suggested by Mr. Atkins is to be adopted; and then "the settlement of losses by fire under average policies" would be in conformity with the conditions, and not in opposition to them.

I am, Sir,
Your obedient servant,
R. RAY.

Atlas Fire Office,
17th November, 1858.

ON CERTAIN ADVANTAGES AFFORDED BY MR. CHISHOLM'S TABLES RECENTLY PUBLISHED.

To the Editor of the Assurance Magazine.

SIR,—In the last Number of the *Assurance Magazine*, your able correspondent, Mr. T. B. Sprague, has given a formula, adapted to the D and N columns, for obtaining the annual premium for a term assurance on two joint lives, under the impression that Mr. David Jones, in his treatise on annuities, had overlooked it. He has evidently not observed, that, in the edition of Mr. Jones's valuable work, published under the superintendence of the "Society for the Diffusion of Useful Knowledge," a list of formulæ is prefixed to the second volume, containing, amongst others, the formula, the supposed want of which excites Mr. Sprague's surprise.

It is as follows—

$$P_{m\,m_1\,\overline{n}|}=r-\frac{N_{m.m_1}-N_{m+n.m_1+n}}{N_{m-1.m_1-1}-N_{m+n-1.m_1+n-1}},$$

and coincides with that given by Mr. Sprague.

Perhaps it may not be out of place here, to allude to the facilities now afforded for the solution of this and cognate problems by the valuable contribution recently made by Mr. David Chisholm to the science of Life Assurance. The M and R columns, as calculated by him, representing the contingency of survivorship, have effected a most material simplification in the methods formerly employed for finding values in which this contingency is involved, and have supplied a want which was most urgently felt. Indeed, by Mr. Chisholm's arduous labours, the commutation system, originated by Mr. Barrett, improved by Mr. Davies, and extended and illustrated by Mr. Jones, has been rendered complete in so far as relates to one and two lives.

But to return to the question alluded to at the commencement of this letter: the columns $M^{1}_{\overline{x}.y}$ and $M^{\;\,1}_{\overline{x.y}}$, as tabulated by Mr. Chisholm, being complementary, may be used for the solution of questions connected with joint life assurances; and as these converse values are placed on opposite pages, the facility of using them is greatly increased. The formula for the annual premium for a term joint assurance is just an extension of that for single lives, being

$$\frac{M^{1}_{\overline{x}.y}+M^{\;\,1}_{\overline{x.y}}-M^{1}_{\overline{x+n}.y+n}+M^{\;\;\;\;1}_{\overline{x+n\,y+n}}}{N_{x.y}-N_{x+n.y+n}}.$$

The expression for the annual premium for an assurance deferred n years is

https://doi.org/10.1017/S2046165800000940 Published online by Cambridge University Press

equally simple, $\frac{M^{1}_{x+n.y+n}+M^{\ \ 1}_{x+n.y+n}}{N_{x.y}}$. If the number of premiums be limited to n, of course $N_{x.y}-N_{x+n.y+n}$ must be substituted in the denominator.

But although these tables considerably lessen the labour of solving such problems as the above, their full value cannot be appreciated, unless we compare the simplicity of the operations now required in the finding of values in which the element of survivance is introduced (which by them is reduced to a simple division), with the laborious methods which were formerly rendered necessary.

Hoping that these few remarks may turn the attention of such of your readers as have not already examined Mr. Chisholm's volumes, to the great facilities which they offer to those who are engaged in intricate calculations.

I am, Sir,
Your obedient servant,

Edinburgh, 27th Nov., 1858. W. F. B.

DEMONSTRATION OF FORMULÆ FOR VALUE OF AN ENDOWMENT ASSURANCE.

To the Editor of the Assurance Magazine.

SIR,—It is very well known that an assurance payable at a given age or at previous death—commonly called an "endowment assurance"—bears a close analogy to an ordinary whole term assurance. This analogy has been pointed out by Mr. Gray (*Tables of Life Contingencies*, art. 233); but he has not given any of the formulæ for the assurance in question. The subject has been also touched upon by yourself—*Assurance Magazine*, vol. i., p. 332—where the proper formulæ are given and demonstrated; a formula is also supplied by Mr. Hardy in his *New and General Notation*, p. 43. The following convenient practical rule is easily seen to follow at once from the reasoning in the passage in the *Assurance Magazine* just referred to:—"To find the annual or single premium for an endowment assurance payable at the age $m+t$ on a life now aged m, calculate the temporary annuity for $t-1$ years on the life m, and enter Orchard's Tables with the result, in just the same way as for an ordinary whole term assurance."

As assurances of the kind in question are not at all uncommon, being granted by most Insurance Companies, and the subject is therefore of some practical importance, I have thought that the following independent proof of the above rule will be interesting to the readers of the *Assurance Magazine.*

Since an endowment assurance on a life m, payable at age $m+t$, is equivalent to a term assurance for t years and an endowment at the end of t years, the single premium for it will be $\frac{M_m-M_{m+t}+D_{m+t}}{D_m}$. The annual premium is got by substituting $N_{m-1}-N_{m+t-1}$ for D_m in the denominator, and is therefore $\frac{M_m-M_{m+t}+D_{m+t}}{N_{m-1}-N_{m+t-1}}$.

But, $$M_m=D_m-(1-r)N_{m-1},$$
$$M_{m+t}=D_{m+t}-(1-r)N_{m+t-1};$$
therefore, $$M_m-M_{m+t}+D_{m+t}=D_m-(1-r)(N_{m-1}-N_{m+t-1});$$

https://doi.org/10.1017/S2046165800000940 Published online by Cambridge University Press