MAGNETIC BRAKING AND ANGULAR MOMENTA OF PROTOSTARS

Telemachos Ch. Mouschovias
University of Illinois at Urbana-Champaign
Departments of Physics and Astronomy

ABSTRACT

A contracting, typical, isothermal interstellar cloud or fragment, magnetically linked to the surrounding medium, loses angular momentum so efficiently that it remains in nearly synchronous galactocentric orbit as long as the field is frozen in the matter. At a high enough density n_{dec}, the field decouples from the matter due to ambipolar diffusion. Thereafter angular momentum is essentially conserved and, consequently, there is a one-to-one correspondence between n_{dec} and the angular momenta of binary and single stars that can ultimately form. Although the calculated rotational velocities (typically, 100 km/sec) are not yet refined enough to account for peculiarities among different spectral classes which may be due to phenomena following protostar formation, they are encouragingly consistent with observations.

1. INTRODUCTION

Under the premise that stellar angular momentum is a remnant of the original angular momentum of the parent cloud, it has been shown that the entire range of periods of binary stars from 10 hours to 100 years (see Abt and Levy 1976) can be accounted for by a single mechanism (Mouschovias 1977; 1978a). The angular momentum of a stellar system (single or multiple) is simply that which is present at the time the magnetic field decouples from the matter and can no longer brake the rotation of a cloud or fragment by transporting angular momentum to the surrounding medium. [Contrary to many statements in the literature, the angular momentum of a parent cloud is much too large to have been stored in orbital motions of individual stars in a stellar cluster (see reviews Mouschovias 1978b; 1981). In this paper we show that the observed angular velocities of single stars (Vogel and Kuhi 1981; Wolff, Edwards and Preston 1982) can be accounted for by the same mechanism.

The observed temperature, ion and neutral densities, mass and magnetic field strength of diffuse H I clouds imply that the field is dy-

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namically important. In fact, clouds should form mainly by material motions along field lines, with lateral contraction setting in only when the density increases sufficiently for gravitational forces to come into play. The density of protons ($n_{p,0}$) at which this occurs is approximately given by the expression

$$n_{p,0} = 2.1 \times 10^2 \frac{(B_{ext}/1\ \mu G)^{3/2}}{(M/1\ M_\odot)^{1/2}} \quad cm^{-3}, \tag{1}$$

where B_{ext} is the "external" (background) field, and M the cloud mass. For the typical values $M = 10^3\ M_\odot$ and $B_{ext} = 3\ \mu G$, we find that $n_{p,0}$ = 34.6 cm^{-3}. The corresponding cloud radius is

$$R_0 = 1.9\left(\frac{M/1\ M_\odot}{n_{p,0}}\right)^{1/3} \quad pc, \tag{2}$$

which yields R_0 = 5.8 pc for the above example. (The constants on the right-hand sides of equations [1] and [2] are slightly different in the case of a disk or cylinder of radius R_0 and half height $Z_0 \approx R_0$. This is unimportant in what follows because the ultimate goal is to obtain the angular momenta of single stars, after <u>several orders of magnitude</u> of angular momentum has been transported away by magnetic braking). Once gravity comes into play, balance of gravitational and thermal-pressure forces is nearly established <u>along</u> field lines, and the cloud contracts only as rapidly as magnetic forces allow it to contract laterally. During such contraction the field strength in a cloud's core (B_c) increases with mass density (ρ_c) as (Mouschovias 1976a,b; also recent review 1982)

$$(B_c/B_0) = (\rho_c/\rho_0)^{1/2}, \tag{3}$$

where the quantities B_0 and ρ_0 refer to values at which gravitational forces become important.

In § 2 below we obtain an expression for the characteristic time of magnetic braking of a cloud whose angular momentum and magnetic field vectors are aligned; the "fanning out" of field lines in the immediate neighborhood of a cloud (or fragment) is now taken into consideration. The density at which the field decouples from the matter due to ambipolar diffusion (Mestel and Spitzer 1956) is calculated in § 3 by equating the time scales for dynamical contraction and ambipolar diffusion. It is also shown that, up to such a density, the time scale for the magnetic braking <u>of a fragment</u> is smaller than the other two time scales. The implied angular velocities of single stars are obtained and compared with observations in § 4.

2. MAGNETIC BRAKING REVISITED

The characteristic time for magnetic braking of an aligned rotator (disk or cylinder) of density $\rho_{c\ell}$ and half thickness $Z_{c\ell}$ is given by $\tau_{\parallel} = (\rho_{c\ell}/\rho_{ext})(Z_{c\ell}/v_{A,ext})$, where $v_{A,ext}$ is the Alfvén speed in the external medium (Mouschovias and Paleologou 1980a,b). This is an exact expression for a rigid rotator, and an excellent approximation (except for short-lived transient effects) for a differentially rotating disk. It applies to the case in which the magnetic field strength in the external medium is comparable to that of the interior field. In a more realistic geometry, in which the field strength decreases from its value in the cloud, through a transition region, to that of the background field, the time scale $\tau_{\parallel}$ acquires a correction factor whose value depends on the thickness of the transition region. If the transition region is extended, so that the moment of inertia of the medium therein (assumed to have density ρ_{ext}) is larger than that of the cloud, then $\tau_{\parallel}$ is reduced from its above value approximately by the factor 2/15. If, on the other hand, the transition region is characterized by a relatively small moment of inertia, it is the external medium beyond the transition region which accepts the cloud's angular momentum. In such a case, $\tau_{\parallel}$ is given by (Mouschovias 1983)

$$\tau_{\parallel} = \frac{\rho_{c\ell}}{\rho_{ext}} \frac{Z_{c\ell}}{v_{A,ext}} \left(\frac{R_{c\ell}}{R_0}\right)^4 , \tag{4}$$

where R_0 is the original radius of the disk-cloud, when its density was $\rho_0 > \rho_{ext}$ (e.g., see eq. [1]), due to its formation by motions along field lines, and its magnetic field was equal to that of the external medium. Since ρ_{ext} and $v_{A,ext}$ refer to the unperturbed medium away from the cloud, it is clear that $\tau_{\parallel}$ decreases upon contraction (but not indefinitely, unlike the perpendicular-rotator case studied by Mouschovias and Paleologou 1979). Thus, even for an aligned rotator, magnetic braking is a more efficient process than previously realized. Our previous conclusion that the angular momentum problem can be resolved, at least for binary stars, in the early, diffuse stages of cloud contraction is further strengthened.

Equation (4) is, equivalently, written as

$$\tau_{\parallel} = \frac{\sigma_0}{2\rho_{ext} v_{A,ext}} \left(\frac{R_{c\ell}}{R_0}\right)^2 , \tag{5}$$

where $\sigma_0 \equiv M_{c\ell}/\pi R_0^2$ is the original column density of the cloud. It is clear that an upper limit on $\tau_{\parallel}$ for the cloud as a whole can be obtained by setting $R_{c\ell} = R_0$. Typically, one finds that $\tau_{\parallel} \lesssim 10^6$ yr. For a fragment, however, with mass of order $M_{\odot}$, the "external" medium is the surrounding cloud. This, as we shall see below, results in a signifi-

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cant decrease in $\tau_{\parallel}$ even if the factor in parentheses on the right-hand side of equation (5) is ignored. In any case, a cloud contracts at a nearly constant angular velocity until the magnetic field decouples from the matter.

3. AMBIPOLAR DIFFUSION: THE DENSITY AT DECOUPLING

The shortest time scale on which a cloud (or fragment) can contract is the free-fall time, τ_{ff}. Nakano and Tademaru (1972) obtained a density of order 10^9 cm^{-3}, at which the field decouples from the matter, by equating τ_{ff} and the characteristic time for ambipolar diffusion, τ_D. However, the presence of magnetic fields dilutes gravity (Mestel 1965). This is represented roughly as a reduction of the universal gravitational constant G by the factor $(1 - M/|\Omega|)$, where M and Ω are, respectively, the magnetic and gravitational energies of the cloud. Since we are concerned with clouds having $M \sim |\Omega|$, the dilution of gravity can be very significant. For such clouds, detailed, numerical, collapse calculations now under way show that the dynamical contraction time typically exceeds τ_{ff} by a factor of 5 - 30. We therefore take $\tau_{dyn} = a\tau_{ff}$, with $a > 1$, in what follows. We thus have that

$$\tau_{dyn} = 4.36 \times 10^7 \; n_p^{-1/2} \; a \quad yr, \tag{6}$$

where n_p is the number density of protons in the cloud --a 10% helium abundance is accounted for.

The characteristic time for ambipolar diffusion is defined as $\tau_B = R/v_D$, where v_D is the relative drift speed between ions and neutrals, determined by equating the driving magnetic force on the ions and the retarding collisional force on the ions due to neutrals. One finds that (see Spitzer 1978)

$$\tau_B = 8\pi \, R_c^2 \, n_i \, n_c \, m_n \, \langle\sigma w\rangle_{in} \, B_c^{-2}. \tag{7}$$

The quantites n_i and n_c are the ion and neutral densities, respectively; m_n is the mass of a neutral particle (which we take to be H_2) and is assumed to be much smaller than the ion mass (Na^+ or HCO^+, in our case); $\langle\sigma w\rangle_{in}$ is the average ion-neutral collisional rate and is approximately equal to 1.7×10^{-9} cm^3/sec in our example. We have introduced the subscript c because we shall apply equation (7) only to the cloud's core, not to the cloud as a whole, because it is neither necessary nor likely that ambipolar diffusion decouple the field from the matter over the entire cloud for stars to form (see Mouschovias 1978b). We now use equation (3) to eliminate B_c from equation (7), we express R_c in terms of the mass and density of the core, and we evaluate the constants to find that

$$\tau_B = 1.2 \times 10^{10} \, \frac{n_{p,0}}{(B_0/3 \;\mu G)^2} \, \frac{n_i}{n_{p,c}^{2/3}} \left(\frac{M_c}{M_\odot}\right)^{2/3} \quad yr; \tag{8}$$

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where the quantities $n_{p,0}$ and $n_{p,c}$ are, respectively, the density of protons in the original cloud when lateral contraction began and the present density of protons in the core. In the density range $2 \times 10^3 \lesssim n_{p,c} \lesssim 2 \times 10^7$ cm^{-3}, we may relate the ion density to the neutral density by $n_i \approx 9.5 \times 10^{-6}\, n_{p,c}^{1/2}$, which is consistent with Nakano's (1979) Figure 1. To obtain the proton density at which the field essentially decouples from the matter, we set $\tau_{dyn} = \tau_B$. The result is

$$n_{p,dec} = 2.0 \times 10^6 \left(\frac{a}{10}\right)^3 \left(\frac{30\ \mathrm{cm}^{-3}}{n_{p,0}}\right)^3 \left(\frac{B_0}{3\ \mu\mathrm{G}}\right)^6 \left(\frac{M_\odot}{M_c}\right)^2 \quad \mathrm{cm}^{-3}. \tag{9}$$

The sensitivity of $n_{p,dec}$ on the quantities on the right-hand side of equation (9) is only apparent because these quantities cannot be specified independently of one another. Using equation (1) for the cloud as a whole to eliminate $n_{p,0}$ from equation (9), we find that $n_{p,dec} \propto B_0^{3/2}\, a^3\, M_{c\ell}^{3/2}\, M_c^{-2}$. In addition, the larger $M_{c\ell}$ is at a fixed B_0, the smaller the dilution of gravity by magnetic forces, so that the quantity a bears some inverse relation to $M_{c\ell}$ which will be determined by the detailed numerical calculations. Further discussion of these points is presented elsewhere (Mouschovias 1983).

The crucial question still remains, whether magnetic braking remains efficient up to the density $n_{p,dec}$. To check this, we calculate $\tau_\parallel$ (actually, its upper limit) and compare it with τ_{dyn} ($= \tau_B$). A core mass of order 1 $M_\odot$ at the typical density given by equation (9) is contained within a radius $R_c = 4.7 \times 10^{16}$ cm, which we take to be the half thickness of the disk, Z_c, in equation (4), which is applied to the core, not the cloud as a whole. With $B \propto \rho^{1/2}$, the Alfvén speed in the "external" medium, which is now the cloud surrounding the core, remains constant and somewhat larger than 1 km/sec. This gives a strict upper limit on $\tau_\parallel$ of 1.5×10^4 yr. At the same density and mass for the fragment, equation (8) yields τ_B ($= \tau_{dyn}$) $= 3.1 \times 10^5$ yr, which is larger than the upper limit on $\tau_\parallel$. It therefore follows that the angular velocity of a fragment at decoupling can be taken to be comparable with that of galactic rotation, i.e., 10^{-15} rad/sec. (A more detailed calculation shows that the angular velocity of a fragment begins to increase somewhat above 10^{-15} rad/sec before decoupling, but that does not affect our present conclusions much.)

4. BEYOND DECOUPLING: ROTATIONAL VELOCITIES OF SINGLE STARS

At densities in excess of $n_{p,dec}$, the angular momentum of a collapsing blob is essentially conserved, so that the angular velocity of a protostar (ω_*) that will ultimately form is

$$\omega_* = \omega_{dec}\, (\rho_*/\rho_{dec})^{2/3}; \tag{10}$$

where ω_{dec} and ρ_{dec} are, respectively, the angular velocity and matter density of the blob at decoupling, and ρ_* is the mean density of the protostar. Since ω_{dec} is maintained at a value near 10^{-15} rad/sec by magnetic braking, equation (10) implies a linear velocity at the protostellar equator given by

$$v_{*,eq} = 252\ \rho_*^{2/3} \left(\frac{R_*}{R_\odot}\right) \left(\frac{10}{a}\right)^2 \left(\frac{n_{p,0}}{30\ \text{cm}^{-3}}\right)^2 \left(\frac{3\ \mu G}{B_0}\right)^4 \left(\frac{M_c}{M_\odot}\right)^{4/3} \frac{\text{km}}{\text{sec}}. \qquad (11)$$

As explained below equation (9), the sensitive dependence of $v_{*,eq}$ on the quantities a, $n_{p,0}$ and B_0 is only apparent. For main-sequence stars of spectral type between O5 and F5, the product $\rho_*^{2/3}$ $(R_*/R_\odot)$ varies by less than a factor of 2 (e.g., see Allen 1974); its extreme values are 0.82 $g^{2/3}$ cm^{-2} for O5 stars and 1.33 $g^{2/3}$ cm^{-2} for F0 stars. It therefore follows that, since protostars lose typically half of their angular momentum (compared to a few orders of magnitude already lost) between the onset of the Hayashi phase and their arrival on the main sequence, typical equatorial stellar $\langle v_{*,eq} \sin(i)\rangle$ should be around 100 km/sec. This is surprisingly (but encouragingly) close --given that three independent physical processes (collapse, magnetic braking and ambipolar diffusion) worked together to give this prediction-- to the observed rotational velocities for such stars, which vary from about 25 to 160 km/sec (Vogel and Kuhi 1981; Wolff et al. 1982).

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DISCUSSION

MESTEL: I would not quarrel with your picture as something that *can* happen, if the parameters — i.e. the initial conditions — are appropriate. A crucial point is that the mass of your cloud is extremely close to the critical mass for the cloud's flux, so that the magnetically diluted contraction time is 20 times the free-fall time. This certainly enables the magnetic stresses to maintain corotation with the background. But if the cloud is only a little more massive, then centrifugal forces will instead be maintained close to gravity (with the contraction of the cloud being *determined* by the rate of braking). Subsequent loss of flux by ambipolar diffusion will leave a rapidly rotating cloud of only moderate density. On the other hand, if the cloud mass is below the critical mass, it will indeed be kept in corotation with the background, but it can contract only through slow flux leakage. My point is that one will want to satisfy oneself that the choice of parameters leading to a line of evolution is not implausibly restrictive.

A further physical question is that of the topology of the field lines. Once detachment of much of the cloud field from the background field has occured, then angular momentum transport will be much removed. But perhaps this rather subtle question should be left for the next Symposium on Star Formation.

MOUSCHOVIAS: As I am sure Dr. Mestel knows, I agree with his first point, but not with the implication that I have unduly restricted the range of the input parameters. I have let observations suggest a "typical" case. Of course, in our detailed calculations we perform a much more complete parameter study. In the ten minutes available to me, I can describe only the typical case and even that, only in an incomplete way. In any case, the equations in my paper show explicitly the dependence of the solution on the relevant parameters.

On the magnetic detachment of the cloud from the background, our calculations suggest that it takes place at a much later stage of contraction, long after the angular momentum problem is resolved. More calculations, aiming specifically at resolving this issue, are needed.