12. THE GAS DYNAMICS OF ACCRETION*

Introductory Report

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1. Introduction

The term accretion originally referred, in astronomical contexts, to the capture of mass by stars, and, later, to mass capture by other centers of gravitational force. As such, the process has not proved to be of general importance, in spite of early hopes. However, there are other aspects of the problem which may yet prove worthy of attention in interstellar gas dynamics. In particular, the effects of stars, galaxies, or even clusters of galaxies, on ambient matter streaming by them may be detectable, directly or indirectly, and it is on such possible effects that I shall concentrate here. These effects are related to the original accretion problem but may be thought of separately; nevertheless, I retain the use of the term accretion to refer to all aspects of the motions induced in an ambient medium by a gravitating object.

In a certain sense, a disturbing gravitating object can be thought of as a probe which may be of use in the diagnostics of the ambient medium. However, the distinction from plasma probes is that we concentrate here more on the effects produced in the medium than on the response of the probe. Anyone who wishes to go deeply into the subject would do well to study the literature on plasma probes. In the list of references, I give a sampling of seven papers.† But here, I shall only go into the astronomical literature, which provides a suitable introduction.

2. Traditional Theory

To provide a background to the fluid dynamical problems, I should like to mention briefly some of the early work on the traditional accretion process. A detailed review has been given by Lyttleton (1953) and it is not necessary to go into details. The simplest case to consider is that of a stationary star embedded in a uniform medium. Bondi (1952) gave a gas dynamical treatment of this problem, the equations of which are now familiar since they are essentially those of Parker's solar wind theory (1958). Bondi found the rate of accretion

$$A = \lambda\pi \left(\frac{2GM}{c^2}\right)^2 \varrho_0 c \tag{1}$$

where M is the mass of the star, c is the speed of sound in the medium whose density

* During the Symposium this Report was presented later than its present location suggests. (Ed.)

† These are marked with asterisks in the bibliography.

Habing (ed.), Interstellar Gas Dynamics, 201–214.

infinitely far from the star is ϱ_0, and where λ is a pure number which varies from 0.25 to 1.1 as the ratio of specific heats, γ, varies from $\frac{5}{3}$ to 1. For $\varrho_0 \approx 10^{-24}$ g cm^{-3}, $M = M_\odot$, $c = 1$ km sec^{-1}, we find that $A \approx 5 \times 10^{-12} M_\odot$ yr^{-1}, which indicates that under normal conditions, spherically symmetric accretion is not a significant process. Of course, one can imagine extreme situations where accretion is more important, but they may also involve rather different conditions than contemplated here. Moreover, the Bondi theory does not delve into the physical conditions at the surface of the star, which can make a serious difference. Some additional work along these lines was done by Mestel (1954) who found that unless the ambient medium is dense enough, the H II region around the star will prevent any rapid accretion; he finds the critical density to be 10^3 cm^{-3} for $M \leqslant 1.75 M_\odot$. This would seem to preclude accretion by early-type stars. Late-type stars, on the other hand, would be expected to have strong winds, thus they too would not be likely to accrete. However, we might still consider a small accumulation of matter in a shell around the star, where the wind and accretion inflow bring one another to rest. Such a shell is probably unstable and I do not know whether there would be any observable effects.

A more extensive literature exists on the capture of mass by stars moving with respect to the ambient medium. The simplest case is that in which the medium is cold, uniform, collisionless, and non-gravitating, and the star moves uniformly through it. In the frame of the star a gas particle has the orbit

$$\frac{1}{q} = \frac{R_A}{2s^2}(1 + \cos\alpha) + \frac{1}{s}\sin\alpha, \tag{2}$$

where q is distance from the star, α is the angle measured from the downstream symmetry axis, or accretion axis, s is the impact parameter, and

$$R_A = 2GM/V_0^2, \tag{3}$$

where V_0 is the velocity at upstream infinity (see Figure 1). It is easy to see that the

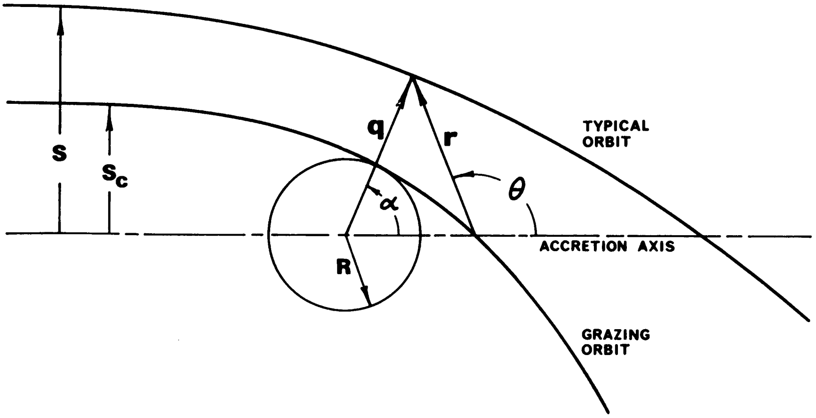

Fig. 1. Illustrating the notation. s is impact parameter; s_c is the critical impact parameter of Equation (5). Coordinates (q, α) are used in this section. Coordinates (r, θ) are introduced in Section 3.

smallest value of q for given s is

$$q_{\min} = 2s^2/[R_A + (R_A^2 + 4s^2)^{1/2}]. \tag{4}$$

This formula can be used to determine whether the particle can strike the surface of the star, whose radius is R. We readily find that particles for which

$$s \leqslant s_c \equiv [R(R + R_A)]^{1/2} \tag{5}$$

strike the star. If we assume that all such particles are captured, we then obtain the accretion rate

$$A = \pi s_c^2 \varrho_0 V_0. \tag{6}$$

With the same assumptions as above, and with $R = R_\odot$ and $V_0 = 10$ km sec^{-1} we obtain $R_A = 7 \times 10^{14}$ cm and $A = 10^{-18} M_\odot$ yr^{-1}.

Lyttleton (1953) has argued that inelastic collisions among the streaming particles could lead to an enhanced rate of accretion. The argument is based on the recognition that if the incident gas is cold, a large density develops on the accretion axis. In the limit of zero temperature, the density distribution becomes

$$\varrho(q, \alpha) = \tfrac{1}{2}\varrho_0 csc\, \frac{\alpha}{2} \left(\frac{R_A}{q} + \sin^2 \frac{\alpha}{2}\right)^{1/2} \times \left[\frac{R_A}{2q} + \sin^2 \frac{\alpha}{2} + \sin \frac{\alpha}{2} \left(\frac{R_A}{q} + \sin^2 \frac{\alpha}{2}\right)^{1/2}\right], \tag{7}$$

where the contribution of particles which have already crossed the axis is neglected. (For a detailed discussion of the density distribution see Danby and Camm, 1957.) We see that for $\alpha = 0$, $\varrho = \infty$; the fact that in this flow rings of gas squeeze down into points on the axis is associated with the singular behavior of ϱ. Hoyle and Lyttleton argued that in this high density region, inelastic collisions will occur and annihilate the transverse momentum of the particles. The downstream velocity is approximately conserved, and particles which cross the axis close enough to the star move too slowly to escape and are accreted. By conservation of angular momentum we find that if no collisions occur, the α-velocity of a particle is

$$v_\alpha = -\, sV_0/q, \tag{8}$$

while energy conservation implies that its q-velocity is

$$v_q^2 = V_0^2 \left(1 + \frac{R_A}{q} - \frac{s^2}{q^2}\right). \tag{9}$$

From Equation (2) we see that the particle crosses the accretion axis ($\alpha = 0$) at a distance $q = s^2/R_A$ from the star so that at $\alpha = 0$, $v_q = V_0$, for all particles. Thus particles which cross the axis at distances less than about R_A from the star, on having their α-momenta destroyed in collisions, will be below escape velocities. From Equation

(2) we see that a particle for which $q=R_A$ at $\alpha=0$ has $s=R_A$, so that we obtain an accretion rate

$$A \approx \pi R_A^2 \varrho_0 V_0\,, \tag{10}$$

which exceeds the estimate of Equation (6) by a factor of about R_A/R_0, but still does not give encouragement.

The theory has been further elaborated in the particle picture by Bondi and Hoyle (1944; see also Lyttleton, 1953) and a picture of the flow in a wake of free particles has been developed. However, the estimated capture rate is not appreciably modified. Various details of these calculations have been cause for debate, and a list of some of the more important papers is given in a brief review by McCrea (1955).

Here I want to mention what appears to be the most serious objection, which has been raised by Danby and Camm (1957) and Danby and Bray (1967). These authors have considered the effect of a finite temperature on the density distribution in the wake of a star and have shown how the singular behavior near the axis is averted. Under typical interstellar conditions, they contend, the density is lowered sufficiently so that collisions will not be important, and they conclude that the mechanism of Hoyle and Lyttleton will not operate. On the other hand, they do not include self-interaction of the gas either through plasma effects or gravitation, and it seems likely that such effects will be of importance. Just what the details of the motion become then is not clear, but if we consider that different 'streams' interpenetrate as they cross the axis, we might speculate on the possibility of collective instabilities which perhaps act like collisions. Whether the collective energy resulting from instabilities is carried away in waves or results in plasma instabilities probably depends on the Mach number, but there does seem to be a case for treating the region near the accretion axis as a continuum to gain some impression of the general nature of the flow. At any rate, with only these vague assurances, I shall devote the rest of this discussion to a gas dynamical discussion of accretion processes.

3. Linearized Gas Dynamics

The treatment of the gas dynamics of a flow past a center of gravitational force is sufficiently difficult that some drastic approximations have been made in this subject. One quite tempting, but not necessarily justifiable, approach is linearization, in which the perturbing body is assumed to produce small deviations from uniformity. This approach was used by Dokuchaev (1964) who considered the uniform motion of a point mass through a uniform medium; this problem is closely similar to that of the motion of a charged satellite moving through a plasma as treated by Kraus and Watson (1958). Dokuchaev also considered the effects of a uniform magnetic field along the motion and the effects of mass loss from the star; but let us discuss here the purely gravitational case, as it stems from the classical problem of the previous section. (I shall also mix into the discussion some unpublished results on this problem obtained by Prendergast, Ruderman, and me.)

Let the density in the ambient medium be $\varrho=\varrho_0+\delta\varrho$ where ϱ_0 is a constant and $|\delta\varrho|\ll\varrho_0$. Similar remarks apply to the other state variables and it is assumed that the gas velocities are small in the stationary frame. We also assume isentropic motion. Then, on linearizing the equations of motion and performing the usual manipulations of acoustics (Ward, 1955), we obtain the wave equation (Appendix A)

$$\frac{\partial^2 \Psi}{\partial t^2} - c^2 \nabla^2 \Psi = 4\pi GM\,\delta(\mathbf{r} - \mathbf{V}_0 t), \tag{11}$$

where

$$\Psi = \delta\varrho/\varrho_0, \tag{12}$$

and where c is the speed of sound and δ is the Dirac function. This kind of problem is very like that of Cherenkov radiation and procedures for solving it are standard (Landau and Lifshitz, 1960). If we ignore the homogeneous solutions of Equation (12) and consider the forced solution, we obtain for $M\equiv V_0/c<1$

$$\Psi = \frac{R_A M^2}{q(1 - M^2 \sin^2\alpha)^{1/2}}; \tag{13}$$

while for $M>1$,

$$\Psi = \begin{cases} \dfrac{R_A M^2}{q(1 - M^2\sin^2\alpha)^{1/2}} & \text{for} \quad \alpha < \arcsin(1/M) \\ 0 & \text{for} \quad \alpha > \arcsin(1/M). \end{cases} \tag{14}$$

These solutions are expressed in the frame of the star, where they are time-independent. The supersonic solution has a discontinuity on the Mach cone, which indicates the formation of a shock. However, on the cone the density is singular which is a standard difficulty in the linear theory of supersonic motion of point sources (Ward, 1955). It can be removed by considering a mass of finite size (Huebner and Herring, unpublished report).

If we have the density, we can then find the velocity from the equations of motion. In the star's reference frame this gives

$$u_{\parallel} = -V_0 - \tfrac{1}{2}R_A V_0\left[\frac{1}{q} - \frac{2}{q(1 - M^2\sin^2\alpha)^{1/2}}\right] \tag{15}$$

$$u_{\perp} = -\tfrac{1}{2}R_A V_0\left[\frac{1+\cos\alpha}{q\sin\alpha} - \frac{2\cot\alpha}{q(1 - M^2\sin^2\alpha)^{1/2}}\right]$$

behind the shock and

$$u_{\parallel} = -V_0 - \tfrac{1}{2}R_A V_0 q^{-1}, \qquad u_{\perp} = \tfrac{1}{2}R_A V_0\,\frac{1+\cos\alpha}{q\sin\alpha} \tag{16}$$

ahead of the shock, where $u_{\parallel}$ and $u_{\perp}$ are the velocities parallel and perpendicular to the original uniform flow, $\mathbf{V}_0$. We see that behind the shock $u_{\parallel}=0$ on the surface

$$(q + \tfrac{1}{2}R_A)^2 = \frac{1}{1 - M^2\sin^2\alpha}. \tag{17}$$

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Between the shock cone and this 'accretion' surface, $u_{\|}$ is starwards; while downward of the surface, $u_{\|}$ is downstream. This is in line with what one would expect, but $u_{\perp}$ has an unpleasant feature. Behind the shock $u_{\perp}$ is negative, whereas ahead it is positive. This means that the velocity is into the shock from both sides, and this does not seem to be physically meaningful. However, mass is conserved to the accuracy of linear theory and to get an idea of the flow we must consider the linearized mass flux: $\mathbf{V}_0(\varrho_0+\delta\varrho)+\varrho_0\mathbf{u}$. This gives the flow pattern shown in Figure 2. Mass capture does not occur in this

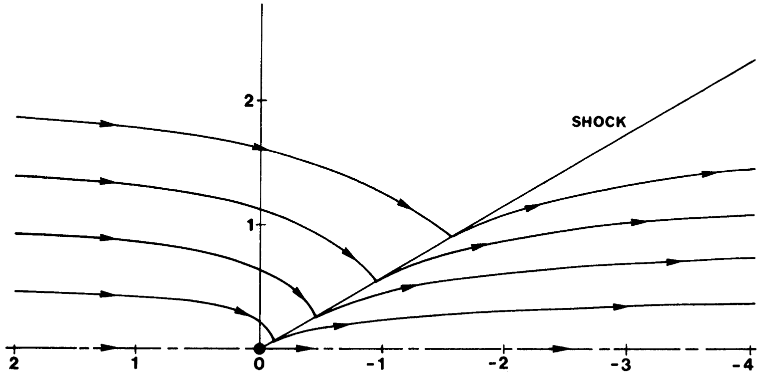

Fig. 2. The mass current as given by linear theory. Unit of length $= R_A$.

model unless a sink is introduced in the continuity equation; the strength of such a sink would have to come from aspects not yet considered, such as conditions on the stellar surface.

Another result of interest from linear theory is the drag, F_D, on the body. This results from the gravitational force of the disturbed gas on the star. If we calculate this drag by Fourier transform techniques we find that the Fourier inversion gives a divergent integral, as is usual with Coulombic potentials; indeed the calculations are much like those encountered in Cherenkov radiation. As in that problem, the limits of integration must arbitrarily be cut off at large and small distance. The long range cut-off is not needed, however, if the self-interaction of the gas is included, and a sort of gravitational shielding occurs. On the other hand, the introduction of self-gravity of the gas brings Jeans instability into the problem so that the results with self-gravity are only suggestive of what happens. For $M = V_0/c > 1$ and with

$$k_J^2 = 4\pi G\varrho_0/c^2 \tag{18}$$

we find, for $M > 1$,

$$F_D = \pi \ln\left[\frac{\pi}{Rk_J}\left(\frac{M}{\sqrt{(M^2-1)}}\right)\right] R_A^2 \varrho_0 V_0\,, \tag{19}$$

where R is the stellar radius, introduced as the short range cut-off.

4. Galactic Wakes*

The linear theory gives some inkling of the difficulties involved in the gas dynamics of accretion, but it is rather unsatisfactory in many ways, and it seems worthwhile to attempt a nonlinear theory. In doing this it is helpful to distinguish two cases $R \gtrsim R_A$ and $R < R_A$, where R is the radius of the moving body. For our Galaxy, $M \approx 2 \times 10^{44}$ g, and if we assume $V_0 \approx 200$ km sec^{-1}, we find $R_A \approx 20$ kpc which is about the radius of the Galaxy. Thus, galaxies belong typically in the class where $R \gtrsim R_A$. This means that gas focused into the wake of a galaxy has already begun to leave the zone of gravitational influence, and this permits a simplification of the problem which is not usually possible in the stellar case.

We treat the Galaxy as spherical and its motion as hypersonic so that the upstream gas is effectively cold. The gas is also collisionless upstream. We assume that particles which strike the Galaxy are absorbed so that no pressure builds up ahead of the Galaxy and we may ignore bow shocks. An orbit which just grazes the Galaxy crosses the accretion axis at a distance, R_0, from the Galaxy (Figure 1), where

$$R_0 = R(1 + \delta^{-1}), \qquad \delta = R_A/R, \tag{20}$$

so that $R_0 > 2R \geqslant 2R_A$. If no collisions occurred, we could completely describe the density and velocity fields by means of orbit theory; for our purposes such a description is most effectively expressed in spherical polar coordinates with origin at $q = R_0$, $\alpha = 0$. Orbits which miss the Galaxy converge conically onto the axis about an apex at this point. We introduce spherical polar coordinates (r, θ) and express quantities in this system. Then density [Equation (7)] is quite complicated in these new coordinates, but for $r \ll R_0$ it simplifies to

$$\varrho = \varrho_0 R(1 + \delta)^{1/2}/(2r \sin\theta), \tag{21}$$

while the velocity becomes

$$\begin{aligned} u &= V_0[1 + \delta(1 + \delta)^{-1}] \cos(\theta + \theta_i), \\ v &= V_0[1 + \delta(1 + \delta)^{-1}] \sin(\theta + \theta_i), \end{aligned} \tag{22}$$

where u and v are the r- and θ-components and

$$\tan\theta_i = \frac{\delta}{(1 + \delta)^{1/2}}. \tag{23}$$

This describes the flow near the apex of the convergence onto the axis. We assume that this flow encounters a (collisionless) shock and that behind the shock the motion obeys the equations of gas dynamics, but that by now we are far enough downstream to neglect the gravity. We also treat the flow as adiabatic and steady in the Galaxy's frame of reference. Equations (21) and (22) suggest that we look for similarity solutions

* This section is based on an unpublished MS of M. A. Ruderman and the author, as is Section 5C.

of the form

$$\varrho = \tfrac{1}{2}\varrho_0 (R/r) f(\theta), \qquad p = \tfrac{1}{2}\varrho_0 V_0^2 (R/r)\, g(\theta),$$
$$u = V_0 U(\theta), \qquad v = V_0 V(\theta). \tag{24}$$

For adiabatic flow the equations admit such solutions (Appendix B) and we obtain a set of ordinary differential equations for U, V, g, and f. These equations can be solved numerically but I will not discuss such solutions here. It suffices to point out that near the axis ($\theta \ll 1$) the solutions behave like

$$U = W_0 [1 - A\theta^{2(\gamma-1)/(2\gamma-1)} + \cdots], \qquad V = -\frac{2\gamma-1}{2\gamma}\theta + \cdots,$$
$$f = f_0 \theta^{-2(\gamma-1)/(2\gamma-1)} + \cdots, \tag{25}$$
$$g = g_0 W_0^2 \left[1 + \frac{f_0}{2g_0\gamma}\left(\frac{2\gamma-1}{2\gamma}\right)^2 \theta^{2\gamma/(2\gamma-1)} + \cdots\right],$$

where W_0, f_0, g_0, and A are constants with the constraint

$$A = \frac{g_0}{f_0}\left(\frac{\gamma}{\gamma-1}\right),$$

and where we have assumed $V(0)=0$. We see that even in the gas dynamical case the density is singular on the axis, as it is in the collisionless case. (For $\gamma=1$, f is not singular, but V is.)

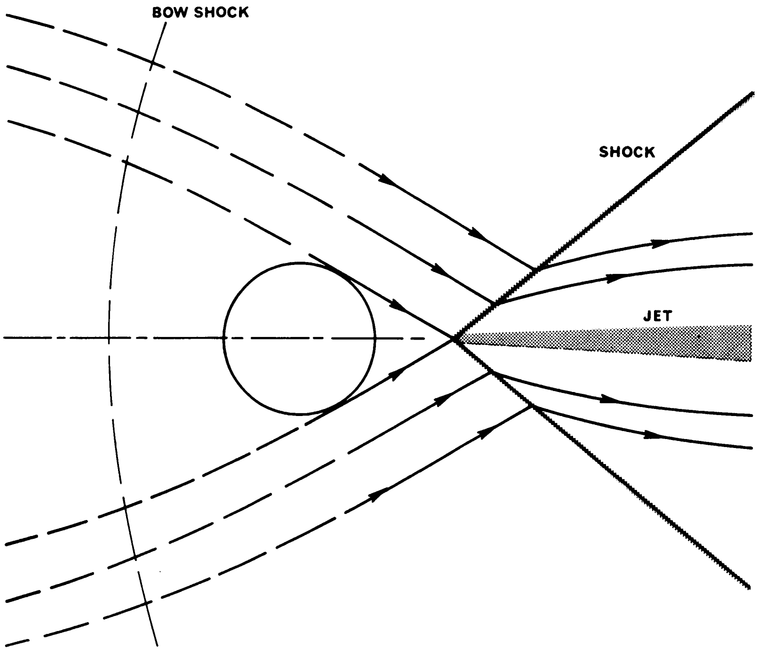

Fig. 3. Solid flow lines indicate the flow sketched from the theory. Dashed lines are the orbits which these should join onto; the bow shock is not in the theory but is indicated.

The gas flow of Equations (25) must now be matched to the incoming flow with the usual conservation conditions for strong shocks, and from this matching we obtain

$$A = \left(\frac{2\gamma - 1}{2\gamma}\right)^2 \frac{1}{2\gamma}\, \theta_s^{2\gamma/(2\gamma-1)}, \qquad \theta_s = \frac{2\gamma(\gamma - 1)}{3\gamma - 1}\,\theta_i,$$
$$W_0^2 = 1 + \delta^2(1+\delta)^{-1}, \qquad f_0 = \left(\frac{\gamma + 1}{\gamma - 1}\right)(1+\delta)^{1/2}\,\theta_s^{-1/(2\gamma-1)}, \tag{26}$$

where θ_s is the shock angle. Thus the solution is determined for given δ. Figure 3 shows a typical flow pattern. The picture we have here is only schematic since many simplifying assumptions have been built into the calculation, but it shows how one might begin to get an idea of which way the flow goes. To do better one should probably try to integrate the equations numerically, but in view of the uncertainties in the plasma effects, perhaps the time for this is not yet ripe.

5. Some Applications

Accretion has at times been suggested as the cause of a variety of phenomena including heating of the solar corona, X-ray sources, galaxy formation, spiral arm formation, comet formation, and novae. I would not like to try to evaluate all of these, and instead, will compound the list a bit.

A. JETS

We know that a variety of galaxies and quasars reveal jet-like appendages, and we might wonder whether these could be accretion wakes. After all, whether you treat the accretion flow as collisionless or gas dynamical, you seem to get evidence of jets. However, it is unlikely that the ambient medium is anywhere dense enough to give rise to an observable galactic jet. On the other hand, the flow we considered in Section 4 need not be the result of accretion, but simply a portion of a non-spherical collapse or implosion. To this extent, the accretion work points a moral. If we want to make a jet we must focus momentum. We would not find this easy to do with ordinary explosions, but the possibility of jet formation in asymmetric collapse does arise. Thus some observed galactic jets might be produced by asymmetric implosions, just as collapsing bubbles often produce prolonged jets (Benjamin and Ellis, 1966), and the kind of convergent flows considered in accretion theory may be quite relevant to this problem, but in a rather different context.

B. THE TAIL OF BETELGEUSE (α ORIONIS)

The star α Orionis shows a jet-like appendage with what looks like a dust core (Morgan *et al.*, 1955). Very little is known except for the picture shown in Figure 4; the tail just barely shows up on the Sky Survey, but it is highly unlikely that even this faint object could be produced by accretion alone. However, the accretion wake might be rendered visible if it were seeded by dust escaping from the star, and the following picture seems possible.

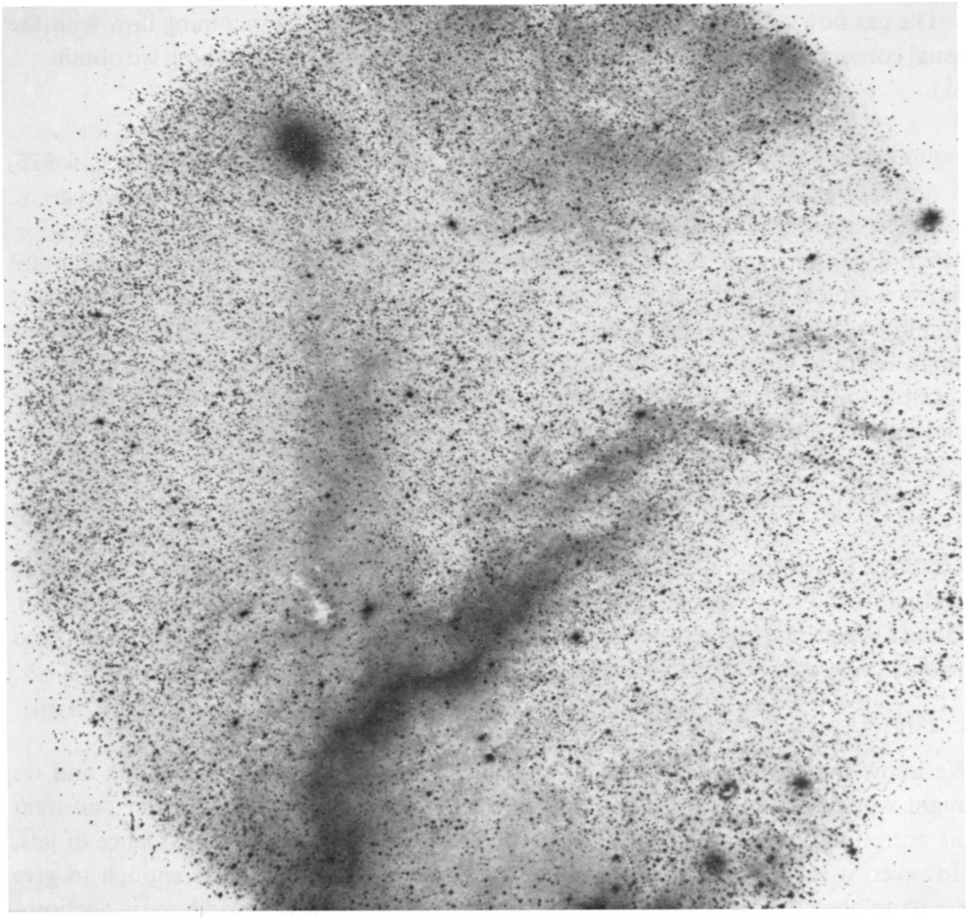

Fig. 4. The Tail of α Orionis: Yerkes Observatory photographs by H. M. Johnson with Meinel 8 in. $f/1$ Schmidt camera and Hα interference filter; diameter of field, 12° (cf. Morgan *et al.*, 1955). Prof. Morgan, who kindly provided the plate reproduced here points out that it was made from a lantern slide, as the original is missing. The tail is nevertheless visible, especially when the plate is held at arm's length.

Suppose that grains form in the atmosphere of α Orionis at some constant rate and that they are driven off by radiation pressure. Inevitably, they must drag some gas with it and thus produce a stellar wind. The extensiveness of such a wind is a matter of debate (Weymann, 1962; Wickramasinghe *et al.*, 1966) and depends very much on the nature of the dust particles. However, that dust particles do exist around α Orionis seems strongly indicated by its infrared excess. The dust wind would behave much like an ordinary wind and have a sonic transition just as in Parker's theory. We would then expect the wind to be arrested in a shock. The shock occurs in the gas which is kept cool by the dust, so that the gas comes virtually to a stop. The gas then drags the dust to a stop and this gives rise to a dusty shell around the star. This shell probably has a radius larger than R_A, so that the theory outlined above is relevant. But it must be modified to take account of the fact that material flowing by the star pulls dust into it so that the flow is enriched. This dust is then squeezed onto the downstream axis to produce the observed tail.

Of course, only detailed observations can decide the correctness of this picture, but at least it provides us with one instance where we may be witnessing an interaction of a star with the interstellar medium.

C. HEATING OF INTERGALACTIC GAS

The preceding example depended on an understanding of the details of the accretion flow. But there are gross considerations which are relevant such as the heating of an ambient gas by supersonically moving objects. The rate of working by such an object is $F_D V_0$, where F_D is the drag force. If the radius of the object exceeds R_A, this formula must be amended to include the geometrical drag which would result from a bow shock. Such corrections for galaxies are not appreciable if V_0 is not in excess of 500 km sec^{-1}.

The heating of the intergalactic gas is important in setting limits on detectability and perhaps the most promising place to look for intergalactic gas is in clusters of galaxies. Typical clusters do not seem gravitationally bound and it has been speculated that the binding is accomplished by intergalactic matter. This is by no means a reliable prediction, but it might be testable if the gas is heated by shock waves generated by the galaxies (Ozernoi and Zasov, in preparation). For example, in the Coma cluster the number of galaxies $N \approx 10^3$ and the dissipation rate by accretion shocks is

$$\varepsilon \approx N \pi C_A R_A^2 (n m_{\rm H}) V_0^3 , \tag{27}$$

where $V_0^2 = 2GM_c/R_c$ and C_A is the logarithmic factor in Equation (19). Subscript c denotes values of R and M for the whole cluster. With $M_G = 4 \times 10^{44}$ g, $M_c = NM_G$, $R_c \approx 5 \times 10^{24}$ cm, we find $V_0 \approx 300$ km sec^{-1} and

$$\varepsilon \approx 5 \times 10^{47} n \text{ erg sec}^{-1} . \tag{28}$$

Now the cluster has a gravitational energy $GM_c^2/(2R_c) \approx 10^{63}$ erg and this would be dissipated in a time of $2 \times 10^8\ n^{-1}$ yr. If, as seems plausible, we assume that the clusters are at least 10^{10} yr old we must have $n < 10^{-2}$ cm^{-3}, which is not inconsistent with the densities required for binding ($n \lesssim 3 \times 10^{-3}$). The gas would be raised to a temperature somewhat less than

$$T_0 = \frac{m_H}{k} \frac{GM_c}{R_c} \approx 10^8 \text{ K} \tag{29}$$

so that temperatures on the order of 10^7 K or a little greater would be expected. In these circumstances we would expect to see thermal X-rays generated at a rate $1.4 \times 10^{-27} n^2 T^{1/2}$ erg sec^{-1} cm^{-3}. The emission comes from the wakes of galaxies which occupy a volume which may be estimated from the theory of the accretion shock; but crudely it is of the order of a few times N times a galactic volume, so that about

10^{-5} of the gas radiates the X-rays. We obtain then an X-ray luminosity of $\approx 10^{48}\, n^2$ erg $\sec^{-1}$. If we compare this with Equation (28) we see that the X-ray efficiency is fairly low at intergalactic densities. (Of course, the formulae cannot be used at high densities where self-absorption enters.) If we take $n \approx 3 \times 10^{-3}$ cm^{-3}, we can get an X-ray luminosity for the Coma cluster of about 10^{43} erg sec^{-1}. This should be readily detectable and would seem to militate against the assumption that there is intergalactic matter in clusters sufficient to bind them.

Similar examples can be worked out for other clusters, individual galaxies, clusters of galaxies moving through the intergalactic medium, and even star clusters in dense galactic nuclei and perhaps quasars.

Appendix A

The equations of isentropic motion are

$$\varrho \left(\frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = - \nabla p + \varrho \nabla \varphi , \tag{A1}$$

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \mathbf{u}) = - A , \tag{A2}$$

$$\nabla^2 \varphi = - 4\pi G (\varrho + \varrho_*) , \tag{A3}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = c^2 \frac{\mathrm{d}\varrho}{\mathrm{d}t} , \tag{A4}$$

where ϱ_* is the density of the moving object and A is the rate of accretion, counted positive if the object is a sink. If the object is a source of matter, then it would not be correct to use (A4) for both the ambient and injected matter since in general c^2 will differ for the two. If we treat the star as a point mass moving with velocity $\mathbf{V}_0$, we have

$$\varrho_* = M \, \delta(\mathbf{r} - \mathbf{V}_0 t) \tag{A5}$$

and

$$A = \lambda \pi R_A^2 V_0 \varrho \tag{A6}$$

where λ is not yet specified, but could depend on the solution. Let $\varrho = \varrho_0 + \delta\varrho$ with $| \delta\varrho | \ll \varrho_0$ and let $\mathbf{u}$ be small (as in usual acoustics). Because of the self-gravity, ϱ_0 is not a constant, but we make the usual Jeans swindle, and treat it so. Then, on dropping nonlinear terms, we obtain

$$\varrho_0 \frac{\partial \mathbf{u}}{\partial t} = - \nabla \, \delta p + \varrho_0 \nabla \, \delta\varphi , \tag{A7}$$

$$\frac{\partial}{\partial t} \delta\varrho + \varrho_0 \nabla \cdot \mathbf{u} = - \lambda \pi R_A^2 \varrho_0 V_0 \, \delta(\mathbf{r} - \mathbf{V}_0 t) , \tag{A8}$$

$$\nabla^2 \, \delta\varphi = - 4\pi G (\varrho_* + \delta\varrho) , \tag{A9}$$

$$\delta p = c^2 \, \delta\varrho , \tag{A10}$$

where λ is now a constant. These equations are readily combined into a wave equation for $\delta\varrho$ (with $k_J^2 = 4\pi G\varrho_0/c^2$):

$$\Box\,\Psi + k_J^2 c^2 \Psi = 4\pi GM\,\delta(\mathbf{r} - \mathbf{V}_0 t) + \lambda\pi R_A^2 \varrho_0 V_0^2\,\delta(z - V_0 t)\,\delta(x)\,\delta(y). \tag{A11}$$

When $k_J = 0$ and $\lambda = 0$, this is Equation (11) of the text. The criterion for neglecting the k_J term is roughly that $k_J R_A \ll 1$, but even in that case certain long range effects are not correctly represented when k_J is set equal to zero. It also is clear that if we solve Equation (11) with $\lambda = 0$, the correction for $\lambda \neq 0$ is just to add a term $\lambda R_A\,\partial\Psi_0/\partial z$ to Ψ_0, where Ψ_0 is the solution with $\lambda = 0$.

We readily verify that Equations (14), (15), and (16) represent solutions of the linear equations.

Appendix B

In the frame of the star we assume steady fluid motions. If we can neglect gravity in the wake, we have the equations

$$\varrho\left(u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial\theta} - \frac{v^2}{r}\right) = -\frac{\partial p}{\partial r}, \tag{B1}$$

$$\varrho\left(u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial\theta} + \frac{uv}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta}, \tag{B2}$$

$$\varrho\left(\frac{\partial u}{\partial r} + \frac{2u}{r} + \frac{1}{r}\frac{\partial v}{\partial\theta} + \frac{v}{r}\cot\theta\right) + \left(u\frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial\theta}\right)\varrho = 0, \tag{B3}$$

$$\left(u\frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial\theta}\right)p = \frac{\gamma p}{\varrho}\left(u\frac{\partial p}{\partial r} + \frac{v}{r}\frac{\partial}{\partial\theta}\right)\varrho. \tag{B4}$$

With the ansatz of Equations (24) we find

$$\begin{aligned} &VU' - V^2 = g/f \\ &VV' + UV = g'/f \\ &U + V' + V\cot\theta + Vf'/f = 0 \\ &(\gamma - 1)\,U = V\left(\frac{\gamma f'}{f} - \frac{g'}{g}\right) \end{aligned} \tag{B5}$$

where prime denotes differentiation with respect to θ. From (B5) we obtain two integrals:

$$g = Kf^{\gamma}(-fV\sin\theta)^{\gamma-1} \tag{B6}$$

and

$$\tfrac{1}{2}(U^2 + V^2) + \left(\frac{\gamma}{\gamma - 1}\right)\frac{g}{f} = \tfrac{1}{2}W_0^2, \tag{B7}$$

where W_0 and K are arbitrary constants. With these results the expansion of Equations (25) is readily obtained, and with this behavior near $\theta = 0$, we can find the solution of (B5) numerically.

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