THE NON LINEAR DYNAMIC OF THE ZZ CETI VARIABLE GD 66

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ABSTRACT. We present here a new method for analysing stellar variability : a geometrical study of the attractor associated with a variable star. In contrary to Fourier analysis, which focuses on times scales, this approach attemps to define the shape of the attractor in its phase space. It presents the advantage of being almost unsensitive to gaps in the data as long as uninterrupted sequences contain several tens of characteristic times. But, its noise sensitivity calls for very accurate photometry (10^{-4}), still extremely difficult to perform up to now.

A preliminary test has been done on the variable white dwarf GD 66 on data obtained by classical photometry from the ground.

Many white dwarfs, since their discovery, are known to present variable and complex Fourier spectra. The interpretation of the observed frequencies in terms of linearly unstable eigenmodes have lead to the conclusion of the non-radial nature of these modes.

The identification of the modes have been subject to some difficulties, as the density of the g-modes spectrum can lead to multifold solutions. For instance, in the case of GD 66, Fontaine et al. (1985) (referred to as FWBLLS in the following) criticize the interpretation given by Dolez et al. (1983) (referred to as DVC in the following).

It is also assumed that the observed frequencies correspond to modes which are unstable at the linear approximation, but generally the most unstable modes do not correspond to the observed frequencies (see i.e. DVC).

Our aim is not only to determine the frequencies of the eigenmodes but to obtain more precise insight on the dynamic of the star in order to construct models, as simple as possible, which describe it.
From this point of view the model is necessarily non-linear to account for at least the amplitude saturation.

Irregularities in the ligth curve, amplitude limitation, complexity and variability of the Fourier spectrum, mode selection are strong indicators of other type of non linearities at work in these objects. One is consequently led to question the validity of the linear analysis

J. Christensen-Dalsgaard and S. Frandsen (eds.), Advances in Helio- and Asteroseismology, 339–342.

https://doi.org/10.1017/S0074180900158334 Published online by Cambridge University Press

and to develop tools which give insight into possible non-linear effects.

GD 66 has been reobserved in this framework.

1. LUMINOSITY VARIATIONS IN GD 66. INDICATIONS OF A NON LINEAR DYNAMIC.

In addition to the observations described in DVC, reanalysed by Auvergne and Baglin (1986 referred to as AB) and the four runs reported in FWBLLS, we have reobserved GD 66 with the same equipment as described in Vauclair and Bonazzola (1981) at the Observatoire de Haute Provence on february 1986 6th (run 1) and february 12th (run 2).

Light curves are given in Fig. 1. Fourier spectra are displayed in Fig. 2 in natural and logarithmic scales.

Several remarks can be made which will serve as a guide for the modelisation and the interpretation of the pulsation.

- a long term amplitude variations is discovered : the amplitude has increased by a factor 4 between run 1 (0.03) and run 2 (0.15), and the shape of the light curve is different in the small and large amplitude regime.
- a continuous component is present at frequencies smaller than 6 to 8 mHz, with a variable intensity. This is more easily seen in the logarithmic power spectrum (Figs. 2.1.2 and 2.2.2).
- a low amplitude peak ($f_1 = f_3/3$) is observed in run 2 and is also present in one run of FWBLLS. On Fig. 1.2 one sees clearly the discontinuities in the light curve each three periods, generating this frequency.
- the frequency f_4 at 5.1 mHz seen also in some runs by FWBLLS and in DVC is associated to the phases of small amplitude whereas the main frequencies f_2 and f_3 correspond to the large amplitude phases.
- the frequency at 3.90 mHz proposed by DVC is not confirmed; the spectrum is noisy in this domain but sometimes very flat as in run 1.

These remarks lead to the idea that the oscillation implies at least 2 modes with strong non-linear interaction.

2. THE VARIABLE STAR AS A DYNAMICAL SYSTEM.

The theory of dynamical systems has told us that a peak in the Fourier spectrum does not necessarily correspond to an eigenmode of the system. It has developed tools appropriate to discover order behind apparent irregularities.

A dynamical system known through a single observable x(t) can be studied using the usual time delay method (Takens 1980). A m dimensional portrait of the system topologically equivalent to the one constructed from the physical variables is given by the set :

$$x(t_i), x(t_i+T), \ldots . x(t_i+(m-1)T)$$

On this portrait, the topological properties can be studied through, for example projections, Poincaré sections of the phase space, and dimension calculation. The applicability of these methods to data undergoing astronomical constraints has been studied by AB.

Contrary to Fourier analysis, they have the advantage to be almost unsensitive to gaps in the data, as long as uninterrupted sequences contain several tens of characteristic times. Then, nights can be added if they are free from systematic bias. The discovery of a large amplitude variation on GD 66 over a characteristic time of the order of one day or more, did not allow us to add our three runs. We need data which cover the overall amplitude variation.

As these methods aim at a local geometrical description of the attractor associated to the variable star, it can be easily understood that they are very sensitive to external noise. As the definition of the attractor needs very long time series (several hundred timescale), very low frequency noise has a crucial effect. High frequency noise (due essentially to photon noise) swappes out small details, whereas low frequency noise (atmospheric one) introduces a global distorsion of the attractor. So that a special care has to be taken to the acquisition and the reduction of the data. Fourier filtering is possible for both high and low frequencies, but only if the frequencies of the star are located in a limited bandpass.

Using filtered data, AB obtained a correct behaviour of the correlation integral on the set of data published by DVC. But the amplitude variations discovered here on a longer time scale may imply a higher dimension for the complete attractor, and require more observations to conclude.

3. CONCLUSION.

When correctly applied with appropriate data, these methods will give a complete description of the dynamics at work. However, the strong requirement on the noise level will necessitate to perform observations at a level of accuracy never reached up to now.

In the case of a linear oscillator they will confirm and reinforce the results of the Fourier analysis, making sure that the observed frequencies correspond to eigenmodes of the system. And, when the phase space has a strong non-linear structure, they will provide the information necessary to modelize the dynamics, and in particular indicate the number of degrees of freedom.

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https://doi.org/10.1017/S0074180900158334 Published online by Cambridge University Press

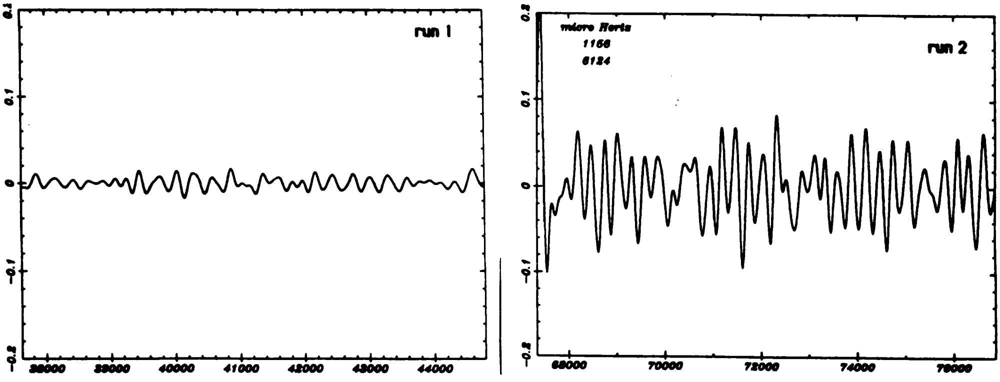

Fig. 1 : Light curves of GD 66 in white light. Relative amplitudes are in intensity, time in second.

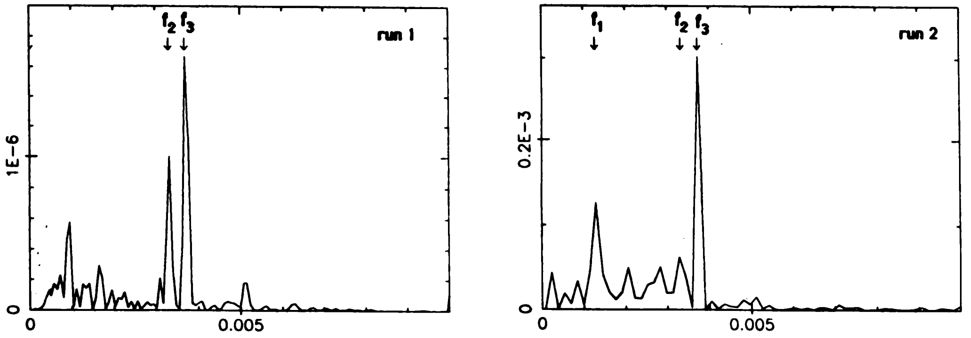

Fig 2.1 : Power spectra of the same light curves Frequencies are in Hz.

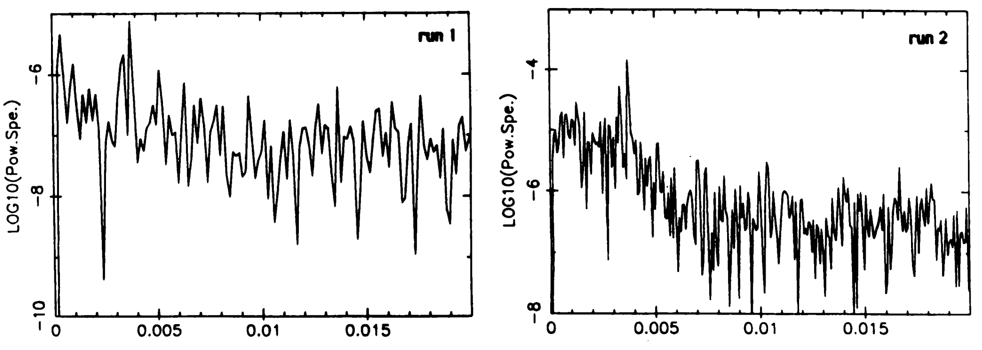

Fig. 2.2 : Power spectra in logarithmic scale Frequencies are in Hz.

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