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Pulsating Stars and the Distance Scale - II

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Abstract. The current Cepheid zero-point is equivalent to an LMC distance modulus of 18.57 ± 0.10. The zero-point from corrected Baade-Wesselink data is probably not significantly different from this. A re-examination of the Baade-Wesselink data for RR Lyrae variables leads to an LMC modulus of 18.51, an age difference between β- and α-group galactic globular clusters of $+1.46 \pm 0.70$ Gyr, and an M_V - [Fe/H] slope in agreement with theory. Other questions discussed include; Avoiding bias in using the Cepheid PL relation; Metallicity spread amongst Cepheids; Cepheids and H_0.

1. Introduction

Cepheids remain central to the distance scale problem. This paper is concerned with them and with the RR Lyrae variables.

For well observed Cepheids with multicolour photometry (e.g., BVI) a PLC relation gives relative distances accurate to about 2 percent (0.04 mag). These good relative distances are satisfactory for some problems; for instance, to investigate the 3-D structure of the Magellanic Clouds (Caldwell & Coulson 1986). But for many problems we need accurate absolute distances. One such problem is the determination of the distance to the centre of the Galaxy (R_o) and other galactic constants from Cepheid distances and motions as in the continuing work described by Caldwell et al. (1992) or that of Pont et al. (1994) (see also Fernie 1995). Cepheids are likely to remain important for studies of galactic structure and dynamics, partly because follow-up studies of the near-infrared sky surveys, DENIS and 2MASS, should lead to the identification of significant numbers of distant Cepheids, possibly increasing the number of known variables of this type by a factor of 5 to 10 (Feast 1994a). Accurate Cepheid distances are of course needed to determine the distances of the Magellanic Clouds and more distant galaxies, with the aim of investigating the structure and dynamics of the local group, calibrating other distance indicators, and providing a basis for the estimation of H_o.

2. Cepheid PL and PLC relations

PL and PLC relations with arbitrary zero points are best set up in the Large Magellanic Cloud (LMC). PL relations at V,J,H and K have recently been derived by Laney & Stobie (1994) who combined LMC and other data. The scatter

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about their relation at K is small (0.17 mag) but not as small as in a PLC relation (negligible intrinsic scatter). Laney & Stobie do not include a rederivation of the PL relations at R and I and in fact the photometry of Magellanic Cloud Cepheids at these wavelengths is considerably less extensive than one would wish. Much of the existing LMC I data, for instance, was obtained with the limited objective of having a few observations per star so as to derive individual (BVI) reddenings (see, Martin et al. 1979; Caldwell & Coulson 1985). This is somewhat unfortunate since current work on extragalactic Cepheids with the HST is at V and I, and ground-based work (e.g., Pierce et al. 1994) is at R. Reasonably satisfactory PL relations at R and I are available (e.g., Madore & Freedman 1991) but this is an area which could do with further work.

It was established a long while ago from BVI photometry of LMC Cepheids (Martin et al. 1979) that the scatter in the PL relation at V was intrinsic and not due either to observational error or differential reddening as had at one time been suggested. This showed, empirically, the need for a PLC relation, a result confirmed by Laney & Stobie (1986) who demonstrated the existence of a PLC relation in the infrared. A further, useful, confirmation comes from the results of the EROS project (Beaulieu 1995, Beaulieu et al. 1995) which show rather clearly that an instability strip of finite width is necessary to interpret overtone pulsators in the LMC. It has not so far been possible to establish an independent PLC relation using Baade-Wesselink (BW) type distances to galactic Cepheids. Amongst reasons for this is the fact that the instability strip narrows towards shorter periods (see e.g., Caldwell & Coulson 1986) and the galactic BW sample tends to have somewhat shorter periods than the LMC sample on which the derivation of the PLC relation depends. However, as was recognised by Gieren et al. (1993), the main reason for their failure to find a PLC relation is the uncertainty of the individual BW distance estimates.

Determining unbiased values for the coefficients in the PLC relation is more difficult than simply establishing that a PLC relation exists. This is so because the problem is multivariant and the results depend on how one chooses to deal with the problem of observational errors. Combination of the EROS work on the colours of LMC overtone and fundamental Cepheid pulsators with the MACHO data on double-mode Cepheids may perhaps allow an estimate of the PLC coefficients free of some of the uncertainties of bias. This is an important consideration if the aim is to compare empirical coefficients with theoretical predictions. It is much less important if one is using the PLC relation for distance estimates since in a particular solution a change in one PLC coefficient is largely offset by a change in the others. Thus the difference in derived absolute magnitudes between two rather extreme solutions is less than 0.1 mag even at the edges of the instability strip (Feast & Walker 1985).

Whether one employs a PL or a PLC relation must depend on the particular application and the extent and accuracy of the available observations. The sensitivity of the various relations to a variety of effects is discussed by Feast (1991).

3. The Cepheid zero point

The zero-point calibration of the PL and PLC relations depends primarily on Cepheids in galactic clusters. The distances to these clusters are based on main-sequence fitting procedures calibrated using field main-sequence stars with relatively well determined trigonometrical parallaxes or adopting distance estimates for the Hyades and/or Pleiades clusters. The available evidence was reviewed recently (Feast 1993). That discussion can now be up-dated by including the following revisions and additions; A revised mean trigonometrical parallax for the Hyades (Gatewood et al. 1992) giving a distance modulus of 3.20 ± 0.06; A new moving cluster solution for the Hyades (Zhao & Chen 1994) giving a modulus of 3.29 ± 0.03; A preliminary determination of the trigonometrical parallax of the Pleiades from HIPPARCOS data (Penston 1994) giving a modulus of 5.67 (uncertainty unstated). Treating all the data as in Feast (1993) leads to an effective modulus for the Pleiades of 5.62 ± 0.05 and an LMC modulus of 18.57 ± 0.10. From a slightly different treatment of the cluster and association data, Laney & Stobie (1994) obtained a result which leads to an LMC modulus of 18.60 when referred to the new Pleiades modulus. These results are very close to the modulus derived from the SN1987A ring, $18.50 \pm \geq 0.13$ (Panagia et al. 1991; Dwek & Felten 1992; Plait et al. 1994). In the above discussion I have not included a recent determination of the Pleiades modulus based on surface brightnesses to obtain stellar angular diameters together with star-spot rotational periods and stellar rotational velocities to obtain linear diameters (O'Dell et al. 1994). This interesting new method gives a Pleiades modulus of 5.60 ± 0.16 and so is not yet competitive with other methods. A trigonometrical parallax of δ Cephei (Gatewood et al. 1993) which leads to a distance modulus of 7.61 ± 0.67 is not inconsistent with a PLC (V,B–V) value based on an LMC distance of 18.57 (=7.19). Weidemann's (1993) suggestion that the Hyades modulus is 3.35, based on white dwarf masses and surface gravities, is compatible with the value adopted here.

Obviously one hopes for improved Hyades and Pleiades moduli from HIPPARCOS. A useful main sequence may also be derived from HIPPARCOS parallaxes of nearby stars provided it can be decided which stars are on the zero-age main sequence and what their metallicities are. In addition in the future we may hope to find more galactic clusters and associations containing Cepheids. Catalogues of galactic clusters are at present quite incomplete beyond about 3 kpc. The infrared sky surveys, DENIS and 2MASS may be expected to find new OB star clusters and associations in a volume near the galactic plane which is about a factor five larger than that currently surveyed (see Feast 1994a). A few of these will, one hopes, contain Cepheids.

Several groups have put a very large effort into the determination of Baade-Wesselink type radii and distances for galactic Cepheids. Gieren et al. (1993) obtained a PL relation based on BW analyses of V,R photometry and radial velocities for 100 galactic Cepheids. When this is applied to the V_o data for LMC Cepheids as tabulated by Laney & Stobie (1994) one finds an LMC modulus of 18.80 or, following the discussion of Laney & Stobie and applying a small metallicity correction, 18.83 (± 0.04 internal). However a recent detailed analysis, also by Laney & Stobie (1995), establishes that their BW work based on infrared photometry is free of certain problems which affect radii and distances determined from optical photometry. A comparison of the Laney-Stobie radii obtained from K,J–K or V,V–K, for 29 stars in common with Gieren et al. and reddening for 24 in common between Gieren et al. and Laney & Stobie (1993), shows that the Laney-Stobie radii are a factor 0.94 smaller and their $E_{(B-V)}$ values 0.022 mag less. Other things being equal this will make their distance scale 0.20 mag smaller. A discussion of infrared and optical BW data for the single Cepheid U Sgr (Welch 1994) agrees, within the uncertainties, with this difference in distance scales. Thus a preliminary conclusion is that the Laney-Stobie BW scale gives about 18.6 for the distance modulus of the LMC, very close to the value based on the cluster distance scale. The definitive analysis of the Laney-Stobie data is awaited with interest (see also Laney 1995). (Note that the zero-point of the BW scale adopted by Di Benedetto (1994) is based on the Gieren et al. radii)

Various attempts have been made to apply the BW technique directly to Magellanic Cloud Cepheids. Further work in this direction is desirable. The estimates so far available do not compete in accuracy with those just discussed. In view of Laney & Stobie's discussion, the photometric part of this work will best be carried out in the infrared.

4. Comparison with the RR Lyrae scale

One is obviously interested in a comparison of results from Cepheid and RR Lyrae zero points. Extensive BW work on RR Lyraes shows good evidence of an M_V - [Fe/H] relation. However the slope of this relation derived by Carney et al. (1992) (= 0.15) was considerably less than that predicted by Sandage (e.g., 1993) from an explanation of the Oosterhoff effect (=0.30). In addition the BW zero point, when applied to RR Lyraes in LMC clusters gave an LMC distance modulus 0.35 mag less than the Cepheid scale adopted above (see Walker 1992). Both these discrepancies have been somewhat alleviated in recent times. Skillen et al. (1993) revised the M_V - [Fe/H] slope to 0.21 using new BW data and their zero point was revised by Fernley (1994) using a new limb-darkening coefficient. Fernley then finds an RR Lyrae distance modulus for the LMC of 18.43. This is less than the Cepheid modulus, though perhaps not seriously so. However both the slope and zero point problems can be made to go away entirely by the following consideration.

The M_V - [Fe/H] relation generally adopted from BW data is obtained from a regression of M_V on [Fe/H]. However use of this relation is subject to bias of the generalised 'Malmquist' type. For objects which are all at the same true distance this bias is avoided by using the inverse regression. (The bias correction factor for the inverse relation (equation (2) of Feast (1994b) becomes unity in this case except for the exponential part which is the (small) correction in converting mean modulus to mean distance and which is not directly relevant here). The data used by Fernley (1994) give the following two regressions:

Direct

$$M_v = 0.97 + 0.21[Fe/H] \quad (1)$$

Inverse

$$M_v = 1.13 + 0.37[Fe/H] \quad (2)$$

The second equation and Walker's (1992) LMC data give an LMC modulus of 18.51 in almost embarrassingly good agreement with the Cepheid modulus (18.57) or the SN1987A modulus (18.50). (Note that this RR Lyrae result may not be entirely free from bias since the method adopted here assumes no selection bias in [Fe/H] for the galactic calibrators and this may not in fact be the case). The slope of the relation required for a comparison with Sandage theory will lie close to midway between the two regressions if the errors in the two variables are similar as is suggested by the work of Skillen et al. (1993, Table 16). Thus theory and observation are evidently in good agreement.

There is an interesting application of the inverse relation in connection with the distances and corresponding ages of galactic globular clusters. Lee et al. (1994) have developed an age criterion based on metallicity and horizontal branch morphology. On this basis metal-poor clusters can be divided into an coeval group (mainly in the inner Galaxy, the β group of van den Bergh 1993) and a group (mainly in the outer Galaxy, the α group of van den Bergh) which is about 2 Gyr younger. Using RR Lyrae absolute magnitudes from the inverse relation above, assuming bias effects will be roughly equal for the two groups, and taking other data from Sandage (1993b), one finds an age difference between ten β-clusters and six α-clusters of $+1.46\pm0.70$ Gyr which is marginally significant and close to the theoretical value of Lee et al.

5. Overtones, metallicities and reddenings

In applying Cepheid PL and PLC relations to specific problems the effects of overtone pulsators and of metallicities and reddenings must be taken into account.

It has long been known that some low amplitude Cepheids in the Magellanic Clouds stand above the PL and PLC relations and that these are best interpreted as overtone pulsators. The work of Smith et al. (1992) on Cepheids in the SMC showed that at short periods the overtone pulsators were an appreciable fraction of the total number and the EROS and MACHO projects are finding large numbers of overtone pulsators, mostly at shorter periods, in the LMC. It seems likely that the overtone pulsators can be identified with good success from their light curves (see for instance Beaulieu et al. 1995).

The effects of metallicities and reddenings have been discussed previously (Feast 1991). Metallicity effects on the PL relation arise from changes in line blanketing, which are particularly important at shorter wavelengths, and from temperature shifts of the instability strip (see for instance Laney & Stobie 1994). Direct tests of the latter effect at metallicities above solar are particularly desirable. If metallicity dependent temperature shifts of the instability strip are uncertain (e.g., at high metallicities) a PLC relation (with blanketing corrected colours) may be safer to use than a PL relation.

In principle BVI photometry of a group of Cepheids at a common distance allows one to estimate their mean metallicity, reddening and distance (Feast 1988). It is however clearly desirable to have independent spectroscopic evidence for Cepheid metallicities. The data of Luck & Lambert (1985,1992) yield the results in Table 1. The final column lists the mean values for young objects in the Magellanic Clouds adopted by Feast (1989). The large scatter in the Luck and Lambert results for LMC Cepheids may be due to problems in the correct phasing of the spectra with the IR photometry used to estimate temperatures (see Feast 1995).

Table 1. Cepheid metallicities

System	[Fe/H][a]	s.e.[b]	N[c]	[Fe/H][d]
The Galaxy	0.00	0.03	15	(0.0)
LMC	−0.30	0.16	5	−0.2
SMC	−0.48	0.04	5	−0.6

[a][Fe/H] from Luck and Lambert (1985,1992)
[b]s.e. = Internal standard error of the mean
[c]N = The number of Cepheids observed
[d][Fe/H] for young objects from Feast (1989)

Recently Andrievsky et al. (1994) have presented spectroscopic results suggesting that galactic double-mode Cepheids have a large range in [Fe/H] (−0.43 to −0.03) and that this correlates with P_1/P_0. This result is particularly surprising in view of the MACHO work (Alcock et al. 1995) which shows narrow P_1/P_0 versus P_o sequences for the Galaxy and the LMC respectively, displaced from one another in the sense expected for the lower metallicity of the LMC Cepheids. This strongly suggests, at least, that at a given period there is a very small range of metallicities in the samples from either galaxy.

6. Magnitude bias

Sandage (1988) was probably the first to discuss in a systematic way the bias in the use of PL relations with a magnitude limited sample of Cepheids all at the same true distance. The basic reason for this bias is the finite range in absolute magnitude at a fixed period. For reasons similar to those given in section 4, this problem can be circumvented in certain cases by using a PL relation derived from a regression of log P on magnitude which is the inverse of the procedure normally adopted. As an illustration the data at B for the galaxy NGC3109 (Sandage & Carlson 1988, and Capaccioli et al. 1992) have been analysed in this way. Table 2 shows the slopes, α, of the inverse regression derived for this galaxy and for the LMC, using in the latter case the data listed by Sandage (1988) restricted to approximately the same range in M_B as in the NGC3109 sample. The difference in slope between the LMC and NGC3109 is not significant in the case of the Capaccioli et al. data. The difference in the case of the Sandage & Carlson data suggests a possible scale error in this early photometry. Using the inverse relation, the distance modulus of NGC3109 from the data of Capaccioli et al. is 0.23 mag less than that given by the direct method.

Effects such as these will be less significant at longer wavelengths since the width of the PL relation decreases with increasing wavelength. However they should be taken into account for the highest accuracy.

Table 2. Inverse Regression Slope Test in NGC3109

Reference	α
LMC	-0.310±0.036
Sandage & Carlson 1988	-0.467±0.044
Capaccioli et al. 1992	-0.270±0.072

7. Cepheids and H_0

The detection and photometry of three Cepheids in NGC4571, a probable member of the Virgo cluster by a group using the CFHT (Pierce et al. 1994) was a major technical achievement and shows the sort of work that is now becoming possible with ground-based telescopes. This work raises an interesting problem. We are used to expressing the intrinsic scatter in a PL relation by a Gaussian standard deviation, (σ). However the scatter about Cepheid PL relations is not Gaussian but approximates to a uniform distribution through a strip of the expected width ($\sqrt{12}\sigma$). For instance the frequency distribution of residuals from a PL(V_0) relation in the LMC (restricted to logP $\geq$ 0.8 since the instability strip narrows at shorter periods) is non-Gaussian at the 1 percent level whilst a uniform distribution is a reasonable fit. The NGC4571 observations were made at R where the PL relation in the LMC has a width characterised by $\sigma \sim 0.25$ (Madore & Freedman 1991) so that the total width ($\sqrt{12}\sigma$) is about 0.87 mag. It is perhaps surprising in view of such a width that the three Cepheids discovered in NGC4571 lie almost exactly along a line of the same slope as the mean LMC PL relation. However given that they do, a uniform distribution of residuals about this relation implies that the observed NGC4571 PL line can lie anywhere in the PL strip over an absolute magnitude range of 0.87 with equal probability. Thus simply adopting the other assumptions that go into the derivation of H_0 by Pierce et al. one finds that this constant can lie anywhere in the range 71 to 106 km/s/Mpc with equal probability, and outside this range with some decreasing probability. The value quoted by Pierce et al. (87±7) is only the centre of the possible range. The alternative to the above interpretation would of course be that for some reason the Cepheid instability strip in NGC4571 was very narrow. One might then worry about the appropriate zero point of the PL relation.

We come finally to the HST work on extragalactic Cepheids. Table 3 lists the salient results from the papers known to me. These papers contain a large amount of important material. For instance it is very informative to compare the rather different methods used by the various groups to obtain their basic photometric zero point, on which of course everything else depends. Freedman et al. (1994a) give an extensive and valuable discussion of photometric and other possible sources of error. One might think that their analysis of the errors applied rather generally to HST work of this kind. Freedman et al. (1994b) obtain their best estimate of H_0 (= 77 ± 16 km/s/Mpc) by taking M100 as a member of the Virgo cluster and assuming a relative distance for Virgo and Coma. (A value of H_0 from Virgo alone is rather sensitive to the (uncertain) cosmological recession velocity of Virgo). Saha et al. (1994, 1995) use their galaxy distances to calibrate

Table 3. Distance Moduli from HST Cepheid Observations

Reference	Galaxy	Modulus	Number
Freedman et al. (1994a)	M81	27.80±0.20	30
Freedman et al. (1994b)	M100	31.16±0.20	20
Saha et al. (1994)	IC4182	28.36±0.09[a]	28
Saha et al. (1995)	NGC5253	28.08±0.10[a]	14
Tanvir et al. (1995)[b]	M96	30.32±0.15	8

[a]Not taking into account errors in the adopted reddening
[b]provisional result

SNIa as distance indicators and deduce $H_0 = 54 \pm 8$. If we take these results at their face value, there seem to be at least two possible ways to proceed. We can note that the quoted values each lie within 1σ of 62 km/s/Mpc. Or, we can allow for 2σ errors, which after all are not so improbable. In that case the two sets of data confine H_0 to the range 45 to 70. Whatever value of H_0 one adopts as a working hypothesis one should bear in mind the wise comment of Freedman et al. (1994b);

"We do not wish to mislead the reader into believing that the problem of determining H_0 has been solved. It has not."

What the recent fine work has shown is that the distance to individual galaxies as far away as Virgo can be obtained with an uncertainty of about 10 percent. The derivation of H_0 requires the use of secondary indicators; SNIa, the Tully-Fisher relation, the various methods used to estimate the depth and structure of Virgo and its distance relative to Coma, etc. None of these indicators have as yet the certainty which we attach to the Cepheid scale. The testing and calibration of these secondary indicators depends heavily on the determination of Cepheid distances to many more galaxies.

8. Addendum

In his paper at this meeting Alistair Walker mentioned a recent determination of the distance to LMC SN1987A by Gould. This depends on a rather detailed modelling of the SN ring emission. A limit to the LMC modulus ($< 18.37 \pm 0.04$) was derived. The quoted error is based on plausible variations in the adopted model. The difference from the result obtained by Panagia et al. (1991) (= 18.50) emphasises the importance of variations in the modelling procedure as was stressed by Dwek & Felten (1992). The work of Gould still leaves some aspects of the SN ring to be understood and it is therefore perhaps unwise to depend solely on the SN distance in setting up an extragalactic distance scale. The best option is probably to average Gould's upper limit (which is essentially also his best estimate) with the Cepheid and RR Lyrae results discussed above. In that case the LMC modulus would be close to 18.5.

If, alternatively, one chose to adopt the view that the difference of Gould's modulus and the Cepheid result (> 0.20 mag) rested entirely with an error in the

Cepheid scale then this could be due either to an error in the absolute calibration or to a difference of unknown origin between Cepheids in different galaxies (in this case the LMC and our Galaxy). This latter possibility would throw doubt on the value of Cepheids as even relative extragalactic-distance indicators. It seems unnecessary to adopt such an extreme position but clearly more work is desirable both on the SN ring modelling and on the Cepheid and RR Lyrae scales.

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Discussion

Stift: (i) You stated that metallicity effects decrease with wavelength. That is at variance with basic astrophysics and I warmly recommend my poster for detailed arguments.
(ii) You use rather wide strip widths (0.8 - 0.9 mag in R). I estimated ~0.5 mag width in V from BV photometry, using a uniform distribution of Cepheids in the instability strip in my numerical simulations.
(iii) The PLC relation is notoriously sensitive to metallicity. Using it for the determination of the 3-D structure of the Magellanic Clouds without metallicity corrections could result in any depth structure being an artefact of metallicity gradients.

Feast: (i) Metallicity effects are more important at B than at V (see section 5 para 3 and Feast 1991)
(ii) In discussing the work of Pierce et al. (section 7) I adopted for consistency the value of σ which they quote (from Freedman & Madore). I quite agree that the width of the strip at R is not particularly well determined. For the LMC data discussed in section 7, the strip width at V is about 0.74 mag.
c) The 3-D structure of the SMC, though seen most clearly in the available BVI(PLC) data, is confirmed by other Cepheid and non-Cepheid work (see, e.g., Laney & Stobie 1986). For discussions of the PLC and PL relations as well as reddening and metallicity determinations see Feast 1988, 1989, 1991.

Barnes: I think you have been unfair to the Baade-Wesselink optical results on Cepheids by comparing $(V - M_V)_{BW}$ *uncorrected* for metallicity with $(V - M_V)_{PLC}$ *corrected* for metallicity. The LMC $(V - M_V)_{BW}$ *corrected* is 18.67, negligibly different from your $(V - M_V)_{PLC}$ of 18.57. If you believe the (preliminary) HIPPARCOS Pleiades result of 5.67, your result would be 18.62. Whatever the relative merits of optical and IR BW methods (and I agree that the IR approach is superior), the optical results agree with the ZAMS method in the Galaxy and in the LMC and in the SMC (Barnes, Moffett & Gieren, 1994, ApJ Letters).

Feast: I have not entirely understood your comment since a metallicity correction to a PL relation at V (Galaxy to LMC) is expected to be small and to increase the adopted LMC modulus (e.g., Laney & Stobie 1994 Table 6). However, this correction is not directly relevant here, since we are in effect simply comparing PL relations derived in different ways. The (uncorrected) PL(V) modulus of the LMC based on clusters and associations is 18.50 (Laney & Stobie 1994) or 18.55 when adjusted to the cluster scale of the present paper. The uncorrected LMC modulus based on the same LMC data and the PL(V) relation from Baade-Wesselink work at V,R (Gieren et al. 1993) is distinctly larger (18.80). This difference is a direct result of the difference between the galactic PL(V) relations derived by the two different methods. It is this difference which now seems understandable in the light of the IR work of Laney & Stobie.

Lamers: Can you explain why it is better to use the 'inverse' relation for the PL relation, rather than the normal relation?

Feast: For Cepheids all at the same distance and selected only by magnitude at the relevant wavelength, the mean logP at a given magnitude is obviously

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independent of selection effects and the 'inverse' relation, by definition, gives a best estimate of mean logP at any given magnitude. This leads to the use made of the inverse relation in the text. On the other hand, the mean magnitude at a given logP is strongly affected by selection according to magnitude. Complications can arise in principle (e.g., varying interstellar extinction from star to star). Also if one worked at a wavelength at which the evolutionary tracks sloped steeply through the instability strip then a variation in mass function from galaxy to galaxy might have some effect on the inverse relation, but this is likely to be small at least at visual wavelengths. (The mass function effect could be important in using the direct relation.)

Hughes: The Malmquist bias in measuring distances to the galaxies using fits to PL relations arises from incompleteness in faint magnitudes, which gives an asymmetric distribution about the PL strip. If this is not corrected, then a regression using either V or logP will still give the wrong fit.

Feast: If the only problem in the case you mention is due to incompleteness or faint magnitudes, then it is correctly dealt with by using the 'inverse' relation (see sections 4 and 6, and my reply to Lamers, also Feast 1994 and Schechter AJ 85, 801, 1980).

Turner: Let me present a less optimistic view on the potential for finding additional Milky Way cluster Cepheid calibrators with fainter searches for Cepheids. Most Cepheids are old enough that their parent clusters are well on the way towards dynamical dissolution. The result is that most calibrating clusters in the local sample are very poorly populated. Distant clusters of the same type would, therefore, be extremely difficult to recognise.

Feast: The importance of the DENIS and 2MASS surveys is that the IR data may well allow clusters and associations of (reddened) early type stars to be picked out rather easily (see Feast 1994a)

Evans: I would just like to make a comment of the use of binary Cepheids for determining the PLC zero point. At the moment we have 9 - 10 with very low reddening. This includes several interesting stars such as a double-mode Cepheid plus two singly-excited overtone Cepheids. The external error as determined by comparing with the Feast & Walker PLC is $\sim$0.35 mag (s.d. per star). Future work is to use IUE spectra of the Pleiades which Ron Pitts and I are working on to directly ratio with the comparison spectra, giving us a distance directly related to the HIPPARCOS results.

Cacciari: The M_{bol} - [Fe/H] relation for RR Lyraes you quoted from Sandage (1990) has a slope 0.37 which is based entirely on the assumed mass-metallicity relation. Sandage assumed the relation from double-mode pulsators known in 1990 ($\sim$0.1 $M_{\odot}$ difference between M3 and M15) but later results using OPAL opacities show a much flatter dependence on metallicity, which would lead to a M_{bol} - [Fe/H] slope around 0.25. We (Fusi Pecci et al.) have very recent HST data of globular clusters in M31, to obtain CM diagrams and the luminosity level of the horizontal branch. From this data we should be able to define independently, at least, the slope of the M_{bol} - [Fe/H] relation very soon.