SECTION VII

COMET P/HALLEY AND FUTURE MISSIONS TO COMETS

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THE DYNAMICAL HISTORY OF COMET HALLEY

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ABSTRACT. The history of the attempts to predict the motion of comet Halley is outlined and the importance of the so-called nongravitational forces acting upon this comet is emphasized. Recent orbital work of the International Halley Watch Astrometry Network is reviewed. Comet Halley's transverse nongravitational parameter is positive and nearly constant with time suggesting that the comet is in direct rotation without precession of the spin pole. The nongravitational effects are consistent with the vaporization of water ice from the comet's nucleus and long term integrations suggest that the comet has been in its present orbit for at least 16,000 years and probably much longer.

I. THE HISTORY OF COMET HALLEY THROUGH THE 1909-1911 APPARITION

1.1. The Prediction of Future Perihelion Passage Times

Since 240 B.C., Chinese observers have documented a nearly unbroken record of scientifically useful observations of comet Halley (Ho Peng Yoke, 1964; Ho Peng Yoke and Ang Tian-Se, 1970). After the probable 240 B.C. apparition, only the 164 B.C. return went unrecorded by the Chinese and with the exception of occasional Korean and Japanese sightings, useful comet Halley observations made outside of China were virtually nonexistant for over a millennium thereafter. Beginning with the cometary observations of the Florentine physician and astronomer, Paolo Toscanelli (1397-1482), quantitative and accurate cometary positions became available throughout the West (Celoria, 1921). However, the necessary theory for representing a comet's motion was not available until the publication of Isaac Newton's PRINCIPIA in 1687. Newton (1687) outlined a semi-analytic orbit determination theory and used the comet of 1680 as an example. While Newton never applied the method to another comet, Edmond Halley began what he termed "a prodigious deal of calculation " and applied Newton's method to determine the parabolic orbits for two dozen well observed comets (Halley, 1705). Struck by the similarity in the orbital elements for the comets observed in 1531, 1607 and 1682, Halley suggested that these three apparitions were due to the same comet, and that it might be expected again in 1758. Halley's subsequent

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calculations indicated that a close Jupiter approach in 1681 would cause an increase in the length of the next period. Halley then revised his earlier prediction and suggested in a publication appearing after his death (Halley, 1759) that the comet that was to bear his name would return again in late 1758 or early 1759.

To refine Halley's prediction, Clairaut (1758) used a modified version of his analytic lunar theory to compute the perturbations on the comet's orbital period due to the effects of Jupiter and Saturn over the interval 1531-1759. Noting that calculations over the intervals 1531-1607 and 1607-1682 predicted the 1682 perihelion passage time to within one month, Clairaut stated that his mid-April 1759 prediction should be good to a similar accuracy. The actual time of perihelion passage in 1759 was March 13.1 (Unless otherwise stated, all times are given in U.T.). Beginning with Clairaut's work in 1758, all subsequent work to 1910 on the perturbed motion of comet Halley was based upon the variation of elements technique (Lagrange, 1783). The various works differed only in how many perturbing planets were included, how many orbital elements were allowed to vary, and how many times per revolution the reference ellipse was rectified by adding the perturbations in elements. Until after the 1909-1911 apparition, no attempt was made to link the observations of two or more apparitions into one orbital solution.

In anticipating the 1835 return, Damoiseau (1820) computed the perturbative effects of Jupiter, Saturn and Uranus on comet Halley over the interval 1682-1835. Since the actual time of perihelion passage in 1835 was November 16.4, Damoiseau's initial prediction of November 17.15 was remarkable. However, Damoiseau (1829) later added the perturbations due to the earth and revised his prediction to November 4.81. De Pontecoulant considered the perturbative effects of Jupiter, Saturn and Uranus over the interval 1682-1835 as well as the earth's perturbative effects near the 1759 time of perihelion passage. His predictions for the 1835 perihelion passage times were successively, November 7.5, November 13.1, November 10.8 and finally November 12.9 (de Pontecoulant 1830,1834,1835). The most complete work leading up to the 1835 return was undertaken by O.A. Rosenberger. After a complete reduction of available observations, Rosenberger recomputed an orbit for the 1759 and 1682 apparitions (Rosenberger 1830a, 1830b). Rosenberger (1834,1835) computed the effect on all the orbital elements from the perturbations of the seven known planets over the 1682-1835 interval. Assuming the comet's motion was unaffected by a resisting medium, Rosenberger's prediction for the 1835 perihelion passage time was November 12.0. Lehmann(1835) also investigated the motion of comet Halley over the 1607-1835 interval taking into account the perturbative effects of Jupiter, Saturn and Uranus. However, his perihelion passage prediction was late by more than 10 days.

In an effort to anticipate the next apparition of comet Halley, de Pontecoulant (1864) took into account the perturbative effects of Jupiter, Saturn and Uranus before predicting May 24.36, 1910 as the next time of perihelion passage. The actual time of perihelion passage turned out to be April 20.18. Cowell and Crommelin began their work with preliminary calculations to see if de Pontecoulant's prediction was approximately correct (Cowell and Crommelin 1907a, 1907b, 1907c, 1908c). Their computations used the variation-of-elements technique, included perturbations by all the planets from Venus to Neptune (except Mars) and predicted a return to perihelion on April 8.5. Cowell and Crommelin (1910) then began a new study on the comet's motion by using numerical integration whereby the perturbed rectangular coordinates are obtained directly at each time step. This time they computed the perturbations from Venus through Neptune and used a time step that varied from 2 to 256 days. They predicted a 1910 perihelion passage time of April 17.11. The 1909 recovery of the comet required that their prediction be corrected by 3 days and they then revised their work by reducing the time steps by one half, carrying an additional decimal place and correcting certain errors in the previous work (Cowell and Crommelin, 1910). Their post recovery prediction was then revised to April 17.51 and they concluded that a least 2 days of the remaining discordance was due to causes other than errors in the calculations or errors in the planetary positions and masses. We note here that the best predictions for the 1835 perihelion passage time by Rosenberger and de Pontecoulant as well as the 1910 prediction by Cowell and Crommelin were too early by 4.4, 3.5 and 2.7 days respectively. As pointed out in Section II, this is just would one would expect since none of these predictions included the effects of the so-called nongravitational forces.

1.2. The Identification of Early Comet Halley Apparitions

Until the 20th century, all attemps at identifying ancient apparitions of comet Halley were done by either determining orbits directly from the observations or by stepping back in time at roughly 76 year intervals and testing the observations with an approximate orbit of comet Halley. Pingre (1783-84) confirmed the suspicion of Halley (1705) by showing that the comet of 1456 was an earlier apparition of comet Halley. Biot (1843) pointed out that an orbit by Burckhardt (1804) for the comet of 989 closely resembled that of comet Halley and Laugier (1843,1846) correctly identified as comet Halley the comets seen by the Chinese in 451, 760 and in the Autumn of 1378. Laugier (1842) also noted that four of the five parabolic orbital elements for the comet seen in 1301 were close to those of comet Halley. By stepping backward in time at roughly 76-77 year intervals and analysing European and Chinese observations, Hind (1850) attempted to identify comet Halley apparitions from 11B.C. to 1301. Approximate perihelion passage times were often determined directly from the observations and an identification was suggested if Halley-like orbital elements could satisfy existing observations. Although many of Hind's identifications were correct, he was seriously in error for his suggested perihelion passage times in 1223,912,837,608,373 and 11 B.C.

Using a variation of elements technique, Cowell and Crommelin (1907d) began the first effort to actually integrate the comet's equations of motion backward in time. They assumed that the orbital eccentricity and inclination were constant with time and the argument of perihelion and the longitude of the ascending node changed uniformly with time - their rates being deduced from the values computed over the 1531-1910 interval. By using Hind's (1850) times of perihelion passage or by computing new values from the observations, they deduced preliminary

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values of the orbital semi-major axis for the perturbation calculations. The motion of the comet was accurately carried back to 1301 by taking into account first order perturbations in the comet's period from the effects of Venus, Earth, Jupiter, Saturn, Uranus and Neptune. Using successively more approximate perturbation methods, Cowell and Crommelin (1907d, 1908a-e) carried the motion of the comet back to 239 B.C. At this stage, their integration was in error by nearly 1.5 years in the perihelion passage time and they adopted a time of May 15, 240 B.C., not from their integration, but rather from their consideration of the observations themselves. After a complete and careful analysis of the European and Chinese observations, Kiang (1971) used the variation of elements technique to investigate the motion of comet Halley over the 240 B.C. - 1682 A.D. interval. By determining the time of perihelion passage time directly from the observations and considering the perturbations from all nine planets on the other orbital elements, Kiang traced the motion of comet Halley for nearly two millennia. Hasegawa (1979) also empirically determined perihelion passage times for comet Halley. For each apparition from 1378 to 240 B.C., he computed several ephemerides using Kiang's (1971) orbital elements, except for the perihelion passage times which were chosen to make the best fit with the observations. Attempts to represent the ancient observations of comet Halley using the numerical integration of the comet's gravitational and nongravitational accelerations are presented in the next Section II.

II. NONGRAVITATIONAL FORCES AND COMET HALLEY

Beginning with the work of Bessel (1835,1836), it became clear that the motion of comet Halley was influenced by more than the solar and planetary gravitational accelerations. Michielsen (1968) pointed out that perihelion passage time predictions that had been based upon strictly gravitational perturbation calculations required a correction of +4.4 days over the past several revolutions. Kiang (1971) determined a mean correction of +4.1 days. In an attempt to account for this 4 day discrepancy between the actual period of comet Halley and that computed using perturbations from the known planets, some unorthodox solutions have been proposed. Brady (1972) suggested the influence of a massive trans-Plutonian planet and Rasmusen (1967) adjusted the ratio of the sun:Jupiter mass ratio from the accepted value of 1047 to 1051. Both of these suggested solutions must be rejected because they would produce effects on the motion of the known planets that are not supported by observation. Rasmusen (1981) derived an 1986 perihelion date of February 5.46 from a fit to the observations in 1835 and 1910 and then added +3.96 days to yield a 1986 perihelion passage time prediction of February 9.42. Brady and Carpenter (1967) first suggested a 1986 perihelion passage time of Feb. 5.37 based upon a "trial and error" fit to the observations during the 1835 and 1910 returns. Brady and Carpenter (1971) then introduced an empirical secular term in the radial component of the comet's equations of motion. Although this device had the unrealistic effect of decreasing the solar gravity with time, it did allow an accurate 1986 perihelion passage time prediction of Feb. 9.39. It is now clear that the actual 1986 perihelion passage time (Feb. 9.44) was accurately predicted by both Rasmusen (1981) and Brady and Carpenter (1971). However if the orbit of the comet is to be accurately computed throughout a particular apparition or if the comet's motion is to be traced back to ancient times, the mathematical model used to represent the obvious nongravitational forces must be based upon a realistic physical model and not upon empirical mathematical devices.

In introducing the icy conglomerate model for a cometary nucleus, Whipple (1950,1951) recognized that comets may undergo substantial perturbations due to reactive forces or rocket-like effects acting upon the cometary nucleus itself. In an effort to accurately represent the motions of many short periodic comets, Marsden (1968,1969) began to model the nongravitational forces with a radial and transverse term in the comet's equations of motion. Marsden et al (1973) modified the nongravitational force terms to represent the vaporization flux of water ice as a function of heliocentric distance. The cometary equations of motion are written;

$$\frac{d^2\vec{r}}{dt^2} = -\mu\frac{\vec{r}}{r^3} + \frac{\partial R}{\partial\vec{r}} + A1\ g(r)\hat{r} + A2\ g(r)\ \hat{T}$$

where $g(r) = \alpha(r/r_0)^{-m}(1 + (r/r_0)^n)^{-k}$

The acceleration is given in astronomical units/(ephemeris day)^2, μ is the product of the gravitational constant and the solar mass, while R is the planetary disturbing function. The scale distance r_0 is the heliocentric distance where reradiation of solar energy begins to dominate the use of this energy for vaporizing the comet's nuclear ices. For water ice, r_0 = 2.808 AU and the normalizing constant α = 0.111262. The exponents m,n,k equal 2.15, 5.093 and 4.6142 respectively. The nongravitational acceleration is represented by a radial term, A1 g(r) and a transverse term, A2 g(r), in the equations of motion. If the comet's nucleus were not rotating, the outgassing would always be preferentially toward the sun and the resulting nongravitational acceleration would act only in the antisolar direction. However the rotation of the nucleus, coupled with a thermal lag angle (θ) between the nucleus subsolar point and the point on the nucleus where there is maximum outgassing, introduces a transverse acceleration component in either the direction of the comet's motion or contrary to it - depending upon the nucleus rotation direction. The radial unit vector ($\hat{r}$) is defined outward along the sun-comet vector, while the transverse unit vector ($\hat{T}$) is directed normal to $\hat{r}$ in the orbit plane and in the direction of the comet's motion. An acceleration component normal to the orbit plane is certainly present for most comets but its periodic nature makes detection difficult in these computations because we are solving for an average nongravitational acceleration effect over three or more apparitions. While the nongravitational acceleration term g(r) was originally established for water ice, Marsden et al (1973) have shown that if the Bond albedo in the visible range equals the infrared albedo, then the scale distance r_0 is inversely proportional to the square of the vaporization heat of the volatile substance.

Using observations of comet Halley over the 1607-1911 interval,

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Yeomans (1977) used a least squares differential correction process to solve for the six initial orbital elements and the two nongravitational parameters A1 and A2. Different values for the scaling distance were tried with the result that r_0 = 2.808 AU was the optimum input value. This suggests that the outgassing causing the nongravitational forces acting on comet Halley are consistent with the vaporization of water ice. This result is a general one for nearly all comets for which nongravitational force parameters have been determined. The positive sign for the determined value of A2 for comet Halley indicates that the comet's nucleus is rotating in a direct sense - in the same direction as the orbital motion. Yeomans (1977) integrated the motion of comet Halley back to 837, and forward to predict a perihelion passage time of 1986 Feb. 9.66.

Brady and Carpenter (1971) were the first to apply direct numerical integration to the study of comet Halley's ancient apparitions. Using their empirical secular term to represent the nongravitational effect, they initiated their integration with an orbit that was determined from the 1682 through 1911 observations and integrated the comet's motion back to 240 B.C. in one continuous run. Because their integration was tied to no observational data prior to 1682, their early perihelion dates diverged from the dates Kiang (1971) had determined directly from the Chinese observations. Using Brady and Carpenter's (1971) orbit for comet Halley, Chang (1979) integrated the comet's motion back to 1057 B.C. However, this integration was not based upon any observations prior to 1909 nor were nongravitational effects taken into account.

Yeomans and Kiang (1981) began their investigation of comet Halley's past motion with an orbit based upon the 1759, 1682 and 1607 observations and numerically integrated the comet's motion back to 1404 B.C. Planetary and nongravitational perturbations were taken into account at each half day integration step. In nine cases, the perihelion passage times calculated by Kiang (1971) from Chinese observations were redetermined and the unusually accurate observed perihelion times in 837, 374 and 141 A.D. were used to constrain the computed motion of the comet. The dynamic model, including terms for nongravitational effects, successfully represented all the existing Chinese observations of comet Halley. This model assumed the comet's nongravitational forces remained constant with time; hence it seems that the comet's spin axis has remained stable, without precessional motion, for more than two millennia. Also implied is the relative constancy, over two millennia, of comet Halley's ability to outgas. This latter result is consistent with the comet's nearly constant intrinsic brightness over roughly the same interval (Broughton, 1979). From the list of Halley's orbital elements given by Yeomans and Kiang (1981) from 1404 B.C. to 1910 A.D., one can make a crude estimate of Halley's minimum dynamic age. The heliocentric distance to the comet's descending node increased from 0.85 AU in 1910 to 1.74 AU in 1404 B.C. If this rate of increase continued back into the distant past then the comet would not have crossed the ecliptic plane near Jupiter's orbit until 14,300 B.C. If Jupiter happened to be near during this nodal crossing, then perhaps comet Halley was captured into its current orbit configuration. Hence in 1986, comet Halley will have been in its current orbit for at least 16,000 years and probably much longer.

III. RECENT ORBITAL WORK ON COMET HALLEY

The recovery of comet Halley on October 16, 1982 at Mt. Palomar showed the comet's image to be only 9 arc seconds away from the ephemeris position provided by Yeomans (Jewitt et al, 1982). At this writing there have been additional accurate astrometric positions provided by astronomers at Kitt Peak Observatory in Arizona, the Canada-France-Hawaii Telescope in Hawaii and from the European Southern Observatory at La Silla, Chile. Recovered at a distance of more than 11 AU from the sun, the comet showed no obvious activity and the initial observational accuracy is not limited by the uncertainty of the comet's center of mass within an extensive coma. The initial astrometric positions of comet Halley are generally accurate to within 1 arc second with a series of 25 positions from La Silla in late January 1984 achieving a heretofore unrealizable root mean square accuracy of less than 0.5 arc seconds in both right ascension and declination.

There are also efforts underway to improve the accuracy of the older data. Morley (1983) has used the SAO star catlog to improve upon the positions taken at Cordoba during the last apparition, West and Schwehm (1983) have remeasured some of the Heidelberg plates and Bowell (1982) has begun to measure some Lowell Observatory plates that were never used for astrometric positions before.

Within the Astrometry Network of The International Halley Watch, the computer software for cometary orbit determination has been improved somewhat. Incoming observations times in UTC are reduced to ephemeris time, the observatory's coordinates are assigned and the right ascension and declination are corrected for the small effects of elliptic aberration. Once verified and weighted, the observations are stored in reverse chronological order on the master data file for use by the orbit determination program. This latter program takes into account the comet's nongravitational perturbations, as well as the planetary perturbations at each time step. The local error allowed at each time step can be input and the time steps of the numerical integration vary to limit the local error to the input tolerance. The partial derivatives of the observables are numerically integrated along with the comet's equations of motion. To be consistent with the reference frames used by the various flight projects to comets Halley and Giacobini-Zinner, the comet's equations of motion also include general relativistic effects by means of the parameterized space-time metric of the Eddington-Robertson-Schiff formalism. Currently this program uses a batch processed, weighted least squares technique for the orbit determination. The program can store and use a priori information matrices and map covariance matrices to specified epochs. For example, the improved orbit determination program was used to establish a prediction for the 1986 perihelion passage time based upon a new fit to the data from the 1759,1835 and 1910 returns. If this program had been available prior to the comet's recovery, the predicted time of perihelion passage would have been 1986 Feb. 9.52. At this writing the most recent orbit for comet Halley is based upon 751 observations over the interval from August 21, 1835 to March 4, 1984. The weighted RMS residual is 1.94" and the orbital elements are given below;

Epoch	1986 Feb. 19.0 E.T.
Perihelion	1986 Feb. 9.43881 E.T.
q (AU)	0.5870992
e	0.9672724
w	111.84657
node	58.14397
I	162.23932
A1	0.1471
A2	0.0155

The angular elements are referred to the ecliptic plane and the mean equinox of 1950.0 and the nongravitational parameters are given in units of 10^{-8} AU/(ephemeris day)^2 .

Alternate nongravitational force models have been tried in an effort to improve upon the existing model developed by Marsden et al (1973). Gas production rates computed by Divine(1982) were evaluated at each integration step using a comet centered rocket-like thrust direction as denoted by Sekanina (1981). Thus this new model allowed for a comet outgassing at a rate that followed the visual light curve and was asymmetric with respect to perihelion. In addition the thermal lag angle (Θ), spin pole inclination(I), and the direction of the comet's subsolar point at perihelion(ϕ) were variables in the model testing procedure. The attempted solutions proved to be insensitive to input values of ϕ and although the final solutions were not completely satisfactory, the optimum values for the spin pole inclination and thermal lag angle were approximately 30 and 5 degrees respectively. No combination of the input variable values could improve upon the existing nongravitational force model of Marsden et al (1973). It seems likely that additional improvements in the solutions using this alternate nongravitational force model will have to await information on the spin pole axis orientation expected from the Halley flight projects in March 1986.

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DISCUSSION

L. Kresak: How do you explain the irregular variationsof the A1 parameter as compared with the conspicuous stability of A2?

D.K. Yeomans: Because of its secular effect in adding or subtracting orbital energy, the transverse nongravitational parameter (A2) is very well determined. On the other hand, the radial parameter (A1) often has an error nearly equal to the determined value itself.

J.A. Fernandez: The fact that you could fit the observed positions of comet Halley to your computed orbit for about 2000 years: would this indicate that random impulses - by outburst for instance - have not played a significant role in the dynamical evolution of comet Halley?

D.K. Yeomans: Yes I think that is an accurate statement. In fact we have conducted a covariance analysis in an effort to assess the effects of unmodeled, stochastic nongravitational forces upon the motion of comet Halley. For reasonable values of these stochastic effects, the long term motion of the comet is affected very little.

J. Lissauer: How can you determine that Halley's orbit did not cross the ecliptic near Jupiter for at least 16,000 years if Halley was strongly perturbed by earth in 1404 B.C.?

D.K. Yeomans: In September 1404 B.C. our calculations suggest that the computed position of comet Halley came within 0.04 AU of the earth. A similar earth close approach took place in 837 A.D. Because of the uncertain initial conditions prior to the earth close approach and the lack of ancient Chinese observations to constrain the comet's motion, we could not continue our integration back prior to 1404 B.C. However, the uniformly increasing distance of the descending node from 1910 back to 1404 B.C. gave us confidence in our extrapolation of this rate back even further in time. The comet could not have crossed the ecliptic plane at Jupiter's distance more recently than 14,000 B.C. and hence the comet must have been in its present orbit for at least 16,000 years.

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