^\circ$  are subject to the simultaneous influence of a large number of secular resonances; the motion of such objects is irregular and shows longperiod oscillations of the eccentricity and the inclination with large amplitudes; the chaotization of the motion may be considerable and the velocity of the chaotization increases with growth of the initial eccentricity.

– The evolution of the near-polar orbits with inclinations  $80^\circ$  and  $90^\circ$  is very complicated but degrees of the orbital destruction can be various (Bordovitsyna & Tomilova 2014 b). For example, the picture of long-time evolution has a catastrophic character for near-circular orbits with  $a = 55\,000$  km and  $i = 90^\circ$ . The lifetime of a model object, which goes through 12 secular resonances, is only 40 years, because the orbit eccentricity tends to unit, the change of inclination leads the object away on orbits with the reverse motion, and the semi-major axis rapidly decreases. The chaotization of the motion takes place with rapidly growing MEGNO parameter.

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