P 86. Let π be a projectivity on a line in the real projective plane. Show that if a single point P has period $n > 1$ under π, then π is periodic of period n, and every non-invariant point has period n.

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We first prove the following result: Let π be a projectivity. If π^n $(n > 1)$ has n invariant points then it is the identity. For $n > 2$ the result follows directly from the fundamental theorem of projective geometry. If $n = 2$ suppose P and Q are invariant points for π^2. Then P and $\pi(P)$ are interchanged by π. Hence π is either an involution or the identity, and π^2 is the identity (by the fundamental theorem and the exchange theorem, $ABCD \barwedge BADC$).

Now let $P = P_1$ and consider the n distinct points P_i with $P_{i+1} = \pi(P_i)$. Now $P_{n+1} = P_1$, π^n has n distinct points and hence is the identity. Thus every point on the line has period $\leq n$. Suppose now there is a point Q_1 with period $m < n (m \neq 1)$. Form as above the sequence $Q_1, Q_2, \ldots, Q_m$. By the same argument every point has period $\leq m$ which is absurd.

Also solved jointly by J. E. Turner and the proposer.

P 88. Let G be a graph with n vertices and more than $k(n-k) + \binom{k}{2}$ edges. Prove that G has a subgraph G_1 each vertex of which has valence $> k$.

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