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Analysis of functional relationships: old and new models

Than Pe

This thesis consists of two parts; the first part deals with model I of the functional relationship where the errors follow normal distribution and the model is additive whereas the second part deals with model II, the multiplicative model and the errors are assumed to follow other distributions from the exponential family where the variables are positive. In particular, we have emphasised the negative exponential and the gamma distributions.

In Chapter I we define the models and the literature on the analysis of model I is reviewed. Model II is new and hence only the work related to the problem is mentioned wherever necessary.

Chapter II is continuation and extension of the work done by Williams [4]. Test of correlation is applied as an indirect way of estimation. Here the correlation between sufficient and ancillary statistics for the incidental parameters is tested. Different definitions of the sufficient and ancillary statistics in the case with incidental parameters are given in Section 2.1. We also looked at the ancillary statistics from the information theory point of view there.

The author has extended the problem for the multivariate case, that is, test of correlation is applied to estimate $p - 1$ structural parameters in $p - 1$ linear relations among p variates.

In Section 2.3, the power of the test of correlation is investigated and we have found the non-null distributions of the test statistics in both two-dimensional and p-dimensional cases. To the author's knowledge, these

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distributions have not been found before.

Tests alternative to the test of correlation are given in Section 2.4. In this section, a conventional test is applied in testing the mean of the ancillary statistic for the incidental parameter. We also applied Singh's approach [2] of using the sum of squares in testing the mean with some modification.

Analysis of model II is done in Chapters III and IV. In Section 3.1, we have applied the classical methods of estimation to estimate the structural parameter under different conditions and we have found that they are not satisfactory. In Section 3.2, we applied the conditional maximum likelihood method and also derived new test statistics to be used as an alternative way of estimation. The application of the test of correlation to the present case is described in Section 3.3. Here we have derived the distribution of the statistics when they are neither sufficient nor ancillary for the incidental parameters. Since there exists no parametric test of correlation for the non-normal case we used the usual F-test and investigated the power of the test by simulation for the exponential and gamma cases. A non-parametric method has been proposed, as an alternative way of estimation, in Section 3.4.

The use of instrumental variables in estimating the parameters has been studied in Chapter 4. The methods of grouping introduced by Wald and Bartlett are applied to our problem and the conditions for the consistency of the estimator and simulated results are given. The conditions for the consistency involve unknown incidental parameters; consequently we suggested the use of the ratio of sample means as a natural estimator and the distribution of the estimator is also investigated in Section 4.2.

In Section 4.3 we have used the method of ranking to the observations and the ratio of the order observations is obtained. Then we proposed the median of the ratios as an estimator which is known to be consistent from the simulated results which are also given.

References

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