A REVIEW OF COMETS AND NONGRAVITATIONAL FORCES

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Abstract. A review is given on the recent advances in understanding the nature of the nongravitational forces that affect the motions of nearly all active comets.

1. Introduction

One of the primary reasons for Whipple's introduction of his icy conglomerate model for the cometary nucleus was to explain the so-called nongravitational accelerations that were evident in the motions of many active periodic comets (Whipple 1950, 1951). That is, even after all the gravitational perturbations of the planets were taken into account, the observations of many active comets could not be well represented without the introduction of additional so-called nongravitational effects into the dynamical model. These effects are brought about by cometary activity when momentum is transferred to the nucleus by the sublimating ices. Whipple noted that for an active, rotating, icy cometary nucleus, a thermal lag between cometary noon and the time of maximum outgassing would introduce transverse accelerations in a comet's motion. In an attempt to model these effects, Marsden (1968, 1969) first introduced a semi-empirical nongravitational acceleration model using what are now termed Style I nongravitational parameters.

Style II parameters were added when Marsden et al. (1973) introduced what has become the standard, or symmetric, nongravitational acceleration model for cometary motions; a rotating cometary nucleus is assumed to undergo vaporization from water ice that acts symmetrically with respect to perihelion. That is, at the same heliocentric distance before and after perihelion, the cometary nucleus experiences the same nongravitational acceleration. The cometary equations of motion are written :

$$\frac{d^2\mathbf{r}}{dt^2} = -\mu\frac{\mathbf{r}}{r^3} + \frac{\partial R}{\partial \mathbf{r}} + A_1 g(r)\hat{\mathbf{r}} + A_2 g(r)\hat{\mathbf{T}} \tag{1}$$

where $g(r) = \alpha(r/r_o)^{-m}(1+(r/r_o)^n)^{-k}$.
The acceleration is given in astronomical units/(ephemeris day)^2, μ is the product of the gravitational constant and the solar mass, and R is the planetary disturbing function. The scale distance r_o is the heliocentric distance inside which the bulk of solar insolation goes to sublimating the comet's ices. For water ice, $r_o = 2.808$ AU and the normalizing constant $\alpha = 0.111262$. The exponents m, n, and k equal 2.15, 5.093 and 4.6142, respectively. The nongravitational acceleration is represented by a radial term, A_1 $g(r)$, and a transverse term, A_2 $g(r)$, in the equations of motion.

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The radial unit vector ($\hat{\mathbf{r}}$) is defined outward along the Sun-comet line, while the transverse unit vector ($\hat{\mathbf{T}}$) is directed normal to $\hat{\mathbf{r}}$, in the orbit plane, and in the direction of the comet's motion. An acceleration component normal to the orbit plane is likely also present for most active comets, but its periodic nature makes a meaningful solution for it difficult in these computations because we are solving for an average nongravitational acceleration effect over three or more apparitions. If the comet's nucleus were not rotating, the outgassing in this model would always be toward the Sun and the resulting nongravitational acceleration would act only in the anti-solar direction. However, the rotation of the nucleus, coupled with a thermal lag angle between the nucleus subsolar point and the point on the nucleus where there is maximum outgassing, introduces a transverse acceleration component in either the direction of the comet's motion or contrary to it – depending upon the nucleus rotation direction.

Equation 2 represents the time derivative of the comet's orbital semi-major axis (a) as a result of radial and transverse perturbing accelerations (R_p, T_p).

$$\frac{da}{dt} = \frac{2}{n(1-e^2)^{1/2}}\left[(e\sin\nu)R_p + \frac{p}{r}T_p\right] \tag{2}$$

In this equation, n, e, ν, and r denote, respectively, the orbital mean motion, eccentricity, true anomaly, and the comet's heliocentric distance, while p is the orbital semi-latus rectum, $a(1-e^2)$. Because of the thermal lag angle, a comet in direct rotation will have a positive transverse nongravitational acceleration component, and from equation 2, it is apparent that the comet's orbital semi-major axis will increase with time (its orbital energy will increase). Likewise, a comet in retrograde rotation will be acted upon by a negative Tp and its semi-major axis will decrease with time. Because the nongravitational acceleration is assumed to act symmetrically with respect to perihelion, the time-averaged effect of the periodic radial acceleration cancels out.

When introducing the standard model, Marsden *et al.* (1973) included possible time dependences in the transverse parameter (A_2). Currently, however, the standard nongravitational acceleration model is most often used solving only for the radial and transverse parameters (A_1 and A_2) over data intervals short enough so that neglected time dependences do not cause systematic trends in the residuals. Solutions for the nongravitational parameters usually require data from at least three apparitions and by comparing the nongravitational parameters determined from several of these short arc solutions, one can determine their change with time.

Largely because of its success in allowing accurate ephemeris predictions, the "standard" nongravitational force model has been in use for two decades. More recently, it has become understood that, while this model is successful in representing the astrometric observational data and allowing the computation of accurate ephemeris predictions, the standard model does not represent a completely accurate representation of the actual processes taking place in the cometary nucleus.

More than a century and a half ago, Bessel (1836) noted that a comet expelling material in a radial sunward direction would suffer a recoil force and if the expulsion of material did not take place symmetrically with respect to perihelion, there

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would be a shortening or lengthening of the comet's period depending upon whether the comet expelled more material before or after perihelion (see equation 2). Although Bessel did not identify the physical mechanism with water vaporization from the nucleus, his concept of cometary nongravitational forces was essentially correct. This review will focus upon this redirection of thought from a view whereby transverse nongravitational effects arise from a rotating comet that outgasses symmetrically with respect to perihelion to a view whereby the radial nongravitational effects are more important and outgassing from surface vents acts asymmetrically with respect to perihelion. For a more comprehensive outline of the earlier work on cometary nongravitational forces, the reader is directed to previously published reviews (Marsden 1968, 1969; Marsden *et al.* 1973; Marsden 1985; Yeomans 1991). Section 2 will focus upon the recent attempts to discern physical characteristics of comets using the results of nongravitational force modeling and Section 3 will outline the efforts to improve upon the standard model for modeling the motions of active comets. Section 4 will examine the long-term behavior of nongravitational effects and the constraints they provide upon the physical characteristics of the cometary nucleus. Section 5 is a summary of these discussions.

2. Inferring Nucleus Characteristics Using Nongravitational Parameters

Rickman (1986) pointed out that radial outgassing forces that act asymmetrically with respect to perihelion were the likely cause of the nongravitational effects upon comets Halley and Kopff and he went on to make estimates of their masses and bulk densities. He noted that the water production curves for Halley and other comets show an asymmetry with respect to perihelion so that the effect of the radial component, integrated over one orbital period, does not cancel out. The nucleus masses were estimated by comparing nongravitational parameters with the rocket-like forces expected from the gas production curves. In turn, the gas production curves were determined from the light curves using an empirical relationship developed by Festou. The bulk density for comet Halley was estimated to be 0.1–0.2 $g\ cm^{-3}$ and that for Kopff was lower still. Rickman *et al.* (1987b) continued this type of analysis and estimated masses for 29 short period comets. The change in the total orbital period per revolution is the sum of the contributions from the radial and transverse rocket effects. The mass of each comet was determined as a function of its estimated thermal and rotational properties. As a group, the bulk densities of these objects were estimated to be less than 0.5 $g\ cm^{-3}$ suggesting that the cometary nucleus is a very porous structure. This type of analysis depends upon the assumption that there is a correlation between the light curve and the assumed gas production curve, that thermal lag angles are present, and that the surface of each object has an un-mantled free sublimating area. Using a similar approach for comet Halley, Sagdeev *et al.* (1988) estimated a bulk density of 0.6 $g\ cm^{-3}$ with error bars of +0.9 and –0.4 $g\ cm^{-3}$. After a rather complete discussion of the method and the uncertainties involved in this type of analysis, Peale (1989) concluded that it is difficult to meaningfully constrain the bulk density of comet Halley.

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Festou *et al.* (1990) established a statistical correlation between the nongravitational parameters and the asymmetry of the gas production rates with respect to perihelion. A linear relationship was devised between the change in the comet's orbital period (ΔP) due to nongravitational effects and the difference (E) between the integrated gas production rate before and after perihelion. A linear relationship should exist between ΔP and E if the radial nongravitational effect is dominant and if the maximum outgassing rate divided by the cometary mass is reasonably constant from comet to comet. Using an early version of the curve of Δ P vs E, Festou *et al.* (1989) predicted a correction to the predicted time of perihelion passage time (ΔP) for P/Brorsen-Metcalf of between –30 and –15 days. When the comet was recovered by Eleanor Helin on July 4, 1989 it soon became evident that the predicted time of perihelion passage required a correction of –15.6 days. In retrospect, Festou *et al.* (1990) noted that their prediction was fortuitous because the prediction should have been –6 to –15 days and the 1989 light curve did not display the same asymmetry they assumed for the 1919 apparition. There remains a question as to whether the magnitude of a perihelion time correction can be accurately predicted from a comet's light curve characteristics. In this regard, one should make note of comet Crommelin. The light curve of this comet reaches a maximum ten days before perihelion yet the motion of this comet suggests that its nongravitational effects are very small. Whether or not the magnitude of a comet's nongravitational effects can be discerned from its light curve, it is evident that the light curve can, in most cases, be used to indicate whether orbital energy is being added or subtracted as a result of the cometary outgassing.

Sekanina (1993a) examined the effects of discrete outgassing sources upon the motions of periodic comets. In a series of arguments, Sekanina pointed out the complex effects that discrete active areas on the nucleus can have upon the cometary nongravitational accelerations. In Sekanina's model, the nongravitational effects depend upon the spin-vector orientation and upon the location of the active areas on the nucleus. These active areas can introduce both radial and transverse nongravitational effects even when the thermal lag angle is assumed to equal zero. Active areas can easily introduce seasonal effects whereby a source is more active before (or after) perihelion. Unlike the standard model, the sign of A_2 has no correlation with the direction of nucleus rotation. The initiation of new active areas, or the dying out of existing areas, is used to explain the time dependence or erratic behavior of the transverse nongravitational parameter (A_2) with time. For example, the transverse nongravitational parameter for periodic comet Giacobini-Zinner, after remaining nearly constant from 1900 through 1959, became erratic thereafter and for comet Comas-Solá, A_2 went from positive to negative over the 1935–1987 interval (see Figure 1).

One of the results of Sekanina's work (1993a) was to define more clearly the deficiencies of the current standard nongravitational force model and to point out how the large set of standard (Style II) nongravitational parameters might be used to provide useful information for modeling comets under the new paradigm. At any given time, the radial, transverse, and normal components of the standard model do not necessarily represent the actual nongravitational accelerations resulting from the outgassing of one or more discrete active sources. However, a successful or-

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bital solution for the standard nongravitational parameters should approximately account for the nongravitational perturbations when these perturbations are integrated completely around the cometary orbit for three or more returns. Attempts to solve for nongravitational parameters over time scales of less than one orbital period can be expected to improve the observation residuals but these parameters are probably meaningless for interpreting the physical nature of the cometary nucleus.

Sekanina noted that the correlation between the sign of A_2 and the perihelion asymmetry of the water production rate is valid for all possible combinations of the rotation parameters as long as only a single region is active on the nucleus. This correlation indicates an insignificant heat transfer lag in the sublimation process. This being the case, there is no correlation between the sign of A_2 and the sense of nucleus rotation. Assuming no heat transfer lags, the sign of A_2 is then to be interpreted in terms of the distribution of active areas with respect to the spin vector orientation. Sekanina noted that for active comets that have been seen at 10 or more apparitions, there is an equal number of comets with A_2 less than zero and those with A_2 greater than zero. The largest absolute values of the A_2 parameter seem to be associated with periodic comets having large perihelion distances (i.e. large q's). From Figure 2, we note that periodic comets Gunn ($q = 2.47$ AU), Schwassmann-Wachmann 2 (2.07), and Brooks 2 (1.84) have the largest magnitudes for their A_2 parameters. Sekanina notes that this may be due to use of an incorrect value for the scale distance r_o in the standard model.

When interpreted in light of Sekanina's model, both negative and positive values of A_1 are realistic; the former corresponds to an advancement in the orbital line of apsides while the latter suggests a regression of this line. For many well observed comets and asteroids that have small semi-major axes and large eccentricities, care must be taken to include the effects of general relativity in the equations of motion because these effects also introduce a non negligible radial acceleration toward the sun (Sitarski, 1983, 1992a).

A nongravitational acceleration acting normal to the comet's orbit plane will affect the longitude of the ascending node and the orbital inclination but neither of these perturbations are secular. Since these perturbations are modulated by either sine or cosine functions of the true anomaly, much of the nongravitational perturbations upon the two orbital elements would average to zero even if the normal perturbative forces remain positive or negative throughout the orbit. Sekanina (1993b) noted that a meaningful solution for the normal nongravitational parameter (A_3) would be possible only for the special case where the perturbations upon the ascending node and the inclination yield a similar value of A_3. Solutions for the A_3 parameter are not usually successful but there are some exceptions. Meaningful values of A_3 were obtained for the 1808–1988 apparitions of P/Grigg- Skjellerup, the 1906–1991 apparitions of P/Metcalf-Brewington and the 1958–1983 apparitions of P/Kopff (Sitarski 1991, 1992b; Rickman *et al.*, 1987a). Over the four returns of periodic comet Clark, Nakano (1992) found a value for A_3 with a formal uncertainty of only 3% and Sekanina (1993b) suggested that for this comet, the effective nongravitational perturbations on the ascending node and the orbital inclination are about equal. The rotation axis at perihelion is located in the plane defined by

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the sun-comet line at perihelion and perpendicular to the comet's orbit plane with the axis inclined about 45 degrees with respect to the orbit plane. As seen from the comet, the rotation axis at perihelion would be pointing in the general direction of the sun but about 45 degrees above it. In this configuration for the spin axis of comet Clark, the cometary outgassing produces a non-canceling nongravitational thrust in a normal direction so that the solution for A_3 would be valid.

3. Modeling the Nongravitational Motion of a Comet

Several efforts have been made to improve upon the standard nongravitational force model either by attempting to change the dynamical model to more closely reflect the physical model of the nucleus or by altering the mechanism by which nongravitational accelerations are introduced.

Froeschlé and Rickman (1986) and Rickman and Froeschlé (1986) used theoretical calculations to examine the secular evolution of the nongravitational parameters as a function of the heliocentric distance for various kinds of short period comets and different assumed thermal inertias. In general, their values of these parameters did not correspond to those computed from the standard model. In fact, there was such a wide variation in the respective behavior of the A_1, A_2, and A_3 parameters that no generally applicable model for the nongravitational effects was suggested. They noted that improved models would likely have to include the effects of rotation pole orientation and seasonal heat flows.

Using the asymmetric light curve of comet Halley, Yeomans (1984) attempted to employ the nucleus rotation parameters introduced by Sekanina (1981) to improve the nongravitational force model for comet Halley. For this latter model, the outgassing is assumed to result from a sub-solar active area which is defined by its cometocentric solar longitude at perihelion (P), the obliquity of the nucleus equator with respect to the orbit plane (I), and by a thermal lag angle measured from the cometary subsolar point to the point of maximum outgassing. Although the optimum lag angle and obliquity turned out to be small, in apparent agreement with subsequent results, the orbital solution did not improve upon the standard nongravitational force model.

For his investigation of the two apparitions of P/Metcalf-Brewington, Sitarski (1992b) found that his nongravitational force model employing a single nongravitational acceleration parameter (A) together with the lag angle, obliquity (I) and cometocentric longitude of the sun at perihelion (P) gave nearly identical results when compared to an orbital solution using A_1, A_2, and A_3 of the standard model. For the investigation of the motion of P/Grigg-Skjellerup and P/Swift-Gehrels, Sitarski (1991) and Bielicki and Sitarski (1991) introduced their model using A, the lag angle, I, and P but found it necessary to also introduce dI/dt and dP/dt for the model of P/Swift-Gehrels' motion over the 1889–1991 interval and dI/dt, dP/dt, and a center-of-light/center-of-mass offset for the motion of P/Grigg-Skjellerup over the 1808–1988 interval.

Yeomans and Chodas (1989) modified the standard nongravitational acceleration model to allow the water vaporization curve to peak a certain number of days either before or after perihelion. They found that for several periodic comets

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that exhibit asymmetric light curves, an asymmetric outgassing model can improve upon the orbital representation of the astrometric data. The asymmetric model, which more accurately mimics the comet's outgassing history, often yields values for the radial and transverse nongravitational parameters that are completely different from the corresponding values derived from using the standard symmetric model. For P/d'Arrest the water vaporization curve (as modeled by the asymmetric nongravitational acceleration function) and the visual light curve reach maxima very close to one another (40 days post perihelion). For P/Giacobini-Zinner, the optimum values in the modeled perihelion offset approximately mimic the observed light curve history with both reaching a maximum at perihelion prior to 1959, then a maximum at 25 days after perihelion in 1959, back to a maximum at perihelion in 1972 and finally to a maximum 15 days prior to perihelion in 1985. For P/Kopff, the computed value of A_2 does not pass through zero with the asymmetric model whereas it does for the symmetric model. Recent results comparing comet Halley orbital solutions using the asymmetric and standard nongravitational acceleration models are given in Table 1.

4. The Time Dependence of Nongravitational Effects

From Figures 1 and 2, it is clear that a number of comets have rather constant values for their A_2 parameters and an equal number of comets have rather time dependent A_2 parameters. In the latter case, the smoothly varying time dependence, and especially the passage of the A_2 parameter through zero, is likely due to major precessional motion of the rotation pole with time. A number of comets like Finlay, Giacobini-Zinner, Brooks 2, and Whipple have rather erratic values of A_2 as a function of time that are perhaps due to the cessation of old active areas or the initiation of new ones (Sekanina 1993a). As an illustration of this process, imagine that the rotation pole of a nucleus is in its orbital plane and aligned perpendicular to the sun-comet line at perihelion. Further imagine that there is an active area on one polar cap so that on the way into perihelion this polar region is active and produces a radial acceleration that would reduce the orbital semi-major axis (see equation 2). On the way out from perihelion, this active polar region is in shadow and thus inactive. Likewise if only the opposite polar region was active, the determined value of A_2 would have an opposite sign because the orbital semi-major axis (and orbital period) would then increase rather than decrease.

If the time dependence of A_2 is smoothly varying, some success has been achieved by introducing time dependencies directly into the nongravitational force model (for example, see Marsden *et al.*, 1973; Sitarski, 1991). However, for so-called erratic comets, solutions have normally been made over short enough time intervals that the nongravitational parameters can be assumed constant. By making several solutions over different time intervals, the changing values of the nongravitational parameters can be noted. The information plotted in Figures 1 and 2 was determined in this way.

For a few of the less active periodic comets (Arend-Rigaux, Neujmin 1, Schwassmann-Wachmann 1), no nongravitational effects could be detected in the orbital solutions. Among the relatively few objects exhibiting time independent

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nongravitational effects are comets Tempel 2, Tuttle, d'Arrest, and Halley. Carusi *et al.* (1991) pointed out an early 1678 apparition of periodic comet d'Arrest and noted that the nongravitational effects for this comet were time independent for nearly 50 revolutions. Yeomans and Chodas (1989) found that the recent astrometric data for this comet could be represented most accurately with an asymmetrical nongravitational acceleration model that reached a maximum some 40 days after perihelion. The light curve of this comet consistently reaches a maximum about this same time. For Halley, Yeomans and Kiang (1981) found the comet's motion to be consistent with time independent nongravitational parameters over two millennia. In addition the absolute magnitude of comet Halley has shown no obvious changes during its two thousand year observational interval (Broughton 1979; Bortle and Morris 1984)

Table 1 outlines a recent orbit determination for comet Halley including the orbital elements for its last four apparitions and its next return in 2061. The asymmetric nongravitational force model of Yeomans and Chodas (1989) was employed and various dates were input for when the modelled outgassing reached its maximum value; the optimum value (in the sense that the weighted rms residual reached a minimum) occurred when the modelled outgassing maximum was taken as approximately 35 days after perihelion. In addition to a solution for the six orbital elements, a solution was made for an offset between the measured photometric center of the comet's image and its center of mass. This offset is assumed to vary along the comet-sun line with an inverse square dependence on the heliocentric distance. The offset at one AU from the sun was determined to be 849 km sunward. This result is in accordance with Medvedev's (1993) estimate of about 1000 km for Halley's post-perihelion offset in the photocenter.

For the standard model, the transverse nongravitational parameter (A_2) is very well determined and it is this transverse acceleration that causes the nongravitational effects. However, for the asymmetric model, the transverse nongravitational parameter (A_2) is often indeterminate and it is the well-determined radial nongravitational parameter (A_1) and the fact that the maximum outgassing is assumed to reach a maximum 35 days after perihelion that introduces the nongravitational effect. Comet Halley's consistent behavior from apparition to apparition is responsible for an increase in the comet's orbital period (4 days) over what might be expected if only gravitational perturbations were taken into account. Various modeling efforts have concluded that the spin state of comet Halley is not in simple rotation. Although not well understood, the rotation characteristics are likely to include both a rotation about its long axis and precession about an axis nearly perpendicular to the rotation axis. However, the long term constancy of Halley's nongravitational parameters do not allow for major changes of these axes with time. Spin axis precession and nutation are likely to be evident but chaotic tumbling over time scales of about two thousand years certainly can be excluded. Over the same time scale, there cannot have been major changes in the location or activity of the outgassing vents.

Yau *et al.* (1993) found the observations of comet Swift-Tuttle in 69 B.C., A.D. 188, 1737, 1862, and 1992-93 to be consistent with no nongravitational effects in this comet's motion and, as was the case for comet Halley, there have been no

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TABLE I

Based on the asymmetric nongravitational acceleration model by Yeomans and Chodas (1989), an orbital solution for comet Halley was computed from 7525 observations over the interval 1759 January 23 through March 18, 1991. The root mean square (RMS) weighted residual reached a minimum (best solution) when the modelled water vaporization curve was assumed to peak 35 days after perihelion (DT = +35 days). A sunward offset of the photometric center-of-light from the comet's center-of-mass was assumed to be operative at each return; at a heliocentric distance of one AU, this offset (S_o) was determined to be some 849 km. Planetary perturbations were computed at each time step using planetary ephemeris DE200 (with outer planet masses improved using the results from the Voyager spacecraft flybys). Full relativistic equations of motion were employed. The final weighted, unweighted, and normalized RMS residuals are given. For comparison, the nongravitational parameters, center-of-light/center-of-mass offset and RMS residuals are also given for a solution using the standard nongravitational force model.

	Asymmetric model	Standard Model
A_1 (10^{-10}AU day^{-2})	3.6038 (± 0.1043)	1.9704 (± 0.2789)
A_2 (10^{-10}AU day^{-2})	0.0532 (± 0.0525)	1.55443 (± 0.00003)
S_o in km	849.2 (± 17)	830.8 (± 17)
DT in days	+35	0
RMS (weighted)	1.141	1.154
RMS (unweighted, arcsec)	5.351	5.562
RMS (normalized)	1.027	1.039

Orbital elements (J2000) from above asymmetric model orbit

Epoch (TDB)	e	q(AU)	ω	Ω	i	T (TDB)
1759 Mar 21.0	0.96768749	0.584473925	110.708762	57.245864	162.372379	1759 Mar 13.059370
1835 Nov 18.0	0.96739544	0.586568623	110.704026	57.518405	162.258718	1835 Nov 16.439845
1910 May 9.0	0.96730219	0.587212031	111.737103	58.562661	162.218514	1910 Apr 20.178242
1986 Feb 19.0	0.96727580	0.587103940	111.865650	58.860054	162.242195	1986 Feb 9.458966
2061 Aug 4.0	0.96657663	0.592780500	112.052286	59.392434	161.965091	2061 Jul 28.719901

obvious changes in this comet's absolute magnitudes over two millennia. Yau *et al.* (1993) list the entire set of orbital elements for this comet from 703 B.C. through A.D. 2392 and Marsden *et al.* (1993) present orbital elements over the 69 B.C.–A.D. 3302 interval. For the similar epochs of osculation, the orbital elements of Yau *et al.* (1993) and Marsden *et al.* (1993) are similar and both groups point out that, over the long term, there are no nongravitational effects evident for this comet.

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Because comet Swift-Tuttle's absolute magnitude has not changed significantly over two millennia and there is a lack of significant nongravitational effects over the same period, additional constraints can be placed upon the model for this comet's nucleus. At one AU from the sun, the outgassing activity of Swift-Tuttle is comparable with that of comet Halley at the same heliocentric distance (A'Hearn 1993). Yet comet Halley experiences an increase in its orbital period of 4 days per revolution due to nongravitational effects while Swift-Tuttle has no perceptible change in its period. If the mass of Swift-Tuttle were significantly larger than Halley's, one would not expect to be able to detect a nongravitational acceleration in its orbital motion. Alternatively, if the outgassing of Swift-Tuttle were directed radially toward the sun and if this outgassing were symmetric with respect to perihelion, there would be no nongravitational effects evident in its orbital motion. However, for this latter case, the rotation axis as well as the size and location of the active regions would have been constant over its entire observational interval.

If comet Swift-Tuttle could be shown to outgas preferentially before or after perihelion, it would strongly imply that the mass of this comet is far larger than that of comet Halley. In this case, although the massive comet would experience a substantial nongravitational force, its nongravitational acceleration would remain undetectable. In the analysis by Green (1993), there was no obvious asymmetry in the comet's visual light curve. Even so, either the string of conditions mentioned in the last two sentences of the previous paragraph are true or the comet is substantially more massive than comet Halley. The latter case seems far more likely. Based upon an analysis of their respective meteor stream characteristics, Hughes and McBride (1989) concluded that the mass of comet Swift-Tuttle is about ten times larger than comet Halley.

5. Summary

Although the notion of an icy conglomerate model for a cometary nucleus (Whipple 1950, 1951) is still in basic agreement with the observations, there have been a number of recent modifications and refinements to this model. Largely as a result of the impressive image of comet Halley's nucleus taken by the Giotto spacecraft, the picture of an outgassing sunlit hemisphere has been replaced by a "vent" model whereby the outgassing activity takes place from discrete active areas.

The observation that the sign of the transverse nongravitational parameter (A_2) is strongly correlated with a comet's outgassing asymmetry with respect to perihelion suggests that the nongravitational accelerations are due to radial outgassing toward the sun. This concept would then replace the earlier notion whereby the nongravitational effects were viewed as resulting from a transverse acceleration introduced by a significant heat lag between the subsolar point of the cometary nucleus and the point of maximum outgassing. Although the widely used nongravitational acceleration model introduced by Marsden *et al.* (1973) is based upon the older model and assumes symmetric outgassing with respect to perihelion, it has been successful in providing improved ephemerides for many comets.

Since a large majority of the orbits for active periodic comets have been computed with the nongravitational acceleration model of Marsden *et al.* (1993), it is

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important to be able to interpret the nongravitational parameters (A_1, A_2, A_3) from this model in terms of the more recent "vent" model. For this latter model, the radial nongravitational parameter (A_1) is the more important nongravitational parameter and it can be either positive or negative. For the older standard model, it was the transverse nongravitational parameter (A_2) that was dominant and a positive value of A_1 was expected. With a few notable exceptions, a solution for the normal nongravitational parameter (A_3) is not appropriate in either the standard or the newer "vent" model.

For the many comets whose nongravitational parameters are time dependent, one can introduce time dependencies directly into the equations of motion if the nongravitational effects vary smoothly with time. Otherwise, sets of nongravitational parameter solutions can be made over short time intervals and the resulting parameters compared to note the time dependencies. Under the older model, the time varying nongravitational parameters were most often interpreted in terms of a precessing rotation axis. For example, a uniform precession of the rotation axis though the comet's orbital plane would result in a smooth sign change in the transverse nongravitational parameter. For the vent model, these time dependencies are still interpreted in terms of rotation axis motion whereas erratic time dependencies are interpreted as the cessation of old vents or the initiation of new ones.

For a comet, like Halley, that is both active and shows no significant time dependencies in its nongravitational effects, one must assume that the same active areas have maintained about the same activity over the comet's observed interval and that its rotation axis has not suffered a major change in orientation. Since an active vent on the surface of comet Halley's nucleus is likely to sublimate tens of meters of ice during each apparition, the vents would appear to get quite deep without changing their outgassing characteristics. This enigmatic behavior is worthy of additional study as is the contrast between the observed, complex rotation state of comet Halley and the apparent long-term stability of its rotation axes.

For a comet like Swift-Tuttle that is active yet whose motion is not subjected to significant nongravitational effects, there are additional constraints upon the characteristics of its nucleus. Either a comet of this type is so massive that nongravitational forces do not affect the comet's orbital motion to any significant degree or the comet's outgassing is directed radially toward the sun and symmetric with respect to perihelion. In the latter case, the activity and location of the active vents, as well as the rotation pole orientation, must have remained rather constant over its entire observed interval.

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https://doi.org/10.1017/S007418090004657X Published online by Cambridge University Press

References

A'Hearn, M. : 1993, Personal communication.

Bessel, F.W. : 1836, "Bemerkungen über mögliche Unzulänglichkeit der die Anziehungen allein berücksichtigenden Theorie der Kometen. *Astron. Nach.*, **13**, 345–350.

Bielicki, M. and Sitarski, G. : 1991, "Nongravitational Motion of Comet P/Swift-Gehrels." *Acta Astron.*, **41**, 309–323.

Bortle, J.E. and Morris, C.S. : 1984, "Brighter Prospects for Halley's Comet." *Sky and Telescope*, Jan. 1984, pp. 9–12.

Broughton, R.P. : 1979, "The Visibility of Halley's comet." *J.R.A.S. Canada*, **73**, 24–36.

Carusi, A., Kresák, L., Kresáková, M., and Valsecchi, G.B. : 1991, "Observations of Periodic Comet d'Arrest in 1678 and Implications for its Evolutionary History." *Astron. Astrophys.*, **252**, 377–384.

Festou, M., Rickman, H., and Kamél, l. : 1989, *IAU Circ.* No. 4804.

Festou, M., Rickman, H., and Kamél, l. : 1990, "Using Light-curve Asymmetries to Predict comet Returns." *Nature*, **345**, 235–238.

Froeschlé, C. and Rickman, H. : 1986, "Model Calculations of Nongravitational Forces on Short Period Comets. I. Low-Obliquity Case." *Astron. Astrophys.*, **170**, 145–160.

Green, D.W.E. : 1993, "The Brightness of P/Swift-Tuttle in 1992-1993." *International Comet Quarterly*, **15**, 182.

Hughes, D.W. and McBride, N. : 1989, "The Mass of Meteoroid Streams." *Mon. Not. R. Astr. Soc.*, **240**, 73–79.

Marsden, B.G. : 1968, "Comets and Nongravitational Forces." *Astron. J.*, **73**, 367–379.

Marsden, B.G. : 1969, "Comets and Nongravitational Forces II." *Astron. J.*, **74**, 720–734.

Marsden, B.G. : 1985, "Nongravitational Forces on Comets : The First Fifteen Years." In *Dynamics of Comets : Their Origin and Evolution* (A. Carusi and G.B. Valsecchi, Eds.), pp. 343–352, D. Reidel Publishing Company, Astrophysic and Space Science Library, vol. 115.

Marsden, B.G., Sekanina, Z., and Yeomans, D.K. : 1973, "Comets and Nongravitational Forces. V." *Astron. J.*, **78**, 211–225.

Marsden, B.G., Williams, G.V., Kronk, G.W., and Waddington, W.G. : 1993, "Update on Comet Swift-Tuttle." *Icarus*, **105**, 420–426.

Medvedev, Yu.D. : 1993, "On the Dust Cloud in a Comet's Head and its Role in Forming the Photocentre Shift Phenomena." *Asteroids, Comets, Meteors 1993 poster paper*, Belgirate, Italy, June, 1993.

Nakano, S. : 1992, "Periodic Comet Clark." *Minor Planet Circular* 20122.

Peale, S.J. : 1989, "On the Density of Halley's Comet." *Icarus*, **82**, 36-49.

Rickman, H. : 1986, "Masses and Densities of Comets Halley and Kopff." In *The Comet Nucleus sample Return Mission Proc. Workshop*, pp. 195–205, Canterbury, UK, 15–17 July 1986, ESA SP-249.

Rickman, H. and Froeschlé, C. : 1986, "Model Calculations of Nongravitational Forces on Short Period Comets. II. High Obliquity Case." *Astron. Astrophys.*, **170**, 161–166.

Rickman, H., Sitarski, G., and Todorovic-Juchniewicz, B. : 1987a, "Nongravitational motion of comet P/Kopff during 1958–1983." *Astron. Astrophys.* **188**, 206–211.

Rickman, H., Kamél, L., Festou, M.C., and Froeschlé, C. : 1987b, "Estimates of Masses, Volumes and Densities of Short-Period Comet Nuclei." *Symposium on the diversity and similarity of comets* (E.J. Rolfe and B. Battrick, Eds.), pp. 471–481, ESA SP-278.

Sagdeev, R.Z., Elyasberg, P.E., and Moroz, V.I. : 1988, "Is the Nucleus of Comet Halley a Low Density Body?" *Nature*, **331**, 240–242.

Sekanina, Z. : 1981, "Rotation and Precession of Cometary Nuclei." *Annual Review of Earth and Planetary Science*, **9**, 113–145.

Sekanina, Z. : 1993a, "Effects of Discrete-Source Outgassing on Motions of Periodic Comets and Discontinuous Orbital Anomalies." *Astron. J.*, **105**, 702–735.

https://doi.org/10.1017/S007418090004657X Published online by Cambridge University Press

Sekanina, Z. : 1993b, "Nongravitational Motions of Comets : Component of the Recoil Force Normal to Orbital Plane." *Astron. Astrophys.*, **277**, 265–282.

Sitarski, G. : 1983, "Effects of General Relativity in the Motions of Minor Planets and Comets." *Acta Astron.*, **33**, 295–304.

Sitarski, G. : 1991, "Linkage of all the Apparitions of the Periodic Comet Grigg-Skjellerup during 1808–1988." *Acta Astron.*, **41**, 237–253.

Sitarski, G. : 1992a, "On the Relativistic Motion of (1566) Icarus." *Astron. J.*, **104**, 1226–1229.

Sitarski, G. : 1992b, "Motion of Comet P/Metcalf-Brewington (1906 VI = 1991a)." *Acta Astron.*, **42**, 49–57.

Whipple, F.L. : 1950, "A Comet Model. I. The Acceleration of comet Encke." *Astron. J.*, **111**, 375–394.

Whipple, F.L. : 1951, "A Comet Model II. Physical Relations for Comets and Meteors." *Astrophys. J.*, **113**, 464–474.

Yau, K., Yeomans, D.K., and Weissman, P.R. : 1993, "The Past and Future Motion of P/Swift-Tuttle." *Mon. Not. R. Astr. Soc.* (in press).

Yeomans, D.K. : 1984, "The Orbits of Comets Halley and Giacobini-Zinner." *Cometary Astrometry* (D.K. Yeomans, R.M. West, R.S. Harrington, and B.G. Marsden, Eds.), pp. 167–175, JPL Publication 84–82.

Yeomans, D.K. : 1991, "Cometary Orbital Dynamics and Astrometry." In *Comets in the Post-Halley Era*, vol. 1 (R.L. Newburn, Jr., M. Neugebauer, and J. Rahe, Eds.), pp. 3–17, Kluwer Academic Publishers.

Yeomans, D.K. and Kiang, T. : 1981, "The Long-Term Motion of Comet Halley." *Mon. Not. R. Astr. Soc.*, **197**, 633–646.

Yeomans, D.K. and Chodas, P.W. : 1989, "An Asymmetric Outgassing Model for Cometary Nongravitational Accelerations." *Astron. J.*, **98**, 1083–1093.

https://doi.org/10.1017/S007418090004657X Published online by Cambridge University Press

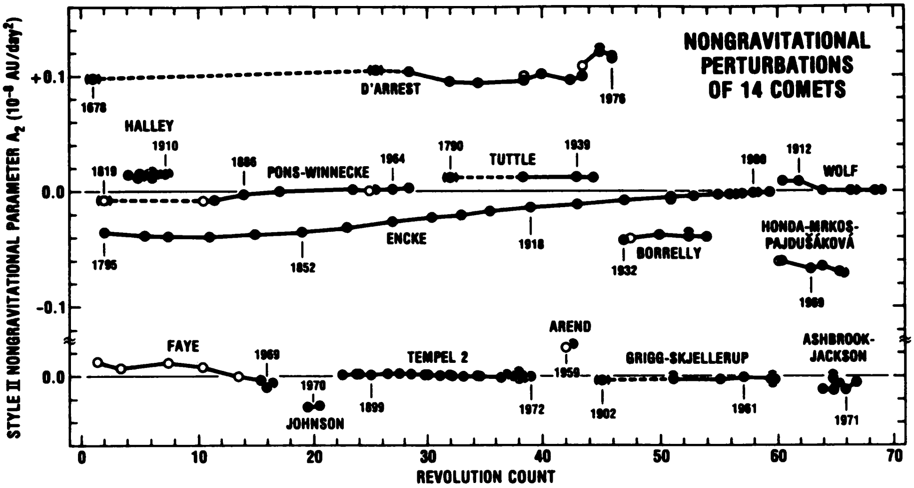

Figure 1. Fourteen Comets whose Nongravitational Effects are either Constant or Slowly Varying with Time. Figures 1 and 2 Courtesy of Z. Sekanina (1993a).

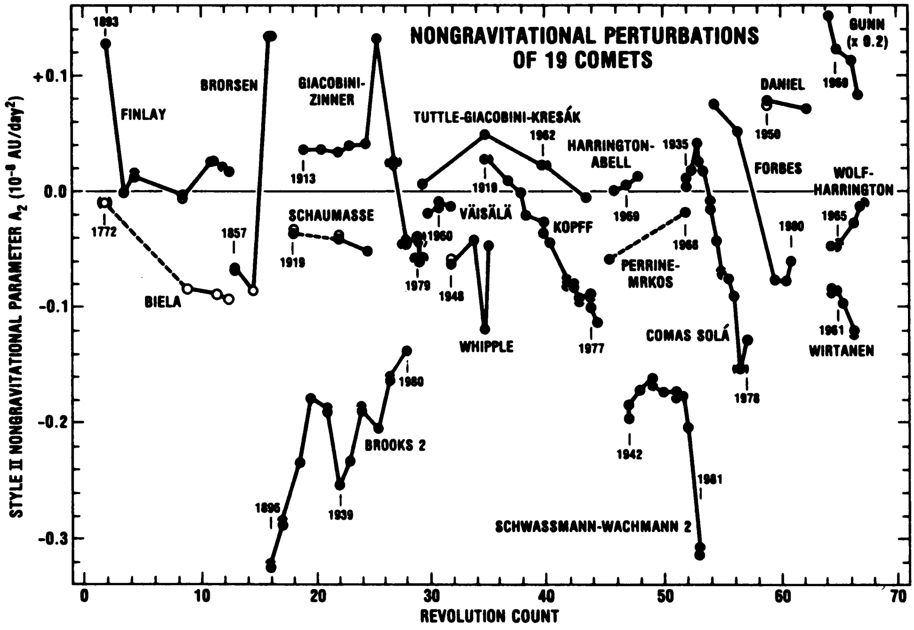

Figure 2. Nineteen Comets whose Nongravitational Effects are Strong Functions of Time.

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