NUCLEAR OPERATORS ON SPACES OF CONTINUOUS VECTOR-VALUED FUNCTIONS

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Let Ω be a compact Hausdorff space, let E be a Banach space, and let $C(\Omega, E)$ stand for the Banach space of continuous E-valued functions on Ω under supnorm. It is well known [**3**, p. 182] that if F is a Banach space then any bounded linear operator $T:C(\Omega, E)\rightarrow F$ has a finitely additive vector measure G defined on the σ-field of Borel subsets of Ω with values in the space $\mathscr{L}(E, F^{**})$ of bounded linear operators from E to the second dual F^{**} of F. The measure G is said to represent T. The purpose of this note is to study the interplay between certain properties of the operator T and properties of the representing measure G. Precisely, one of our goals is to study when one can characterize nuclear operators in terms of their representing measures. This is of course motivated by a well-known theorem of L. Schwartz [**5**] (see also [**3**, p. 173]) concerning nuclear operators on spaces $C(\Omega)$ of continuous scalar-valued functions. The study of nuclear operators on spaces $C(\Omega, E)$ of continuous vector-valued functions was initiated in [**1**], where the author extended Schwartz's result in case E^* has the Radon-Nikodym property. In this paper, we will show that the condition on E^* to have the Radon-Nikodym property is necessary to have a Schwartz's type theorem. This leads to a new characterization of dual spaces E^* with the Radon-Nikodym property. In [**2**], it was shown that if $T:C(\Omega, E)\rightarrow F$ is nuclear than its representing measure G takes its values in the space $\mathcal{N}(E, F)$ of nuclear operators from E to F. One of the results of this paper is that if $T:C(\Omega, E)\rightarrow F$ is nuclear then its representing measure G is countably additive and of bounded variation as a vector measure taking its values in $\mathcal{N}(E, F)$ equipped with the nuclear norm. Finally, we show by easy examples that the above mentioned conditions on the representing measure G do not characterize nuclear operators on $C(\Omega, E)$ spaces, and we also look at cases where nuclear operators are indeed characterized by the above two conditions. For all undefined notions and terminologies, we refer the reader to [**3**].

0. Preliminaries. If X and Y are Banach spaces then $\mathscr{L}(X, Y)$ will stand for the space of bounded linear operators from X to Y. An element T in $\mathscr{L}(X, Y)$ is said to be a *nuclear operator* if there exist sequences (x_n^*) in X^* and (y_n) in Y such that for each x in X

$$T(x) = \sum_{n=1}^{\infty} x_n^*(x)y_n,$$

and

$$\sum_{n=1}^{\infty} \|x_n^*\| \, \|y_n\| < \infty.$$

We say that $\sum_n x_n^* \otimes y_n$ represents the nuclear operator T. The *nuclear norm* of a

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nuclear operator $T: X \to Y$ is defined by:

$$\|T\|_{\text{nuc}} = \inf\left\{\sum_n \|x_n^*\| \, \|y_n\|\right\},$$

where the infimum is taken over all sequences (x_n^*) and (y_n) such that $T(x) = \sum_{n=1}^{\infty} x_n^*(x) y_n$ holds for all x in X. The nuclear operators from X to Y form a normed linear space under the nuclear norm [**3**, p. 170], which we shall denote by $\mathcal{N}(X, Y)$.

If Ω is a compact Hausdorff space and E is a Banach space then $C(\Omega, E)$ will stand for the space of continuous E-valued functions defined on Ω under supnorm. If $E = \mathbb{R}$ or $\mathbb{C}$, we will simply write $C(\Omega)$. The space $M(\Omega, E^*)$ will stand for the space of all regular E^*-valued vector measures μ defined on the σ-field Σ of Borel subsets of Ω that are of bounded variation. We shall use the fact (see [**3**, p. 182]) that $M(\Omega, E^*)$ is a Banach space under the variation norm $\|\mu\| = |\mu|(\Omega)$, and that $M(\Omega, E^*)$ is isometrically isomorphic to the dual space $C(\Omega, E)^*$. When $E = \mathbb{R}$ or $\mathbb{C}$, we will simply write $M(\Omega)$. If $\mu \in M(\Omega, E^*)$ then for each $e \in E$ we will denote by $\langle e, \mu \rangle$ the element of $M(\Omega)$ such that for each $f \in C(\Omega)$,

$$\int f \, d\langle e, \mu \rangle = \mu(f \otimes e),$$

where $f \otimes e$ is the element in $C(\Omega, E)$ such that $f \otimes e(\omega) = f(\omega)e$ for each $\omega \in \Omega$.

If $\nu \in M(\Omega)$ and $x^* \in E^*$, we denote by $\nu \otimes x^*$ the element of $M(\Omega, E^*)$ which to each Borel subset B of Ω associates the element $\nu(B)x^*$ of E^*. If E and F are Banach spaces and Ω is a compact Hausdorff space then we will denote by G the finitely additive $\mathcal{L}(E, F^{**})$-valued measure representing the operator T. Recall that if B is a Borel subset of Ω then

$$G(B)e = T^{**}(\phi_{B,e})$$

for all $e \in E$, where $\phi_{B,e}$ is the element of $M(\Omega, E^*)^*$ such that, for each $\lambda \in M(\Omega, E^*)$,

$$\phi_{B,e}(\lambda) = \lambda(B)(e)$$

and T^{**} is the second adjoint of T.

Finally we recall that a Banach space X has the *Radon-Nikodym property* (RNP) if, for every finite measure space (S, Σ, μ) and every vector measure $m: \Sigma \to X$ of bounded variation that is absolutely continuous with respect to μ, there exists a strongly measurable Bochner integrable function $g: S \to X$ such that

$$m(A) = \int_A f \, d\mu$$

for each $A \in \Sigma$.

1. Some properties of the measure representing a nuclear operator. Throughout, we let Ω be a compact Hausdorff space with Σ its σ-field of Borel subsets and we let E and F be Banach spaces. In what follows, we shall look at some of the properties that a nuclear operator on $C(\Omega, E)$ induces on its representing measure G. In [**2**], it was shown that if $T: C(\Omega, E) \to F$ is nuclear then $G: \Sigma \to \mathcal{N}(E, F)$. In the next proposition we shall show that G enjoys a stronger property.

PROPOSITION 1. *If $T:C(\Omega,E)\to F$ is a nuclear operator with representing measure G then*:

(i) *for each Borel subset B of Ω, $G(B):E\to F$ is a nuclear operator, and*

(ii) *the measure G is countably additive and is of bounded variation as a vector measure taking its values in $\mathcal{N}(E,F)$ under the nuclear norm.*

Proof. If $T:C(\Omega,E)\to F$ is nuclear then there are sequences (μ_n) in $M(\Omega,E^*)$ and (y_n) in F such that, for all $f\in C(\Omega,E)$,

$$Tf=\sum_{n=1}^{\infty}\mu_n(f)y_n$$

and

$$\sum_{n=1}^{\infty}\|\mu_n\|\,\|y_n\|<\infty.$$

In particular the operator T^{**} is nuclear with

$$T^{**}(\phi)=\sum_{n=1}^{\infty}\phi(\mu_n)y_n$$

for all $\phi\in C(\Omega,E)^{**}$. This implies that, for any Borel subset B of Ω and for all $e\in E$,

$$\begin{aligned}G(B)e&=T^{**}(\phi_{B,e})\\&=\sum_{n=1}^{\infty}\mu_n(B)(e)y_n.\end{aligned}$$

Since $(\mu_n(B))$ is a sequence in E^* and $\sum_{n=1}^{\infty}\|\mu_n(B)\|\,\|y_n\|\le\sum_{n=1}^{\infty}\|\mu_n\|\,\|y_n\|<\infty$, this will quickly show that, for each Borel subset B of Ω, the operator $G(B):E\to F$ is nuclear. To prove (ii), note that since $G:\Sigma\to\mathcal{N}(E,F)$, let $|G|_{\text{nuc}}$ denote the extended non-negative function whose value on a set B in Σ is given by

$$|G|_{\text{nuc}}(B)=\sup_{\pi}\sum_{B_i\in\pi}\|G(B_i)\|_{\text{nuc}},$$

where the supremum is taken over all finite partitions π of B. We first show that $|G|_{\text{nuc}}(\Omega)<\infty$.

For this, note that if $\pi=\{B_i\}$ is a finite partition of Ω then

$$\begin{aligned}\sum_{B_i\in\pi}\|G(B_i)\|_{\text{nuc}}&\le\sum_{B_i\in\pi}\sum_{n=1}^{\infty}\|\mu_n(B_i)\|\,\|y_n\|\\&\le\sum_{B_i\in\pi}\sum_{n=1}^{\infty}|\mu_n|(B_i)\,\|y_n\|\\&\le\sum_{n=1}^{\infty}\sum_{B_i\in\pi}|\mu_n|(B_i)\,\|y_n\|\\&\le\sum_{n=1}^{\infty}|\mu_n|(\Omega)\,\|y_n\|\\&=\sum_{n=1}^{\infty}\|\mu_n\|\,\|y_n\|<\infty.\end{aligned}$$

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This of course shows that $|G|_{\text{nuc}}(\Omega)<\infty$. To complete the proof of (ii), we need to show that G is countably additive. First note that since $|G|_{\text{nuc}}(\Omega)<\infty$, it follows from [**3**, p. 7] that G is strongly additive, that is, if (B_i) is a sequence of pairwise disjoint Borel subsets of Ω, we have that the series $\sum_{i=1}^{\infty} G(B_i)$ converges in $\mathcal{N}(E,F)$. To complete the proof, we need to check that the series $\sum_{i=1}^{\infty} G(B_i)$ converges to $G\left(\bigcup_{i\ge 1} B_i\right)$ in $\mathcal{N}(E,F)$. To this end, consider a series $\sum_{n=1}^{\infty} \mu_n \otimes y_n$ representing the operator T such that

$$\sum_{n=1}^{\infty} \|\mu_n\|\,\|y_n\|<\infty.$$

Without loss of generality, we may and shall assume that $\|y_n\|\le 1$ for all $n\ge 1$. Let $\epsilon>0$ and pick $N\in\mathbb{N}$ such that

$$\sum_{n=N+1}^{\infty} \|\mu_n\|<\epsilon/2.$$

Since $\mu_1,\mu_2,\ldots,$ and $\mu_N\in M(\Omega,E^*)$, there exists $K\in\mathbb{N}$ such that

$$|\mu_j|\left(\bigcup_{i\ge K} B_i\right)<\frac{\epsilon}{2^{j+1}} \quad \text{for all} \quad j=1,\ldots,N.$$

This implies that

$$\begin{aligned}\left\|G\left(\bigcup_{i\ge 1} B_i\right)-\sum_{i=1}^{K-1} G(B_i)\right\|_{\text{nuc}} &= \left\|G\left(\bigcup_{i\ge K} B_i\right)\right\|_{\text{nuc}} \le \sum_{n=1}^{\infty}\left\|\mu_n\left(\bigcup_{i\ge K} B_i\right)\right\| \\ &\le \sum_{n=1}^{N}|\mu_n|\left(\bigcup_{i\ge K} B_i\right)+\sum_{n=N+1}^{\infty}|\mu_n|\left(\bigcup_{i\ge K} B_i\right)<\epsilon.\end{aligned}$$

This shows that G is countably additive as a vector measure taking its values in $\mathcal{N}(E,F)$.

The first question that arises at this stage, is when do properties (i) and (ii) above characterize nuclear operators? The next proposition shows that one necessary condition is that F should have the Radon-Nikodym property.

Proposition 2. *If F fails RNP then, for any Banach space E, there is a non-nuclear operator $T:C([0,1],E)\to F$ whose representing measure takes its values in $\mathcal{N}(E,F)$ and is of bounded variation as a countably additive vector measure taking its values in $\mathcal{N}(E,F)$.*

Proof. If F lacks RNP then, by [**3**, p. 175], there is an F-valued countably additive vector measure m on the Borel subsets of $[0,1]$ such that m is of bounded variation, m is absolutely continuous with respect to Lebesgue measure but m admits no Bochner integrable derivative with respect to its variation $|m|$. If we define $T':C[0,1]\to F$ by

$$T'(f)=\int_{[0,1]} f\,dm$$

then T' is not a nuclear operator (see [**5**] or [**3**, p. 173]). Now fix $e\neq 0$ in E; then choose e^* in E^* with $e^*(e)=1$ and define $T:C([0,1],E)\to F$ by

$$T(\phi)=T'(e^*\circ\phi)$$

for each ϕ in $C([0,1],E)$. In particular, for each f in $C[0,1]$ and x in E, we have

$$T(f\otimes x)=e^*(x)T'(f).$$

It is clear that the measure representing the operator T is the measure $G=m\otimes e^*$ which to every Borel subset B of $[0,1]$ associates the one-rank operator such that $G(B)x=e^*(x)m(B)$ for each $x\in E$. On the other hand, for any finite partition $\pi=\{B_i\}$ of $[0,1]$ we have

$$\begin{aligned}\sum_{B_i\in\pi}\|G(B_i)\|_{\text{nuc}}&\le\sum_{B_i\in\pi}\|e^*\|\,\|m(B_i)\|\\&\le\|e^*\|\,|m|\,([0,1])<\infty.\end{aligned}$$

So

$$|G|_{\text{nuc}}([0,1])<\infty.$$

If T were a nuclear operator then there would exist (μ_n) in $M([0,1],E^*)$ and (y_n) in F such that, for each ϕ in $C([0,1],E)$,

$$T(\phi)=\sum_{n=1}^{\infty}\mu_n(\phi)y_n.$$

In particular, for each $f\in C[0,1]$,

$$\begin{aligned}T(f\otimes e)&=T'(f)\\&=\sum_{n=1}^{\infty}\mu_n(f\otimes e)y_n\\&=\sum_{n=1}^{\infty}\int_{[0,1]}f\,d\langle e,\mu_n\rangle y_n.\end{aligned}$$

This of course shows that T' is represented by $\sum_n\langle e,\mu_n\rangle\otimes y_n$; moreover

$$\sum_{n=1}^{\infty}\|\langle e,\mu_n\rangle\|\,\|y_n\|\le\|e\|\sum_n\|\mu_n\|\,\|y_n\|<\infty,$$

which implies that T' is nuclear. This contradiction finishes the proof.

This brings us to ask the following question. Let E and F be Banach spaces such that F has the Radon–Nikodym property. Let Ω be a compact Hausdorff space and let $T:C(\Omega,E)\to F$ be a bounded linear operator satisfying conditions (i) and (ii) of Proposition 2. Is T nuclear? Recently, the first named author has given a positive answer to the above question when F is complemented in its bidual F^{**} (see [**4**]).

2. The Radon–Nikodym property and nuclear operators. Throughout this section Ω is a compact Hausdorff space and E and F are Banach spaces. Every bounded linear operator $T:C(\Omega,E)\to F$ induces a bounded linear operator $T^{\#}:C(\Omega)\to\mathscr{L}(E,F)$, where, for each f in $C(\Omega)$,

$$T^{\#}(f)(e)=T(f\otimes e)$$

for all $e\in E$.

In this section we shall look at the interplay of the two operators T and $T^{\#}$. The next result shows that, when T is nuclear, the range of $T^{\#}$ is in $\mathscr{N}(E,F)$.

THEOREM 3. *If $T:(\Omega, E)\to F$ is nuclear then $T^{\#}$ takes its values in $\mathcal{N}(E,F)$.*

Proof. Let (μ_n) in $M(\Omega, E^*)$ and (y_n) in F be such that $\sum_{n=1}^{\infty}\|\mu_n\|\,\|y_n\|<\infty$ and, for each ϕ in $C(\Omega, E)$,

$$T(\phi)=\sum_{n=1}^{\infty}\mu_n(\phi)y_n.$$

For each $n\geq 1$, define $\mu_n^{\#}:C(\Omega)\to E^*$ by

$$\mu_n^{\#}(f)e=\mu_n(f\otimes e)$$

for each $f\in C(\Omega)$ and $e\in E$. Since, for each $f\in C(\Omega)$ and $e\in E$,

$$\begin{aligned}T^{\#}(f)(e)&=T(f\otimes e)\\&=\sum_{n=1}^{\infty}\mu_n(f\otimes e)y_n\\&=\sum_{n=1}^{\infty}\mu_n^{\#}(f)(e)y_n,\end{aligned}$$

it follows that, for each f in $C(\Omega)$, $T^{\#}(f)$ can be represented by the series $\sum_{n=1}^{\infty}\mu_n^{\#}(f)\otimes y_n$. Moreover, since $\sum_{n=1}^{\infty}\|\mu_n^{\#}(f)\|\,\|y_n\|\leq\|f\|\sum_{n=1}^{\infty}\|\mu_n\|\,\|f_n\|<\infty$, it follows that $T^{\#}(f)$ is a nuclear operator from E to F for each $f\in C(\Omega)$.

The next result illustrates one key relationship between T and $T^{\#}$.

THEOREM 4. *The operator $T:C(\Omega,E)\to F$ is nuclear whenever $T^{\#}:C(\Omega)\to\mathcal{N}(E,F)$ is nuclear.*

Proof. Assume $T^{\#}:C(\Omega)\to\mathcal{N}(E,F)$ is nuclear. Then there exist sequences (ν_n) in $C(\Omega)^*$ and (N_n) in $\mathcal{N}(E,F)$ such that, for each $f\in C(\Omega)$,

$$T^{\#}(f)=\sum_{n=1}^{\infty}\nu_n(f)N_n$$

and

$$\sum_{n=1}^{\infty}\|\nu_n\|\,\|N_n\|_{\text{nuc}}<\infty.$$

Without loss of generality, we may and do assume that $\|\nu_n\|\leq 1$ for all $n\geq 1$. Similarly, since each N_n is a nuclear operator for each $n\geq 1$, there are sequences $(e_{n,m}^*)$ in E^* and $(y_{n,m})$ in F such that, for all $e\in E$,

$$N_n(e)=\sum_{m=1}^{\infty}e_{n,m}^*(e)y_{n,m}$$

and

$$\sum_{m=1}^{\infty}\|e_{n,m}^*\|\,\|y_{n,m}\|\leq\|N_n\|_{\text{nuc}}+1/2^n.$$

Hence

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\|\nu_n\|\,\|e_{n,m}^*\|\,\|y_{n,m}\|\leq\sum_{n=1}^{\infty}\|\nu_n\|\,\|N_n\|_{\text{nuc}}+\sum_{n=1}^{\infty}\|\nu_n\|/2^n<\infty. \tag{1}$$

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Moreover, for each $f \in C(\Omega)$ and $e \in E$,

$$\begin{aligned} T(f \otimes e) &= T^{\#}(f)(e) \\ &= \left(\sum_{n=1}^{\infty} \nu_n(f)N_n\right)(e) \\ &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \nu_n(f)e^*_{n,m}(e)y_{n,m} \\ &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \nu_m \otimes e^*_{n,m}(f \otimes e)y_{n,m}. \end{aligned}$$

Since the set $\{f \otimes e : f \in C(\Omega), e \in E\}$ is total in $C(\Omega, E)$, we can assert that T is represented by the double indexed series $\sum_n \sum_m (\nu_n \otimes e^*_{n,m}) \otimes y_{n,m}$, where $(\nu_n \otimes e^*_{n,m})$ is in $M(\Omega, E^*) \cong C(\Omega, E)^*$ and $(y_{n,m})$ is in F. An appeal to (1) shows that T is nuclear.

In the following, we will show that the converse of Theorem 4 does not always hold. But if in addition E^* is assumed to have the Radon–Nikodym property then a close look at [**1**, Theorem III.4] reveals that any nuclear operator $T : C(\Omega, E) \to F$ will indeed induce a nuclear operator $T^{\#} : C(\Omega) \to \mathcal{N}(E, F)$. Moreover our next result shows how critical the condition on E^* to have the Radon–Nikodym property is in order that T nuclear implies $T^{\#}$ nuclear. As a matter of fact, one can characterize dual Banach spaces with the Radon–Nikodym property as follows.

THEOREM 5. *Let E and F be Banach spaces. The following properties are equivalent*:

(i) *the dual space E^* has RNP*;

(ii) *for every compact space Ω, a bounded linear operator $T : C(\Omega, E) \to F$ is nuclear if and only if $T^{\#} : C(\Omega) \to \mathcal{N}(E, F)$ is nuclear.*

Proof. (ii)$\Rightarrow$(i). If E^* fails RNP, by [**3**, p. 175], there exists $\mu \in M([0, 1], E^*)$ such that the operator $\mu^{\#} : C[0, 1] \to E^*$ defined by

$$\mu^{\#}(g)e = \mu(g \otimes e)$$

for all $g \in C[0, 1]$ and $e \in E$ is not nuclear. Choose $y \in F$ such that $y \neq 0$, and define $T : C([0, 1], E) \to F$ by

$$T(\phi) = \mu(\phi)y$$

for all $\phi \in C([0, 1], E)$. It is clear that T is a rank-one operator; hence it is nuclear. The operator $T^{\#} : C[0, 1] \to \mathcal{N}(E, F)$ induced by T is clearly the operator such that, for each $g \in C[0, 1]$,

$$T^{\#}(g) = \mu^{\#}(g) \otimes y.$$

To see that $T^{\#}$ is not nuclear, note that, for each $y^* \in F^*$, we can define the operator $T^{\#}_{y^*} : C[0, 1] \to E^*$ by

$$T^{\#}_{y^*}(g) = (T^{\#}g)^*(y^*)$$

for each $g \in C[0, 1]$.

If $T^{\#}$ were a nuclear operator then $T^{\#}_{y^*}$ would also be a nuclear operator for each $y^* \in F^*$. This of course follows from the fact that $T^{\#}_{y^*}$ is the composition of $T^{\#}$ and the bounded linear operator from $\mathcal{N}(E, F)$ to E^* which to an element N in $\mathcal{N}(E, F)$

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associates the element $N^*(y^*)$ in E^*. But, for each g in $C[0, 1]$,

$$T^{\#}_{y^*}(g) = y^*(y)\mu^{\#}(g).$$

By the Hahn-Banach theorem, choose y^* in F^* such that $y^*(y) = 1$; then, for this particular y^*, we have

$$T^{\#}_{y^*} = \mu^{\#}.$$

This contradiction shows that $T^{\#}$ can not be nuclear. This proves (ii)$\Rightarrow$(i).

The proof of (i)$\Rightarrow$(ii) is implicit in [**1**, Theorem III.4]. We shall provide a sketch of a proof for the sake of completeness. For this, assume that $T: C(\Omega, E) \to F$ is nuclear and that E^* has the Radon–Nikodym property. By Proposition 1, we know that the measure G representing the operator T is countably additive as a vector measure taking its values in $\mathcal{N}(E, F)$ and $|G|_{\text{nuc}}(\Omega) < \infty$. Here it is easy to note that it follows from general vector measure techniques [**3**, p. 3] that $|G|_{\text{nuc}}$ is countably additive. The proof that $T^{\#}$ is nuclear now follows the proof of the scalar case as given in [**3**, p. 173] with some minor changes. For instance, since E^* has the Radon–Nikodym property, one can proceed to produce a Bochner $|G|_{\text{nuc}}$-integrable function

$$H: \Omega \to \mathcal{N}(E, F)$$

such that, for each Borel subset B of Ω,

$$G(B) = \int_B H(\omega)\, d\, |G|_{\text{nuc}}(\omega)$$

and, for each $f \in C(\Omega)$ and each $e \in E$,

$$T(f \otimes e) = \int_\Omega f(\omega)H(\omega)(e)\, d\, |G|_{\text{nuc}}(\omega).$$

Hence, for each $f \in C(\Omega)$,

$$T^{\#}(f) = \int_\Omega f(\omega)H(\omega)\, d\, |G|_{\text{nuc}}(\omega).$$

Another appeal to [**3**, p. 173] shows that $T^{\#}$ is nuclear.

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