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Distributivity in semilattices

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There are various non-equivalent notions of distributivity in semilattices. The algebra of these notions is discussed and compared. Semilattices are not considered as algebras with one binary meet operation but rather as partial algebras with a binary meet operation and a partial join operation of varying type.

The concepts of ideal system and join partial congruence are central to the work. Examples of ideal systems are listed and some consequences of an ideal system being distributive are given, the most important of these being that the finitely generated ideals form a distributive lattice, and a version of the Prime Ideal Theorem.

A semilattice is called weakly distributive if meets distribute over arbitrary finite joins. The results of this section are from a joint paper written by the author and his supervisor, W.H. Cornish [*1*]. In particular it is seen that weakly distributive semilattices can be characterized in terms of the distributivity of ω-ideals, and as a consequence the ω-free distributive extension of a weakly distributive semilattice is obtained. A semilattice congruence which preserves arbitrary finite joins is called ω-join partial, and the smallest such congruence which identifies two comparable elements of a weakly distributive semilattice is described.

A semilattice is called m-distributive if meets distribute over m-element joins. Results similar to those for weakly distributive semilattices are given for m-distributive semilattices, m-ideals, and m-join partial congruences.

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The term "n-distributive" is used to describe a semilattice which is m-distributive or weakly distributive. There are four equivalent conditions for the n-join partial congruences on an n-distributive semilattice to be the restriction of the lattice congruences on its n-free distributive extension. The most important is that the lattice of n-join partial congruences is distributive. Some necessary and some sufficient conditions for this to happen are presented.

The restriction of the n-join partial congruences on an n-distributive semilattice to a principal filter of the semilattice is a lattice homomorphism provided the semilattice satisfies a certain connectivity condition. To obtain a partial converse, a method of constructing weakly distributive semilattices is described, and by way of contrast, semilattices which are m-but-not-m+1-distributive are shown to be complicated and difficult to construct.

The smallest possible ideal system on a semilattice is the set of all strong ideals. For the class of mildly distributive semilattices, those in which the ideal system of strong ideals is distributive, there is a stronger link between ideals and filters than for the other classes so far discussed.

Semilattices with the upper bound property, that is those semilattices in which each pair of elements with a common upper bound have a supremum, display better properties than those expected of a partial algebra. This is explained by defining a suitable ternary operation which turns this class of semilattices into a congruence distributive variety. An investigation into its subvarieties is commenced and various Mal'cev-type conditions are found to hold.

Reference

[1] W.H. Cornish and R.C. Hickman, "Weakly distributive semilattices", *Acta Math. Acad. Sci. Hungar.* 32 (1978), 5-16.