NONLINEAR CALCULATIONS FOR BL HER STARS

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I. INTRODUCTION

BL Her Stars are primarily Population II Cepheids with periods between one and three days. On the H-R diagram they are about 1^m brighter than the RR Lyrae variables but below the W Virginis variables. They are not as blue as the so-called anomalous Cepheids. Typically these stars are metal deficient, but notable exceptions are the prototype BL Her and V553 Cen with solar type abundances (Smith, et al. 1978). Their evolutionary status given by Sweigart and Gross (1976) puts them on the immediate post-horizontal-branch stage with energy supplied by both He and H burning shells.

BL Herculis stars are of particular interest because they are frequently observed to have light curve bumps on increasing or decreasing light (Stobie 1973). These bumps appear to follow a Hertsprung (1926) progression much like the more massive classical bump Cepheids. Consequently, they may have the same mechanism, either echos from the stellar center (Christy 1968), or a near resonance between the fundamental (period Π_0) and second overtone (period Π_2) radial pulsation modes (Simon and Schmidt 1976) where $\Pi_2/\Pi_0 \cong 0.50 \pm 0.03$. This resonance hypothesis based on nonlinear calculations of Stobie (1969a,b) predicts bumps before maximum light for $\Pi_2/\Pi_0 < 0.50$, and bumps after maximum light for $\Pi_2/\Pi_0 > 0.50$.

II. LINEAR MODELS

We have calculated linear periods and period ratios (Π_2/Π_0) from static models based on linear nonadiabatic pulsation theory (Castor 1971). Results for the King Ia composition ($X = 0.700$ $Z = 0.001$) are plotted on the H-R diagram (Fig.1). The instability region for fundamental mode pulsation is bounded on the left (blue) side for

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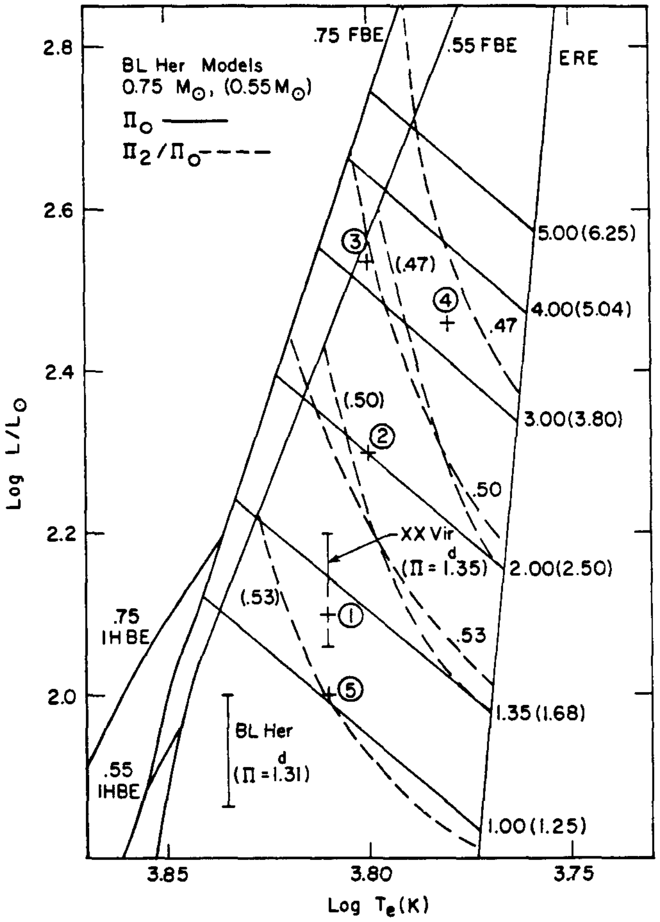

Fig. 1

0.75 $M_\odot$ (.75FBE) and 0.55 $M_\odot$ (.55FBE) and on the right (red) side by the estimated red edge (ERE) where convection carries 50% of the total luminosity in the H ionization region.

Lines of constant Π_0 are indicated by solid lines with values given on the right in days for both masses. (0.55 $M_\odot$ in parenthesis). Lines of constant Π_2/Π_0 are given by curved dashed lines. The curvature is due to the fact that lines of constant Π_2 are less steep than those of constant Π_0, and they converge near the blue edge. Also plotted are the locations of XX Vir and the BL Her according to Smith et al. (1978). BL Her, with a bump on descending light falls outside the resonance band, but with reasonable changes in L and T_e, this star

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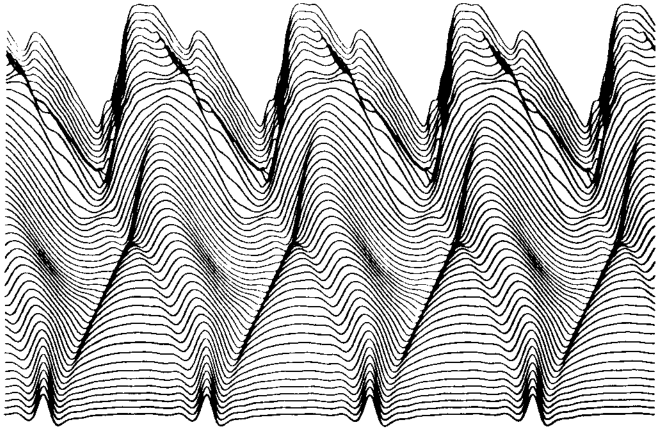
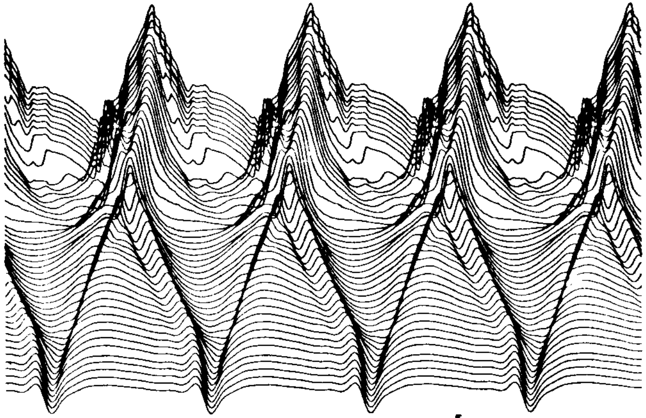


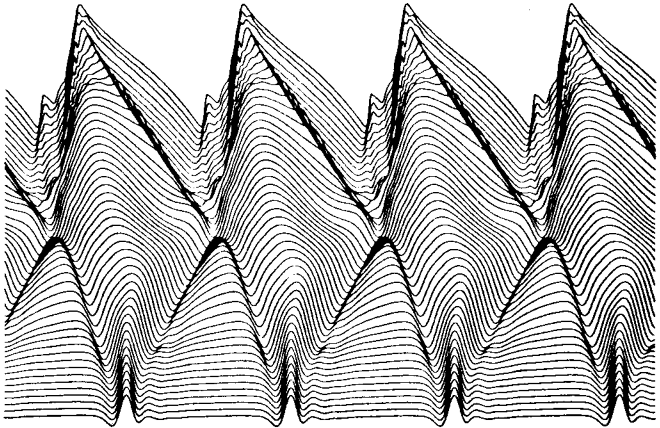
Radial Velocity vs Time (5.42×10^5sec)
V(cm/sec) ib=-2.72×10^2,5.11×10^2 ob=-3.68×10^6,2.55×10^6

Fig. 2



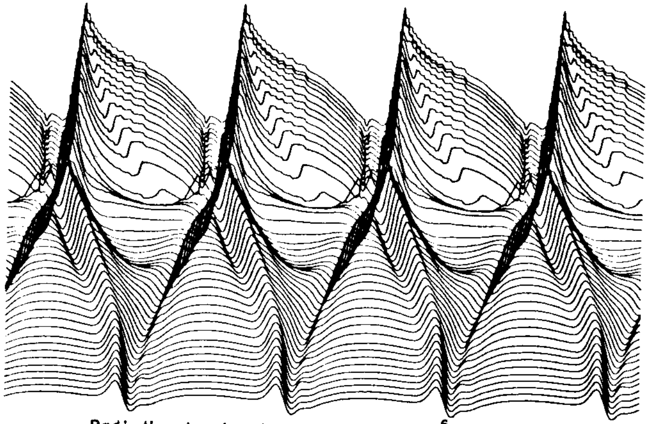
Radiative Luminosity vs Time (5.42×10^5sec)
L(erg/sec) ib=4.91×10^{35} 4.91×10^{35} ob=2.70×10^{35},8.44×10^{35}
BL Her $0.55M_\odot$ 6450K $126L_\odot$ F 50 zones Kingla $\Pi_2/\Pi_0=0.521$

Fig. 3



Radial Velocity vs Time (1.18×10^6sec)
V(cm/sec) ib=-3.53×10^2,7.68×10^2 ob=-4.82×10^6,5.07×10^6

Fig. 4



Radiative Luminosity vs Time (1.18×10^6sec)
L(erg/sec) ib=1.12×10^{36} 1.12×10^{36} ob=4.25×10^{35},2.20×10^{36}
BL Her $0.75M_\odot$ 6025K $288L_\odot$ F 50 zones Kingla $\Pi_2/\Pi_0=0.482$

Fig. 5

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can fall on its $\Pi_0 = 1.3$ line and in the 0.55 $M_\odot$ resonance band with the correct bump phase. XX Vir, with no observed bump, probably has a mass approaching 0.75 $M_\odot$ to be at a luminosity where $\Pi_2/\Pi_0 > 0.53$.

III. NON-LINEAR MODELS

In Fig. 1, the lines of constant Π_2/Π_0 map out a region around 0.5 for both masses where bumps should be expected. Our nonlinear full amplitude models 1 & 3 (0.55 $M_\odot$), 2 & 4 (0.75 $M_\odot$), indicated by crosses and adjacent circled numbers, were chosen to get bumps on both sides of maximum light. Case 5 was reconstructed from model 1 of a Carson, Stothers, and Vemury (1980) preprint to compare with results using a new Carson opacity table with the Los Alamos opacities.

Figures 2 & 3 give plots of radial velocity vs. time and luminosity vs. time for each of the 50 Langrangian zones in our model 1 with $\Pi_2/\Pi_0 = 0.521$. As predicted we get bumps on descending velocity and descending light. One can follow the primary shock reflecting off the core to the surface as a bump, supporting the echo mechanism as well as a resonance. Figures 4 & 5 give the same plots for model 4 at 0.75 $M_\odot$ with $\Pi_2/\Pi_0 = 0.482$, with the bump on rising velocity and rising light. Cases 2 & 3 also show bumps at the appropriate phase (Ap. J., 1981, in press).

Table 1 gives a basic summary of model parameters and results. In column 7, the difference in velocity and light bump phase $\leqslant 4\%$ for all cases, regardless of the phase difference between velocity and light maxima (Col. 8).

TABLE 1
Nonlinear Models
No Convection
King1a

No.	$M/M_\odot$	$T_e(K)$	$L/L_\odot$	$\Pi_0(d)$	Π_2/Π_0	$\Delta\Phi^b_{v-l}$	$\Delta\Phi^p_{v-l}$	Total Ampl (km/s)
1	0.55	6450	126	1.55	0.521	−0.04	−0.09	56
2	.75	6310	200	2.00	.520	−0.04	−0.12	57
3	.55	6310	343	4.04	.480	0.00	+0.09	25
4	.75	6025	288	3.28	.482	−0.02	0.00	82
				$X = 0.745$	$Z = 0.005$			
5a	.60	6457	100	1.22	.531	–	−0.01	66
5b	0.60	6457	100	1.22	0.536	–	+0.01	49

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Model 5a and 5b, using Carson and Los Alamos (Huebner et al. 1977) opacities respectively, show bumps only in the first few surface layers. They are probably not echoes or a resonance. Model 5a has an amplitude almost 30% more than 5b, due to a greater emphasis of bound-free He absorption at ~4000 K resulting in more driving and a larger surface shock phenomenon. From the first 4 cases, it is apparent from the large range of total amplitudes (uncorrected for projection for an observer, 25-82 km/s) that the bumps seen in these models do not depend greatly on amplitude.

IV. CONCLUSIONS

From our linear and nonlinear calculations it is evident that bumps occur in the velocity and light curves when there is a near resonance between Π_0 and Π_2. In addition, these resonance bumps do not seem to depend on the limit cycle amplitude, and cannot be attributed to the surface shock encountered when models are driven to excessively high amplitudes.

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DISCUSSION

SIMON: I want to reiterate. The business of bumps is not a productive way to talk about things. Bumps depend upon the person, whereas the Hertzsprung sequence does not. Whether you see bumps or not is not a crucial thing. In a catalogue you can see two cases with periods very close together with one showing bumps the way the artist has drawn them and the other showing nothing. Fourier components of these light curves are virtually the same. If you try to plot the Hertzsprung sequence as a function of amplitude, you get hash. Whether there is a shoulder or dip is a very unproductive way of looking at things.

HODSON: I didn't go into the observations. It is terrible trying to determine where the bump is. We need better observations.

SIMON: The Carson models are perfectly good, I believe.

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HODSON: If you drive anything to a high enough amplitude, you will get bumps.

SIMON: I'm saying no. If you Fourier analyze these models, and I've done this for classical Cepheids, you can see that they are correct.

LUB: I suggest you use only the modern data.

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