5.4 ORBITAL EVOLUTION OF CIRCUM-SOLAR DUST GRAINS

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Abstract. The orbital evolution of circum-solar dust grains is obtained by numerical integration of the equations of motion which includes the grains' interactions with the solar radiation field and the solar wind. Our past solution (Lamy, 1974) is improved by avoiding a classical approximation for the Poynting-Robertson term and leads to an important revision of the orbital behaviour. Results are presented for obsidian grains whose inward spiraling is stopped by the effect of sublimation.

1. INTRODUCTION

The dynamics of circum-solar dust grains which experience sublimation was first investigated by Lamy (1974), hereafter referred as Paper I, who gave a qualitative description of the orbital behaviour of grains of silicates (obsidian and andesite) and of iron. Independently, Mukai et al. (1974) considered the case of obsidian and graphite. The trajectories were obtained by numerical integrations showing that the inward spiraling of interplanetary grains under the Poynting-Robertson and corpuscular drags are either reduced or counterbalanced by the effect of the net increase of the radiation force caused by the decrease of the grain's radii when sublimating. We show here that the classical approximation for the Poynting-Robertson drag introduced by Wyatt and Whipple (1950)-in which the cross-sections for absorption and radiation pressure are taken to be equal - completely breaks down for sufficiently small grains. We reconsider the case of obsidian to illustrate this point. Our solution for the interaction with the solar radiation field and the solar wind and the corresponding approximations are the same as in Paper I except for the temperature distribution which has been improved. The reader is referred to this article for further details.

2. EQUATIONS OF MOTION

The interaction with the solar radiation field is responsible for the radiation pressure force (with the associated Poynting-Robertson drag) and the thermal equilibrium of grains. The corresponding equations are given in Paper I but the numerical solution was improved. As a consequence, we found the temperature distribution law (in °K)

$$T_g(R) = \exp\left[7.389\ (\mathrm{Ln}\ R)^{-\ 0.1945}\right]$$

with R expressed in solar radii, to be best suited to obsidian grains of radius $s \cong 1$ μm close to the Sun. The temperature controls the sublimation rate as

$$\frac{ds}{dt}\ (\mu m/sec) = 408\ \frac{p}{\delta}\ \sqrt{67/T_g}$$

via the vapor pressure p, expressed in tor and given by Centolanzi and Chapman (1966):

$$\log p = 10.915 - 24928.3/T_g$$

$\delta = 2.37$ gm/cm^3 is the density of obsidian.

The direct impacts of solar wind protons and α-particles onto interplanetary grains give rise to a force

$$\vec{F} = \pi\ s^2\ |\vec{w} - \vec{V}|\ (\vec{w} - \vec{V})\ \sum_{p,\alpha}\ m_i\ n_i$$

(Baines et al., 1965) whose tangential component F_f (the corpuscular drag), alone, is of importance. Here, $\vec{w}$ denotes the solar wind velocity, $\vec{V}$ the grain's orbital velocity and m_i and n_i, the mass and number density of each particle species. The plasma or Coulomb drag was neglected as in Paper I but the sputtering was included since it was found that it is not negligible (Lamy, 1975). Possible electromagnetic intereactions were not considered.

r and f being the usual polar coordinates, the equations of motion can be written:

$$\ddot{r} - r\ \dot{f}^{\ 2} = -\frac{GM_\odot}{r^2} + \frac{3}{4\ s\ \delta\ c}\ \frac{\Omega}{\pi}\ I_p$$

$$- \frac{3}{4\ s\ \ \ c}\ \frac{\dot{r}}{c}\ \frac{\Omega}{\pi}\ (I_p + I_a)$$

$$\frac{1}{r}\ \frac{d}{dt}\ (r^2\ \dot{f}\ ) = -\frac{3}{4\ s\ \delta}\ \frac{r\ \dot{f}}{c^2}\ \frac{\Omega}{\pi}\ I_a + \frac{F_f}{\pi\ s^2}$$

$$\frac{ds}{dt} = \dot{s}_{\ sublimation} + \dot{s}_{\ sputtering}$$

Ω is the solid angle subtented by the sun at the location of the grain, c is the velocity of light and I_p et I_a are two integrals:

$$I_p = \int Q_{pr}(\lambda)\ F_{\odot}(\lambda)\ d\lambda$$

$$I_a = \int Q_{abs}(\lambda)\ F_{\odot}(\lambda)\ d\lambda$$

where the Q's are the efficiency factors for radiation pressure and absorption as given by Mie theory and $F_{\odot}(\lambda)$ is the monochromatic emissive power of the sun. Following the treatment of Wyatt and Whipple (1950), these two integrals have been often taken equal in the litterature, thus implying that $Q_{pr}(\lambda) = Q_{abs}(\lambda)$. This has already been pointed out by Mukai et al. (1974) who used the correct equations. In order to show that the above approximation is invalid for micronic grains, the two integrals are plotted over a large range of values of s (Fig. 1).

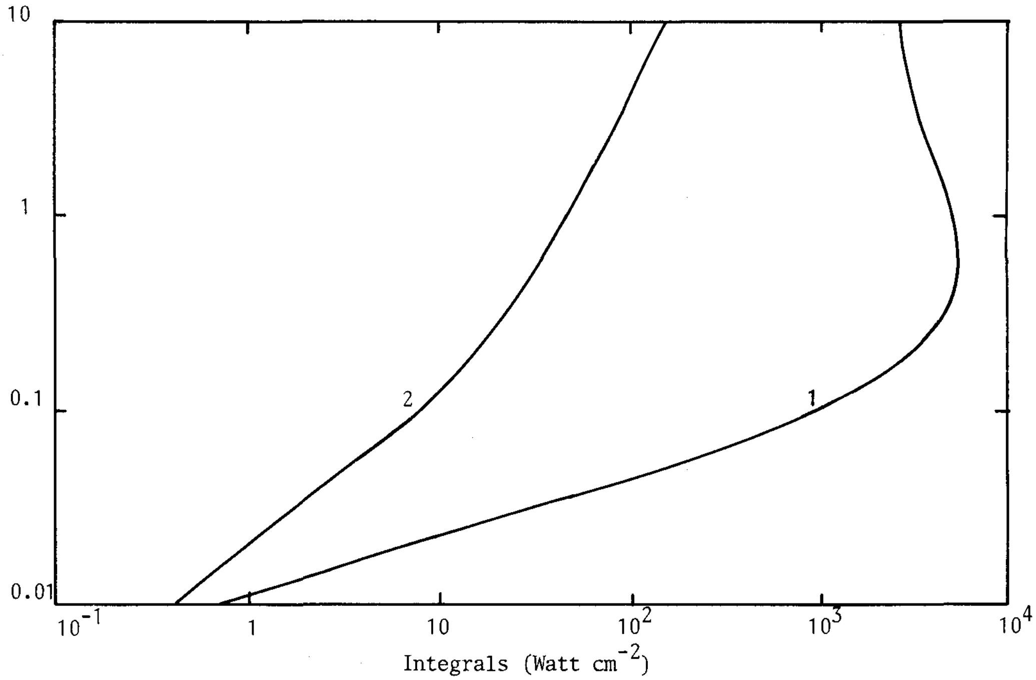

Fig. 1. The integrals for radiation pressure (1) and absorption (2)

For s ≃ 1 μm, the difference reaches two order of magnitudes. Although this does not appear in the above equations, the Poynting-Robertson terms were corrected according to Guess (1968) to take into account the solid angle effect of the sun.

After eliminating the time and switching to the true anomaly as independent variable, the equations were numerically integrated using the Runge-Kutta fourth order method. The program contained a model of the solar wind derived from the measurements of Koutchmy (1972). The integrals I_p and I_a were computed by linear interpolation between two of ten points on a log-log scale. In this present study, the initial conditions include circular orbits only.

3. RESULTS AND DISCUSSION

A simultaneous plot of the aphelion and perihelion distances and of the grain's radius versus the number of revolution as already used by Lamy (1975) was found to best illustrate the orbital evolution.

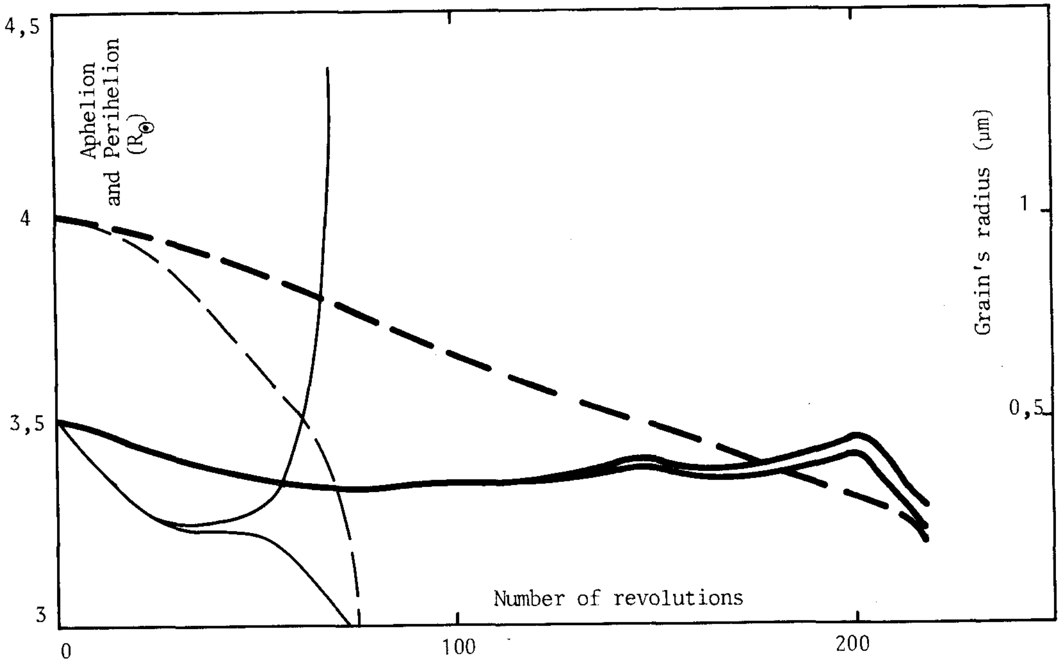

Fig. 2. Orbital evolution of an obsidian grain (s_o = 1μm) starting from an initially circular orbit, 3.5 $R_\odot$ in radius: aphelion and perihelion distances (solid lines) and the grain's (broken line) versus the number of revolution. The Thin lines correspond to the approximation $Q_{abs} = Q_{pr}$ for the Poynting-Robertson drag, and the thick lines, to the exact solution.

Figure 2 shows the result for an obsidian grain of initial radius 1 μm starting on a circular orbit of radius 3.5 $R_\odot$. It is in complete agreement with the theoretical study based on perturbation theory given in Paper I: sublimation stops the inward spiraling and causes the orbit to expand, oscillate and to become slightly elliptic. As stated in Paper I, grains with radius smaller than 0.2 μm have such a short lifetime that they play a negligible role. The solution based on the Wyatt and Whipple approximation is presented on the same graph; since the Poynting-Robertson drag is overestimated, it controls much strongly the orbital evolution - which is drastically different - and shortens the lifetime of the grain. As a final example, the new result for an obsidian grain of initial radius 0.5 μm starting on a circular orbit of radius 3.5 $R_\odot$ is presented in figure 3.

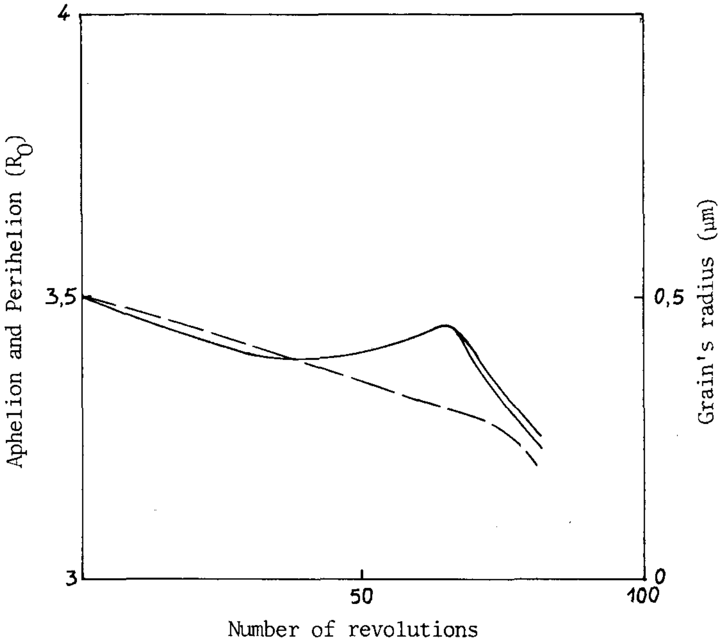

Fig. 3. Orbital evolution of an obisidian grain (s_o = 0.5 μm) starting from an initially circular orbit, 3.5 $R_\odot$ in radius: aphelion and perihelion distances (solid lines) and the grain's radius (broken line) versus the number of revolution.

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