ASTRONOMICAL CONSTANTS AS AN OBJECT OF RECENT RESEARCH

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RÉSUMÉ. — Après avoir exposé les problèmes liés à la construction d'un système de constantes astronomiques, l'auteur décrit le nouveau catalogue FK 4. Puis, il passe en revue les récents progrès dans la connaissance des orbites des planètes principales et dans la détermination de la parallaxe solaire, de la constante de l'aberration et de la constante de la précession.

ABSTRACT. — After a presentation of the problem posed by the construction of a system of astronomical constants, the author describes the new catalogue FK 4. He then reviews recent progress in the knowledge of the orbits of major planets and in the determination of solar parallax, the constant of aberration and the constant of precession.

ZUSAMMENFASSUNG. — Nach Darstellung der mit der Aufstellung eines Systems astronomischer Konstanten verbundenen Probleme beschreibt Verf. den neuen FK 4-Katalog. Sodann gibt er einen Überblick über die Fortschritte, die in letzter Zeit in der Kenntnis der Bahnen der Grossen Planeten und in der Bestimmung der Sonnenparallaxe, der Aberrationskonstanten und der Präzessionskonstanten erzielt worden sind.

Резюме. — Автор излагает сначала задачи возникающие при построении системы астрономических постоянных, а затем описывает новый каталог FK4. Он излагает последние сведения об орбитах главных планет и новейшие определения солнечного параллакса и постоянных аберрации и прецессии.

Introduction. — Astronomical measurements form the basis of theories that are intended to describe the state of matter in the universe at the present time, in the past and in the future. The quality of a set of measurements is judged by the sizes of the internal and systematic errors; the nature of the latter often cannot be recognized until a good theory is available. The quality of a theory is judged according to its ability to represent the measurements accurately; further, it is judged according to the degree to which it is consistent in itself and to how far it is compatible with the laws well-established by other measurements. I shall not deal with those " theories " which cannot be verified by measurements, including those cosmological and cosmogonic theories in which the *Astronomical Constants* vary with time as a consequence of the evolution of the planetary system or of the universe as a whole.

The conventionally adopted values of the astronomical constants resulted from measurements of maximum accuracy at the time of their adoption. The theories in which the constants appear as parameters are considered to be well-founded. One might therefore think that the ephemerides published in the international or national almanacs can serve all desired purposes, and that their verification by observation is unnecessary. From the astronomical point of view, however, the continuous comparison of the ephemerides with the observations is their most essential purpose. Discordances indicate errors in the constants as well as inadequacies in the theories. The improvement of the constants and of the theories has been a lasting object of research, since discrepancies have been found between the observations and the ephemerides that represent the theories, and will certainly continue to be found in the future.

The fact that no changes have been made in the most important astronomical constants for more than 60 years is in apparent contradiction to the declared purpose of the ephemerides. There are, however, important reasons for this conservative attitude. Analysis of the differences between computed and observed values should not be rendered difficult or even impossible by frequent changes in the foundations. The use of different ephemerides in different countries or by different groups of research workers would have a similarly unfavourable effect. I personally concur with what the late Sir Harold Spencer Jones said in Paris in 1950 : " Any astronomer who has had much to do with the discussion of old observations and who has been concerned with the application of corrections required by changes during the period covered by the observations in the adopted values of the constants of precession, nutation, and aberration, is sure to prefer that changes should not be lightly made ". The fact that Newcomb's thoroughly determined values of the solar parallax and of the constants of aberration, nutation and precession are still conventionally adopted is in full accord with this attitude.

Various opinions are held on the question as to which of the basic parameters should be designated as fundamental constants.

This question arises with the problem of the construction of a consistent system of values, which has to satisfy the existing theoretical relations precisely. Several constants will have to be obtained with the greatest accuracy from the observations as directly as possible, and these will be the primary fundamentals, while others may be derived as secondary fundamental constants in using the theoretical relations.

De Sitter's choice of primary constants is mentioned since it is, at least historically, of interest. He considered as primary the values for :

the mean radius of the Earth,

the acceleration of gravity at mean latitude,

the dynamical compression of the Earth,

two small quantities, $\varkappa$ and λ_1, depending on the internal constitution of the Earth,

the velocity of light,

the solar parallax, and

the mass of the Moon.

If one wished to set up a consistent system of constants at the present time, one would have to make a different choice in the light of the most recent findings of research, and it is certain that any choice made now would not be of lasting validity either. In this address I do not intend to go into the question of an appropriate choice of quantities as fundamental constants. Part of the address of Spencer Jones at Paris in 1950 was devoted to this question, and I assume that it will also be a subject of discussion at this conference.

The principal object of my report is recent research that may be regarded as a contribution towards an improvement of important constants. It seems appropriate first to point out certain techniques which prepared the way for progress since the last Paris conference.

New techniques. — Artificial satellites and planetary probes have made it possible to carry out a very effective improvement of certain constants. Within the past few years the external gravity field of the Earth and the Earth's flattening have been determined with an unprecedented accuracy. The coefficients J_n of the zonal harmonics in the Earth's potential have been determined to the twelfth order, and the important form factor J_2, from which the Earth's flattening can be computed, has become known with appreciable accuracy. Combined with the world-wide discussion of astro-geodetic and gravimetric data on the Earth's surface by I. D. Zhongolovich, I. Fischer and W. M. Kaula, a *World Geodetic System* could be established with more accurate values

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for the equatorial radius, the Earth's flattening and the equatorial gravity. The following table shows the new values compared with those of the *International Ellipsoid.*

	World Geodetic System.		International Ellipsoid.
R (metres)	6 378 163	± 21	6 378 388
$\frac{1}{f}$	298.24	± 0.01	297
g_e (gal)	978.0436	± 0.0012	978.0490

In a similarly effective manner the radar techniques have proved to be a powerful tool for determining the parallax of the Moon and of the Sun. As far as the parallax of the Sun is concerned I shall take this up later in more detail.

The traditional astrometric techniques for measuring the positions of celestial bodies have also been enriched by the successful use of new instruments, of which I should like to mention in particular Danjon's impersonal astrolabe for the measurement of stellar positions, and the dual-rate Moon-position camera of Markowitz.

Time, stellar positions and planetary theories. — At the last Paris conference several problems were raised whose solution appeared to be an urgent necessity. These problems concerned the definition and the measurement of time, the systematic accuracy of stellar positions and the improvement of planetary theories.

It was known that the Earth is an imperfect standard time-keeper on account of the irregularity of its rotation. With the introduction of the uniform Newtonian time known as ephemeris time and the publication of ΔT, the correction to be applied to universal time to give ephemeris time, one of the well-known deficiencies has been removed. The second of ephemeris time was formally adopted as the fundamental unit of time by the Comité International des Poids et Mesures in 1956; this means that the motion of the Earth about the Sun has replaced the rotation of the Earth as standard time-keeper.

The correction ΔT is determined most accurately from observations of the Moon which thus acquired outstanding importance for accurate time-keeping. An ephemeris of adequate precision became an urgent need. This demand was met by the *Improved Lunar Ephemeris* which was based on Brown's theory instead of his tables. It has been included since 1960 in the national ephemerides.

As a result of the rapid development of atomic frequency standards, the accuracy with which the second of ephemeris time can be determined from astronomical observations is inferior to that of the second determined from atomic frequency standards. Therefore, the introduction of a new definition of the unit of time must be expected.

The demand for precise positions and proper motions of the fundamental stars has been met — as far as the available observations permit — by the revision of the FK 3. The *Fourth Fundamental Catalogue* (FK 4) has been completed at Heidelberg, and publication has just taken place. In accordance with recommendations of the I. A. U. this catalogue has in future to represent the celestial co-ordinate system. The right ascensions and declinations of the fundamental stars serve to define the mean equator (the fundamental plane of the system) and the equinox (the intersection of equator and ecliptic as zero-point of the right ascensions). Since equator, equinox and stars are in continuous motion, the co-ordinate system at a certain equinox and epoch must be reducible to any other equinox and epoch. This is achieved by the proper motions of the fundamental stars and by the application of the effects of precession. In accordance with international agreement, Newcomb's constant of precession has been used for all reductions in the compilation of FK 4.

The main properties of the new catalogue are summarized in its introduction. Therefore, no details are given here except those which may be of interest in connection with the determination of certain astronomical constants. The equatorial declinations of FK 4 are based on those absolute catalogues with epochs from 1846 to 1956 which include observations of the Sun and, in general, of major planets. In the following table the corrections $\Delta\delta$ to the equatorial declinations of FK 3 are given as they resulted from nine groups of catalogues together with their weights *p*. The catalogue numbers refer to the corresponding numbers of the catalogues incorporated in FK 4 and cited in a list published in FK 4. The observational catalogues Nos. 78 to 149 are recent ones which are not embodied in FK 3.

TABLE I.

Corrections to equatorial declinations of FK 3.

Catalogue numbers.	Mean epoch.	$\Delta\delta$.	*p*.	Sun (S), Moon (M), Maj. Planets (P), Min. Planets (mP).
5, 9, 12	1857	+0.26″	4	S, S, S
10, 13, 15	1869	−0.12	6	S, S, S
17, 20, 23, 24, 28	1884	−0.09	10	S, S, S, S, S
25, 29, 30, 49, 50	1892	+0.02	12	SP, S, S, S, S
33, 40, 45, 51, 54	1906	+0.06	33	SP, S, S, M, S
34, 55, 57, 60, 69, 81	1915	+0.01	47	SP, S, SP, S, SP, SP
65, 70, 76, 94, 96	1927	−0.05	40	S, S, S, SP, SP
99 + 120, 100, 115, 125	1937	−0.08	45	SP, SP, S, SP
110, 133, 140, 144, 149	1949	−0.01	50	SP, SP, P+mP, —, SP

The system of weights is a uniform one over the whole period of time, the weight varies for observational catalogues from about $p = 1$ to 15

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depending on the number and accuracy of the observations of the Sun and of the other bodies of the planetary system. In the last column, the objects of observation are indicated.

The result of the least-squares solution is

$$\Delta\delta_{\rm def} = \begin{cases} -0''.016 \pm 0''.021 \text{ m. e. for ep. } 1922.2 \text{ from all catalogues,} \\ -0''.017 \pm 0''.021 \text{ m. e. for ep. } 1928.4 \text{ from catalogues after } 1900, \end{cases}$$

and a centennial proper motion,

$$\Delta\mu' = -0''.097 \pm 0''.098 \text{ m. e. from all catalogues.}$$

It may be noticed that the definitive correction $\Delta\delta_{\rm def}$ of the equatorial declinations of FK 3 derived from all catalogues from 1846 to 1956 is the same as the correction derived from all catalogues with epochs after 1900. $\Delta\delta_{\rm def}$ as well as the centennial proper motion $\Delta\mu'$ derived from all catalogues is very small and within the limits of the errors.

The correction ΔN to Newcomb's equinox N_1 is defined as the correction to all right ascensions of the fundamental catalogue as the result of the discussion of observations of the Sun, Moon and planets; the value now adopted is $\Delta N = -0^s.050$, which is identical with the correction adopted in FK 3. The large dispersion of the various observed values of the equinox correction prevents the derivation of any reliable time dependence except the variation with time caused by the effects of changes in the method of observing. Equinox corrections observed with the impersonal micrometer vary from $\Delta N = -0^s.028$ to $-0^s.073$.

The derivation of the system of positions in α and δ has been based on absolute observations with epochs after 1900. The system of proper motions in δ has been based on absolute catalogues from 1846 to 1955 and in α on instrumental series of absolute observations from 1897 to 1956. The average values of the mean errors (standard deviations) of the FK 4 system are listed in table II.

TABLE II.

Average values of the mean errors of the FK 4 *system.*

Decl.	Epoch 1935 $\varepsilon_\alpha \cos\delta$.	Cent. $\varepsilon_\mu \cos\delta$.	Epoch 1925 ε_δ.	Cent. $\varepsilon_{\mu'}$.
°	s	s	"	"
≥+80	±0.001	±0.010	±0.017	±0.07
+70	0.002	0.008	0.016	0.07
+50	0.003	0.010	0.016	0.05
+20	0.002	0.008	0.015	0.06
0	0.001	0.006	0.014	0.05
−20	0.002	0.012	0.020	0.08
−50	0.004	0.017	0.030	0.10
−75	0.009	0.024	0.040	0.13

The systematic errors are determined from the dispersion of the systems of absolute catalogues and of the instrumental systems around their mean which represents the system of FK 4. The indicated epochs 1935 and 1925 are the mean epochs of the system of α and δ respectively. The listed errors do not include the mean errors of equinox and equator point and the mean errors of the individual positions and proper motions of the stars within the system of FK 4. The mean errors of the individual corrections for each fundamental star are printed in the catalogue. The table demonstrates that the systematic errors are fairly small except in the southern sky, where the system is essentially based on observations of the Cape only. From $\delta = -20^\circ$ to the south the mean error of the μ-system is the internal error of the Cape system itself.

No observations prior to 1845 have contributed to the system of FK4. Those from 1745 to 1845 have contributed to the individual positions and proper motions within the system with a total weight which approximately corresponds to the weight of one average modern observational catalogue.

There are other improvements which concern the general demand for more accurate positions of the celestial bodies in the national ephemerides. E. W. Woolard completely redeveloped the theory of the rotation of the Earth around its centre of mass on the basis of Brown's lunar theory and Newcomb's theory of the Sun. He revised the numerical expressions for the nutation to replace the expressions derived by Oppolzer from the corresponding theories of Hansen and Leverrier. Woolard's theory has been introduced in the computation of the effects of nutation in the ephemerides since 1960.

Following a proposal made by J. G. Porter and D. H. Sadler, the effect of the annual aberration in the ephemerides is computed more accurately from the actual motion of the Earth referred to the centre of mass of the solar system.

Important improvements have been accomplished in our knowledge of the orbits of the major planets. In order to provide a uniform system of co-ordinates for several large-scale research problems P. Herget computed, on the basis of Newcomb's theories, the tables of Venus and of the Sun for 1800 to 2000. The meridian observations of Mercury have been discussed by G. M. Clemence, of the Sun by H. R. Morgan and F. P. Scott, and of Venus by R. L. Duncombe. The relativistic effects in the motion of the inner planets have been confirmed to a high degree of accuracy. The outstanding discordance between gravitational theory and observations among the secular variations of the orbits of the inner planets was the unexplained difference between the observed and computed motion of the node of Venus left in the results of Newcomb. This is now removed by Duncombe's discussion of the observations from 1750 to 1949. The origin of the observed corrections to Newcomb's secular change in the obliquity of the ecliptic has been found. A numerical integration for 1920-2000 by Herget taking into account the squares and products of the perturbing forces revealed that Newcomb's inadequate treatment had produced this error in his theory.

G. M. Clemence completed a new general theory of the motion of Mars in Hansen's co-ordinates and made a comparison with a numerical step-by-step integration of the orbit by P. Herget for the years 1919-1954. The power of this test, executed the first time for a general theory, lies in its complete independence and its precision which is much greater than a comparison with observations. The new theory of Mars can be regarded as the most significant achievement in the construction of accurate planetary theories. Furthermore, we may expect soon the completion of Clemence's new theory of the motion of the Earth; this has been a desideratum for some decades.

With the aid of electronic computers, W. J. Eckert, D. Brouwer and G. M. Clemence succeeded in tracing the motions of the five outer planets Jupiter to Pluto from 1653 to 2060 by simultaneous numerical integration of the equations of motion with a precision much greater than that of current observations. These integration orbits have been included in the ephemerides since 1960. Brouwer and his collaborators have started investigations to give general theories for these bodies by new methods, and I hope that their efforts may soon be successful.

In the following, I should like to confine myself to an account of the progress made in determining the solar parallax, the constant of aberration and the constant of precession. Apart from the undeniably great importance of these constants, this selection gives me an opportunity to take up some of the current issues which are likely to play a considerable role in the discussions at this conference.

Solar parallax. — From the measurement of interplanetary distances by means of radar echoes the number of kilometers in the astronomical unit has been determined with great accuracy. The inferred value of the solar parallax corresponds approximately to the arithmetic mean of the trigonometric parallax derived by Spencer Jones and the dynamical parallax derived by Witt and Rabe. The latter determinations are those with the smallest probable errors among the determinations carried out by traditional astronomical methods. In table VI of the survey of determined values compiled by S. Böhme and myself (1), determinations of the solar parallax from various methods of observation are listed. Attention should be drawn first to the trigonometric determination made by Spencer Jones on the basis of the triangulation of Eros in the years 1930-1931. Twenty-three observatories in five continents

(1) *See* p. 277.

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contributed to this observational program. The result deviates appreciably from all previous trigonometric determinations which, however, were based on far fewer observations. It also deviates from the dynamical results which are based on the perturbations on Eros and which were obtained by Witt in 1933 and by Rabe in 1950. The deviation is much larger than could have been expected from the probable errors of the trigonometric and dynamical determinations. The agreement of Witt's and Rabe's results is remarkable, and I shall return to this fact a little later.

All recent measurements are determinations by means of radar techniques. There is first the value resulting from the measurement of radial velocities of *Pioneer V* by radar. This value is only a little smaller than the dynamical values, and the published accuracy does not yet indicate any advantage over the traditional methods. A completely new situation has arisen by virtue of the results of the radar echoes from Venus in 1961. The measurements made at five different laboratories led to values which are almost in full agreement with one another within the limits of their cited accuracy. The probable errors of the values of the astronomical unit obtained at MIT and at JPL correspond to $\pm$ 400 and $\pm$ 250 km respectively.

If we wanted to decide upon the best value of the solar parallax according to the amounts of the published errors, i. e. to the internal accuracy, our decision would doubtless have to be made in favour of the radar echoes from Venus. Then we are faced with the question of the sources of the systematic deviation of the trigonometric and dynamical results.

Let us first consider very briefly Spencer Jones' result in comparison with the dynamical value. What is striking here is the wide dispersion of the individual values for the trigonometric parallaxes from measurements of right ascensions with sixteen instruments. Furthermore, it is almost solely the high weight of two instruments at the Cape which determines Spencer Jones' weighted mean value $8''.7900 \pm 0''.0013$ from right ascensions. The confidence in the significance of this value comes from the theoretically independent determination of the parallax from all declination observations combined, which resulted in $8''.7907 \pm 0''.0011$. The declination observations can be utilized only by comparing the observations at two observatories sufficiently well separated in latitude. Because of the favourable latitude of the Cape the observations at two Cape instruments play an overwhelming rôle for the combined solution. It seems to me that the unavoidable strong dependence on the Cape observations is a weakness of the whole undertaking. No other weakness can be seen in the very thorough discussion by Spencer Jones.

In this respect, the situation is entirely different for the dynamical method. Accidental and systematic errors in the observed positions of

Eros of likely amounts cannot play a similarly important rôle, when the determination of the orbit is based on observations over several decades.

The discoverer of Eros, G. Witt at Berlin, was the first to make considerable efforts to secure the orbit of the planet with high accuracy. In his last paper on the subject published in 1933, Witt gives the result of the discussion of the observations from 1893 to 1931 which include a few prior to the discovery of the planet in 1898. An improvement of the orbital elements is presented together with a determination of the mass of the Earth-Moon system. Witt's result for the mass of the Earth-Moon system (328 390 ± 103 m. e.) was in close agreement with a result obtained earlier by Noteboom on the basis of the observations from 1893 to 1914 only. At the end of his paper Witt states : " There are enough questions still to be settled before we can tackle the evaluation of the observations for the purpose of determining the solar parallax and the mass of the Moon ". Witt himself refrained from stating the value of the solar parallax which corresponds to his result for the mass of the Earth-Moon system.

After Stracke had computed excellent perturbations of Eros by all the major planets from 1926 to 1945, Rabe attempted an improved representation of the observations for this period using most of Stracke's normal positions. In the co-ordinates of Eros and the Sun, Rabe took into account corrections for ephemeris time. Rabe was guided by the idea that the observations of Eros should best be represented by including as unknowns in the equations of condition — besides corrections to the orbital elements of Eros — corrections to the masses of Mercury to Mars, to the four elements of the orbit of the Earth, and $\Delta\alpha$, $\Delta\delta$, the corrections to equinox and equator. Rabe's solution for 16 unknowns, indeed, produced a better representation of the observations than any other before. His result for the mass of the Earth-Moon system turned out to be very near to Witt's value; the difference is within the limits of the standard deviations of both values. In the following table all results obtained from the discussion of the motion of Eros are summarized.

TABLE III.

Mass Earth-Moon from perturbations on Eros.

Observations.	Author.	m^{-1} p. e.
1893–1907.............	G. Witt (1908)	328 659 ± 82
1893–1914.............	E. Noteboom (1921)	328 370 ± 68
1893–1931.............	G. Witt (1933)	328 390 ± 69
1926–1945.............	E. Rabe (1950)	328 452 ± 43

Disregarding the first value obtained by Witt in 1908 on the basis of at least partly inaccurate observations and special perturbations by

Venus to Saturn only, all the other three determinations are in fair agreement. The fact that the probable error of Rabe's value is about two-thirds of the two previous ones is not surprising in view of an increase of the number of unknowns from seven to sixteen.

In conclusion, I should like to summarize my opinion on the dynamical method in saying that it has proved to be a powerful one and that it may turn out to be capable of providing an improved and reliable value of the solar parallax, if the solution is not overstrained by too many unknowns.

The discordance between the result of the dynamical determination and the measurement of the astronomical unit from radar echoes obtained from Venus at its inferior conjunction in 1961 is so far unexplained. An attempt to find a solution to this problem must be made in the near future. The possibility of slight systematic errors in both results cannot be excluded. There is, however, hardly any hope of achieving by means of the dynamical method such high internal accuracy as is indicated by the published errors of the radar results in 1961. A discussion of all observations of Eros on the basis of reliable elements of the Earth's orbit and reliable masses of some inner planets might result in a value of the mass of Earth-Moon, which may differ from Witt-Rabe's result by more than the cited internal error.

On the other hand, also in the papers published by the research workers at the Jet Propulsion Laboratory, California, and the Lincoln Laboratory, Massachusetts, suspected sources of certain systematic errors are pointed out in the measurements of the radar echoes. The radar measurements have to be repeated at future inferior conjunctions of Venus and have to be extended over intervals of time which are as long as possible. It is evident that the ideal method consists of radar observations of a planet through complete revolutions.

I should like to stress the necessity of publishing in full all details of the radar observations. The measurements carried out at 440 Mc/s (68 cm) over more than two months in 1961 showed a systematic variation of the determined values of the astronomical unit with time within a range of about 600 km. W. Priester and his collaborators at Bonn found a correlation between this variation and the solar 20-cm flux as measured at the Heinrich-Hertz-Institut, Berlin, during the same period. The 20-cm flux is a good indicator of solar activity. It is related to the extreme ultraviolet flux of the Sun and to the solar wind intensity. If the correlation between the 20-cm flux and the measured values of the astronomical unit should be real, then we would be dealing with a new phenomenon which cannot easily be understood. If we assume that Venus has an ionosphere changing in height with solar activity, an electron density as high as about 10^9 cm^{-3} would be required to reflect the 440 Mc/s radar signal from this ionosphere.

It is admitted by the Bonn astronomers that the indicated correlation may be completely fortuitous. However, the suspicion that it is real is in accord with a statement made independently by W. B. Smith of the Lincoln Laboratory, which reads : " Certain inconsistencies in the data on range, Doppler shift, and Doppler spectral width cast some doubt on the assumption of free-space propagation and the model of a uniformly rough spherical surface for the planetary target ".

A. A. Mikhailov, in his recent article on the astronomical unit of length, reminds us that all methods " have their inherent difficulties and sources of systematic errors ". At best, may I add, systematic errors which are at first unexplainable may lead to the discovery of so far unrecognized physical effects. There seems to be some indication that a real physical background exists for the discordance between the dynamical and the radar echo values of the solar parallax, and that the results might be reconciled either by a better theory of motion for Eros or by the elimination of so far undetected effects which influence the radar measurements, or by both.

Constant of aberration. — The determination of an accurate value of the constant of aberration has been a matter of concern for a long time. The reasons for the difficulties in the direct determination of this constant from observations were described by Spencer Jones in a few words at the Paris conference in 1950, and these are still valid today. He remarked : " The constant of aberration is perhaps the most difficult of all astronomical constants to determine free from systematic errors. Being dependent upon observations taken at different seasons of the year and at different times of the day, it is peculiarly liable to systematic errors of a seasonal or diurnal nature ".

There is the alternative of inferring the value of the constant from the adopted value of the solar parallax. In order to reach conformity with the solar parallax $8''.80$ adopted in 1896, the constant of aberration should be $20''.48$ instead of the conventional value $k = 20''.47$. If the recent independent parallax determinations (dynamical, trigonometric, and by means of radar echoes) indicate the range in which the true value of the solar parallax lies, then the two values $k = 20''.49$ and $20''.50$, corresponding to $\pi_{\odot} = 8''.7965$ and $8''.7923$, indicate the likely range for the constant of aberration.

As far as the direct determinations of k from various observations are concerned, I refer to table IX of the survey of determined values compiled by S. Böhme and myself (2). This list is far from complete. It contains only those determinations which, as far as we could see, were intended by their authors to furnish an improved value of the constant. I need not remark that in this connection values that were inferred from the

(2) *See* p. 284.

various determinations of the solar parallax are of no interest. The remaining great number of published values of k is mainly due to the fact that in the discussion of latitude observations the constant is derived from the closing errors with each instrument and at each station. In 1953, T. Hattori determined the value of the constant from all available latitude observations made at the international latitude stations, as he says " for the sake of comparison ". The value for each of seven stations was derived from observations of periods of at least 15 years up to 49 years. The individual results lie between about $20''.46$ and $20''.57$ where the quoted extreme values are those of maximum mean error, namely $\pm 0''.017$. The mean of all stations is $20''.527 \pm 0''.004$. While considerable discrepancies appear between various stations, two different instruments at the same station (Mizusawa) produce concordant results.

The values of k obtained at Greenwich and Washington were questioned by Spencer Jones who remarked : " It is believed that the probable errors assigned to them do not represent the real probable errors because of a tendency for the position of the zenith to change in the same direction during the course of any night ". In accordance with this, Hattori concludes that a diurnal variation of latitude, possibly caused by a diurnal variation of the apparent zenith during a night, should be determined from the local discrepancies of the derived values of the aberration constant. What he implies is that one should not expect an accurate value of the constant from latitude observations.

There remains, however, the undeniable fact that all astrometric determinations taken together lead to a value near to $20''.51$ or even $20''.52$ corresponding to a solar parallax of $8''.788$ or even $8''.784$. These two quantities are beyond the range in which the solar parallax is very likely to lie, but, like the recent determinations of the solar parallax, they indicate that the adopted value of the constant of aberration is too small.

We shall have to consider, therefore, the possibility of adopting a revised value. There is another reason for an early revision of the conventional value of the constant of aberration, which has long been recognized as being incorrect. The effect of errors in this constant is detrimental to an evaluation of the differences between computed and observed data. It cannot easily be eliminated later on. The deviation of 2 or 3 hundredths in this constant is sufficiently large to influence the residuals. What action is to be taken will be an issue for discussion at this conference.

Constant of precession. — It has been realized for a long time that Newcomb's constant of precession needs an appreciable correction. The premises on which Newcomb had based his determination are known to be invalid, since the positions of the stars do not represent the ideal frame of reference as had to be assumed in his determination. In deducing a value of the precessional constant from stellar positions at various epochs, effects have to be taken into account which result from all kinds of systematic motions of the stars such as star streaming and differential galactic rotation, and from various kinds of systematic errors of the underlying co-ordinate system. Certain hypotheses on the motion of the stars cannot be excluded in the derivation, and it is hard to estimate their effect on the results. The idea of replacing the stars as representatives of the co-ordinate system by extragalactic nebulae is just as attractive as it is hard to put into practice. Since the time when the idea was first written down by Simon Laplace in his *Exposition du Système du Monde*, the difficulties of measurement of precise positions of nebulae and of proper motions of stars with respect to these have been realized. Great efforts are being made at present to secure such proper motions which then may be related to the system of fundamental stars. Once this has been achieved, a new basis for the derivation of the constant of precession will be available. Although the solution of this problem is to be expected in future we are not relieved of the task of continuing on the lines followed up to now, first of all, because they also lead us towards our goal and, secondly, because they are likely to provide valuable contributions to the theory of stellar motions and to that of planetary motions.

Corrections to Newcomb's lunisolar precession have, in practice, been determined either from proper motions of stars, or from the motions of inner planets. Table XI of the paper by S. Böhme and myself contains eighteen determinations, two of which are based on planetary motions. The latter two determinations are not influenced by any systematic effects in the proper motions of stars.

Since most of the various results obtained from the discussion of proper motions are based on different selections of stars and on different proper motion systems, and since they were derived by methods of varying suitability, the dispersion among the resulting values is not surprising. (I do not need to give reasons here for the inadmissibility of forming any mean value from the list).

May I single out a few of these determinations for a few comments. There is the determination made in 1935 by N. Pariisky, K. Ogrodnikov and V. Fessenkov from the material used by Newcomb for the derivation of his constant. The material consists of Auwers' proper motions of Bradley's stars in the right ascension system N_1 and the declination system of Newcomb. It is remarkable that a careful study of the effects of systematic motions of the stars and of the effects of systematic errors in proper motions resulted in a correction ($\Delta p_1 = +0''.84 \pm 0''.09$ p. e.) which is likely to be near the truth.

The results obtained by D. Brouwer from the investigation of secular motions of Mercury and the Earth, and by G. M. Clemence from the observed motions of the perihelia of both planets are in close agreement with each other.

Finally, may I draw attention to the value obtained by H. R. Morgan and J. H. Oort as a result of an investigation made on the recommendation of the Paris conference in 1950. Here they attempted to arrive at the best possible value by a compilation of those available determinations that appeared to be based on the best suitable material for proper motions. They also attempted to take into account the influences of systematic errors in the proper motion systems. Good reasons were given for the selection that they made concerning the suitability of certain available determinations.

The other crucial question as to the influence of systematic errors of the proper motion system was answered too firmly on the basis of a suspected error in the μ'-system of FK 3 which should arise from the neglection of the latitude variation in observations before 1890. The effect of these unknown latitude variations was estimated to result in an error $+0''.14 \cos \alpha$ in the μ'-system of FK 3. It is this large amount which, subtracted from the actually determined Δn, produces the low value $\Delta n = +0''.30$ corresponding to $+0''.75$ for the correction to Newcomb's general precession in longitude.

At present there is no reason to argue about the validity of the applied correction for the latitude variation, since the material incorporated in FK 4 has offered an opportunity to investigate this effect anew, and I hope to furnish a solution to this question very soon.

Though the constant of precession is one of those quantities which deserves to be regarded as a primary fundamental constant, and although the necessary amount of the correction to Newcomb's value is large, we should not be overhasty in adopting a new value.

For the time being, the value of the correction Δp_1 proposed by Morgan and Oort may well be used in discussions of proper motions. May I add that in such discussions no averages of different systems of proper motions should be taken, since this practice would obscure the influences of the systems on the quantities to be determined. The differences of the systems may be of the order of these quantities. Newcomb's constant of precession should be maintained in the reduction of all observations and in the ephemerides until an improved value is more definitely established.