BULL. AUSTRAL. MATH. SOC. 53C20
VOL. 60 (1999) [521–528]

RIGIDITIES OF ASYMPTOTICALLY EUCLIDEAN MANIFOLDS

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Let M be an n-dimensional compact Riemannian manifold. We study the fundamental group of M when the universal covering space of M, $\widetilde{M}$ is close to some Euclidean space $\mathbb{R}^s$ asymptotically.

1. INTRODUCTION

Let (M,g) be an n-dimensional compact Riemannian manifold with metric g. For fixed $p \in \widetilde{M}$, let $B(p,R)$ and $B(0,R)$ be R-balls in $\left(\widetilde{M},\widetilde{g}\right)$ and $(\mathbb{R}^s,\delta)$, respectively. We define M to be an asymptotically Euclidean manifold if

$$\lim_{R\to\infty} \frac{d_H\big(B(p,R),B(0,R)\big)}{R} = 0,$$

where d_H is the Gromov-Hausdorff distance.

It is easy to show $\left(\widetilde{M},\varepsilon_i^2\widetilde{g}\right) \to (\mathbb{R}^s,\delta)$ for any positive sequence $\varepsilon_i \to 0$ with respect to the pointed Gromov-Hausdorff distance [**3**], where δ is the Euclidean metric, that is, the R-balls centred at p in $\left(\widetilde{M},\varepsilon_i^2\widetilde{g}\right)$ converge to the Euclidean R-ball with respect to the Gromov-Hausdorff distance for any $R > 0$. The notion of the asymptotic cone of $\widetilde{M}$ in [**5**] is similar to $\lim\limits_{i\to\infty}\left(\widetilde{M},\varepsilon_i\widetilde{g}\right)$. We prove the following theorem.

THEOREM 1.1. *Let (M,g) be an n-dimensional compact Riemannian manifold with metric g. If M is an asymptotically Euclidean manifold, then $\pi_1(M)$ is an almost Abelian group.*

REMARK 1.2. For an asymptotically Euclidean space M, if we can observe $\widetilde{M}$ at infinite distance from $\widetilde{M}$, $\widetilde{M}$ looks like a Euclidean space. If there exists a positi sequence $\{R_i\}$ such that $\lim\limits_{R_i\to\infty} d_H\big(B(p,R_i),B(0,R_i)\big)/R_i = 0$, then $\widetilde{M}$ looks like a Euclidean space if we observe at infinite distance from $\widetilde{M}$. We call such a space a *weakly asymptotically Euclidean space.*

Then we obtain the following corollary immediately:

Received 7th April, 1999
Supported by KIAS and grant No. 1999-2-102-002-3 from the interdisciplinary Research program of KOSEF.

COROLLARY 1.3. *Let M be a weakly asymptotically Euclidean manifold. If $\pi_1(M)$ is an almost solvable group, then $\pi_1(M)$ is an almost Abelian group.*

If M is an n-dimensional compact Riemannian manifold with no conjugate points and $\pi_1(M)$ is almost solvable, then $\pi_1(M)$ is a Bieberbach group [**2**]. With the Hopf conjecture proved in [**1**], we obtain the following rigidity theorem:

THEOREM 1.4. *Let M be an n-dimensional compact Riemannian manifold without conjugate points. If $\pi_1(M)$ is an almost solvable group, then M is isometric to a flat torus up to finite cover.*

We apply a method similar to that for Corollary 1.3 to manifolds with no conjugate points and so give a geometric proof for this theorem.

2. PRELIMINARIES

For the proof of Theorem 1.1, we use the following generalised Bieberbach theorem and corollary as in [**3**].

THEOREM 2.1. [**3**] *Let G be a closed subgroup of the group of isometries of $\mathbb{R}^n$. Then $\pi_0(G)$ contains a finite index, free Abelian subgroup whose rank is not greater than $\dim(\mathbb{R}^n/G)$.*

COROLLARY 2.2. [**3**] *Suppose, in addition, that the quotient space $\mathbb{R}^n/G$ is compact. Then there exists a normal subgroup G' of G such that*

(1) *$[G : G'] < w_n$, where w_n is a number depending only on n,*
(2) *$\mathbb{R}^n/G'$ is isometric to a flat torus.*

We denote by (X, Γ, p) a pointed length space (X, p) on which a group Γ acts isometrically. Put

$$\Gamma(R) = \{\gamma \in \Gamma \mid d(\gamma p, p) < R\}$$

and let $B_q(R, X)$ be the R-ball centred at q in X. The notation

$$\lim_{i\to\infty} (X_i, \Gamma_i, p_i) = (Y, G, q)$$

means that (X_i, Γ_i, p_i) converges to (Y, G, q) with respect to the equivariant pointed Hausdorff distance. (See [**3**] for the precise definition of the equivariant Hausdorff distance.)

THEOREM 2.3. [**3**] *Let (X_i, Γ_i, p_i) and (Y, G, q) be such that*

$$\lim_{i\to\infty} (X_i, \Gamma_i, p_i) = (Y, G, q);$$

let G' be a normal subgroup of G. Assume that

(1) *G/G' is discrete and finitely presented,*

(2) *Y/G is compact,*

(3) *Γ_i is properly discontinuous and free, X_i is simply connected,*

(4) *there exists R_0 such that G' is generated by $G'(R_0)$ and such that $\pi_1(B_q(R_0, Y))$ surjects to $\pi_1(Y)$.*

Then there exists a sequence of normal subgroups Γ'_i of Γ_i such that

(5) $\lim_{i\to\infty}(X_i, \Gamma'_i, p_i) = (Y, G', q)$,

(6) *Γ_i/Γ'_i is isomorphic to G/G' for each sufficiently large i,*

(7) *there exists R_i independent of i such that Γ'_i is generated by $\Gamma'_i(R_1)$.*

We observe an asymptotically centralising property for a nilpotent group. We assume that $\pi_1(M)$ is not an Abelian group but a nilpotent group. We denote by Z the centre of $\pi_1(M)$. Let $\pi : \pi_1(M) \to \pi_1(M)/Z$ be the natural quotient map and Z' be the centre of $\pi_1(M)/Z$. For $h'_1 \in Z'$, we denote $\pi^{-1}(\langle h'_1\rangle)$ by Z_1. Although Z_1 is not the centre, it is an Abelian subgroup of $\pi_1(M)$. By the following easy example, we see an asymptotically centralising property of the nilpotent group.

EXAMPLE 2.4. Let $G = \langle e_1, e_2, e_3\rangle$ with the relation $[e_2, e_3] = e_1^r$ and $[e_1, e_j] = 0$ for $j = 1, 2, 3$. Then G is a nilpotent group and $\langle e_1\rangle$ is the centre of G. Every element g in G can be represented by $e_1^{l_1}e_2^{l_2}e_3^{l_3}$. If we write $e_1^{l_1}e_2^{l_2}e_3^{l_3}$ as (l_1, l_2, l_3), then we have the following product in $\mathbb{Z}^3$;

$$(m_1, m_2, m_3) \cdot (l_1, l_2, l_3) = (l_1 + m_1 + rm_3l_2, m_2 + l_2, m_3 + l_3).$$

So we have $(0, 0, 1) \cdot (0, l, 0) = (l, l, 1)$. Then $\lim_{l\to\infty}(0, 0, 1) \cdot (0, l, 0)/l = (1, 1, 0)$ and we obtain a non trivial central component by G-action asymptotically.

For non central element h_1 in Z_1, there exists a g such that $[g, h_1] \neq 0$. Let $[g, h_1] = z$, where z is contained in the centre. By the same reason as in the above example, we have $g \cdot h_1^l = h_1^l \cdot z^l \cdot g$. So it has also an asymptotically centralising property. We shall prove Theorem 1.1 by a contradiction from this asymptotically centralising property.

3. Proof of Theorem 1.1

We only need to consider the case that $\pi_1(M)$ is an almost nilpotent group since an asymptotically Euclidean manifold has a polynomial volume growth. Then we can find a nilpotent subgroup N with $[\pi_1(M) : N] < \infty$. Let $\varepsilon_i > 0$ be a sequence such that $\varepsilon_i \to 0$. We rescale the metric of $\widetilde{M}$ by multiplying by ε_i^2. Let X_i be the rescaled manifold. Then

$$(X_i, \pi_1(M)) \to (\mathbb{R}^s, G)$$

https://doi.org/10.1017/S0004972700036686 Published online by Cambridge University Press

for some isometry group G of $\mathbb{R}^s$ from our assumption. Since $\pi_1(M)$ is almost solvable, if we apply Corollary 2.2 and Theorem 2.3 to some finite covering space of M inductively, we obtain that there exists a subgroup $\Pi_i \in \pi_1(M)$ such that

$$(X_i, \Pi_i) \to (\mathbb{R}^s, \Pi), \tag{3.1}$$

where Π is a translational isometry group and $[\pi_1(M) : \Pi_i] < \infty$. We may assume that $\Pi_i = \Pi_L$ for $i \geqslant L$. Then

$$[\pi_1(M) : \Pi_L \cap N] \leqslant [\pi_1(M) : \Pi_L][\pi_1(M) : N] < \infty.$$

For brevity, we write N instead of $N \cap \Pi_L$, which is also a nilpotent group. From the choice of Π_i, we know that N is a torsion free nilpotent group. Let Z be the centre of N. By considering the orbit of a given point $p \in \widetilde{M}$ under the isometric action of N, we regard N as a subset of $\widetilde{M}$. Then $N = N_i \subset X_i$ converges to $\mathbb{R}^s$ since Π is a translational isometry group of $\mathbb{R}^s$. Also Z converges to Ξ for some Euclidean space $\Xi = \mathbb{R}^k$, $k \leqslant s$. If we show that $k = s$, this will complete the proof for the nilpotent case. Since Z is the centre, the subgroup $\langle\{Z, g\}\rangle$ generated by Z and g is also an Abelian group for any $g \in N$. So $\langle\{Z, g\}\rangle$ also converges to the Euclidean space $\mathbb{R}^{k+1}$.

Now we study the isometric action of $\pi_1(M)$. Since $\pi_1(M)$ act isometrically on $\widetilde{M}$, also it acts on $\mathbb{R}^s$ isometrically [4]. Consider $\Xi^{k+1} := \lim_{i\to\infty} \langle\{Z, h_1\}\rangle = \lim_{i\to\infty} Z_1$ and the isometric action by g as in section 2. Let v be a vector orthogonal to Ξ in Ξ^{k+1}. By (3.1), we may assume that $v = \lim(v_1, v_1^2, \cdots)$ for some $v_1 \in Z_1$. From the above computation, we have

$$g(v) = \lim_{l\to\infty} g \cdot v_1^l = \lim_{l\to\infty} z^l \cdot v_1^l \cdot g = z' + v,$$

where $z' = \lim(z, z^2, \cdots) \in Z$.

We shall show that g does not act isometrically on X, which is a contradiction. We consider the Euclidean space Ξ^{k+1}. For $z = \lim(z_1, z_1^2, \cdots) \in \Xi$,

$$d(gz, z) = \lim_{l\to\infty} \frac{1}{l} \|z_1^{-l} g z_1^l\| = \lim_{l\to\infty} \frac{1}{l} \|g\| = 0,$$

where $\|g\| = d(p, g(p))$. So Ξ is invariant under the action of g.

Since g is an isometric action, $g(v)$ is orthogonal to Ξ. Then

$$0 = \langle g(v), g(\Xi)\rangle = \langle g(v), \Xi\rangle = \langle z', \Xi\rangle \neq 0, \tag{3.2}$$

which is a contradiction. ($\langle , \rangle$ is the inner product on $\mathbb{R}^s$.) This completes the proof of Theorem 1.1.

Now we prove Corollary 1.3. We may assume that $\pi_1(M)$ is a strongly polycyclic group which is not a nilpotent group. It is not known that a weakly asymptotically Euclidean manifold has a polynomial growth. We can only obtain that it has a non-exponential growth. So we need some further arguments.

As above, there exists a subgroup Π_L of $\pi_1(M)$ satisfying (3.1). Let Γ_0 be $\pi_1(M)$ and $[\Gamma_i, \Gamma_i] = \Gamma_{i+1}$. Then $\Gamma_N \neq \{0\}$ and $\Gamma_{N+1} = 0$ for some N. If Γ_N is the centre of $\pi_1(M)$, then we can apply the same proof as above. Assume that $\Gamma_N \simeq \mathbb{Z}^k$ is not the centre of $\pi_1(M)$. We also denote the limit of Γ_N as $X_i \to \mathbb{R}^s$ by $\Xi = \mathbb{R}^k$. Also we have that $g \in \Gamma_{N-1}$ can be considered as an isometry from Ξ to Ξ. Let $v_1 \in \Gamma_N$. We may assume that there exists $g \in \Gamma_{N-1}$ such that $v_1^{-1} g v_1 g^{-1} = [v_1^{-1}, g] = w \neq 0$ for $w \in \Gamma_N$. In fact, if $[\Gamma_{N-1}, \Gamma_N] = 0$, we may consider $\langle \{\Gamma_N, g\} \rangle$ for $g \in \Gamma_{N-1}$ instead of Γ_N. Then $\langle \{\Gamma_N, g\} \rangle$ is an Abelian normal subgroup of Γ_{N-1}. In this way, we may assume $[\Gamma_{N-1}, \Gamma_N] \neq 0$. Then $g v_1 = v_1 w g$ and $g v_1^2 = v_1 w g v_1 = v_1 w v_1 w g = v_1^2 w^2 g$ since Γ_N is an Abelian group. So we have that

$$g v_1^n = v_1^n w^n g.$$

Let $v = \lim \left(v_1, v_1^2, \cdots \right)$, where we consider v_1^i and v as elements of X_i and Ξ, respectively. We know that $g(v) = \lim_{i \to \infty} g v_1^i = v + w'$, where $w' = \lim \left(w, w^2, \cdots \right)$. If we take a basis $\{ \lim_{n \to \infty} \gamma_i^n \}$ of Ξ for generators $\{\gamma_i\}$ of Γ_N, then $g \in \mathrm{GL}(k, \mathbb{Z})$. Using the packing arguments, we easily show that $g^m = \mathrm{Id}$ for sufficiently large m, since g is an isometry of $\mathbb{R}^k$. So we obtain that $[g^m, \Gamma_N] = 0$. This is a contradiction to $[g, \Gamma_N] \neq 0$ since we may assume that Γ_{N-1}/Γ_N is a torsion free Abelian group. This completes the proof of Corollary 1.3.

4. An asymptotic approach to Theorem 1.4

Let Z_0 be an Abelian subgroup of $\pi_1(M)$. With the same notation as in the previous sections, we only need to prove the following Lemma:

Lemma 4.1. *We denote Z_0 as a subset of X_i by Z_i. If M has no conjugate points, Z_i converges to the k-dimensional vector space with the standard Euclidean norm, where k is the rank of the centre of $\pi_1(M)$.*

We use the same notations as the previous section, that is, $\Gamma_N \simeq \mathbb{Z}^k$ and $[\Gamma_{N-1}, \Gamma_{N-1}] = \Gamma_N$. If we apply the proof in Corollary 1.3 to the Γ_{N-1} and Γ_N with Lemma 4.1, we obtain Theorem 1.4.

Proof of Lemma 4.1: We denote the limit of Z_i by Z. We can easily verify that Z is a vector space with a norm. Basically we shall follow the proof of [**1**]. We shall prove this lemma by the following step. It is a slight modification of the proof in [**1**].

STEP 1. Assume that $\delta_i = 1/i$. Since Z is a k-dimensional vector space, we take k-linearly independent vectors $\lambda_1, \cdots, \lambda_k$. Let $\overline{Z} = Z/\langle\lambda_1, \cdots, \lambda_k\rangle$. Then $\overline{Z}$ is diffeomorphic to a k-dimensional torus.

For $p \in Z$, there exists a geodesic $\gamma_p \subset \widetilde{M}$ such that $\gamma_p(i) \to p$. In fact, such a geodesic γ_p is not necessarily unique. The following arguments are about the choice of such a geodesic. We know that $\gamma_p(it)$ converges to a geodesic tp in X. Let F and F_0 be unit balls in X and Z, respectively. For n-independent vectors $p_j \in F$, we construct global coordinates for $\widetilde{M}$ with the Busemann functions

$$y_j = \lim_{t\to\infty} d\Big(\gamma_{p_j}(t), \cdot\Big) - t,$$

where $p_1, \cdots, p_k \in F_0$. Then we obtain a vector field v_{p_j} on $\widetilde{M}$ as $\lim_{t\to\infty} \nabla d\Big(\gamma_{p_j}(t), \cdot\Big)$ for the above γ_{p_j}. For any $p \in F_0$, we obtain v_p by the same manner. This limit exists if we take some subsequence by $C^{1,\alpha}$ (or $L^{2,q}$)-boundedness of the distance function r under the Ricci curvature and the injectivity radius bounded below. (Every compact manifold has a lower bound of Ricci curvature. In our case, we take the limit for a fixed manifold X_i. Since $p \in Z$, we know that the v_p are $\pi_1(M)$-invariant vector fields.) By taking a subset $V = \{(y_1, \cdots, y_k, 0, \cdots, 0)\}$, we easily show that V converges to Z as $X_i \to X$. Now we can define an injective map $\iota : F_0 \to UV_x \subset U\widetilde{M}_x$ such that $\gamma_p(i) \to p$ and $\gamma_p'(0) = \iota(p)$, where UV_x and $U\widetilde{M}_x$ are the unit tangent spaces of V and $\widetilde{M}$ at x, respectively. Note that for general manifolds, there are no natural injective maps from F to UM_x. Conversely, we can define $F : S^{k-1} = UV_x \to F_0$ by $F(v) = \lim_{i\to\infty} \exp iv$, that is, $\gamma_{F(v)}(t) = \exp tv$. Then we have $F \circ \iota = \mathrm{id}$.

Define the Busemann function of the geodesic tp by $B_p(q) = \lim_{t\to\infty} d(tp, q) - t$ for $p \in Z$. As in [**1**], every horosphere of a ray pt is a translation of the tangent cone to F_0 at $-p$ so B_p is a linear function and F_0 has a unique supporting hyperplane at p. (See [**1**].)

We use similar notation to that in [**1**]. Denote by UZ and $U\overline{Z}$ the unit tangent bundles of Z and $\overline{Z}$, respectively. Let (v, p) be a unit tangent vector at $p \in Z \simeq \mathbb{R}^k$, where v is a vector in R^k and p is a base point. We define $D : UZ \to F_0$ by $D(v, p) = v$. Then for $q \in F_0$,

$$\begin{aligned} B_p(q) &= -\lim_{i\to\infty} i^{-1} \int_0^i \langle\gamma_q'(t), v_p\rangle dt \\ &= -\lim_{i\to\infty} \int_0^1 \Big\langle i^{-1}\frac{d}{dt}\gamma_q(it), v_p\Big\rangle dt. \end{aligned}$$

So we have

$$B_p(q)^2 \leqslant \liminf_{i\to\infty} \int_0^1 \Big\langle i^{-1}\frac{d}{dt}\gamma_q(it), v_p\Big\rangle^2 dt.$$

Using normal coordinates, we easily show that $i^{-1}\frac{d}{dt}\gamma_q(it)$ converges to the geodesic flow on UZ.

We fix some lifting map $L : U\overline{Z} \to UZ$. Let mes be the normalised Liouville measure on $U\overline{Z}$ invariant under the geodesic flows. In fact, we have not proved that $\overline{Z}$ is a Riemannian manifold yet but we know that there exists an invariant measure by the Krylov-Bogolubov theorem. Let $m = D \circ L(mes)$. We denote $i^{-1}\gamma(it)$ by $\gamma^i(t)$. We define a map $C : U\overline{Z} \to \mathbb{R}$ by

$$C(w) := \langle \gamma'_{D\circ L(w)}(0), v_p \rangle^2 = \lim_{i\to\infty} \left\langle \frac{d}{dt}\gamma^i{}_{D\circ L(w)}(0), v_p \right\rangle^2.$$

for a fixed $p \in F_0$. Note that $\gamma'_{D\circ L(w)}(0) = \iota\big(L(w)\big)$. Since $i^{-1}\frac{d}{dt}\gamma_q(it)$ converges to the geodesic flow on UZ, we have

$$\begin{aligned}\int_{F_0} B_p(q)^2 dm(q) &= \lim_{i\to\infty}\int_{U\overline{Z}} B_p\big(D\circ L(w)\big)^2 dmes \\ &\leqslant \int_{U\overline{Z}} \lim_{i\to\infty}\int_0^1 \left\langle i^{-1}\frac{d}{dt}\gamma_{D\circ L(w)}(it), v_p\right\rangle^2 dtdmes \\ &= \int_{U\overline{Z}} \lim_{T\to\infty} T^{-1}\int_0^T \lim_{i\to\infty}\left\langle i^{-1}\frac{d}{dt}\gamma_{D\circ L(w)}(it), v_p\right\rangle^2 dtdmes \\ &= \int_{U\overline{Z}} C(w)dmes,\end{aligned}$$

by the Birkhoff ergodic theorem.

Define $g : S^{k-1} \to \mathbb{R}$ by $g(w) = \langle w, v\rangle^2$. Since $\int_{U_x\overline{Z}} \left\langle \frac{d}{dt}\gamma_{D\circ L(w)}(0), v_p\right\rangle^2 dmes$ does not depend on x and v_p, we have

$$\begin{aligned}\int_{U\overline{Z}} C\, dmes &= \int_{\overline{Z}} \mathrm{dvol}(z)\int_{U_x\overline{Z}}\left\langle \frac{d}{dt}\gamma_{D\circ L(w)}(0), v_p\right\rangle^2 dmes_x(w) \\ &= \int_{U_x\overline{Z}} \langle \iota(w), v_p\rangle^2 dmes_x(w) = \int_{U_x\overline{Z}} \iota^* g\; dmes_x.\end{aligned}$$

Since $F\circ\iota = \mathrm{id}$, we have $\iota\circ F\big(\iota(p)\big) = \iota(p)$ so $\iota\circ F$ is the identity on $\iota\big(U_x\overline{Z}\big)$. Thus we obtain that

$$\begin{aligned}\int_{S^{k-1}} \langle w, v\rangle^2 d\mu \geqslant \int_{\iota(U_x\overline{Z})} \langle w, v\rangle^2 d\mu &= \int_{\iota(U_x\overline{Z})} F^*\iota^* g\; d\mu \\ &= \int_{F\circ\iota(U_x\overline{Z})} \iota^* g\; F(d\mu) = \int_{U_x\overline{Z}} \iota^* g\; dmes_x,\end{aligned}$$

where μ is the standard measure with $\mu\big(\iota(U_x\overline{Z})\big) = 1$ and $v \in S^{k-1}$. Hence we obtain that

$$\int_{F_0} B_p(q)^2 dm(q) \leqslant \int_{S^{k-1}} \langle w, v\rangle^2 d\mu.$$

This value does not depend on $v \in S^{k-1}$ and as in [1], we obtain that

$$\int_{F_0} B_p(q)^2 dm(q) \leqslant \frac{1}{k}.$$

The following step is the same as in [1].

STEP 2. Let F_0^* be the set of all linear functions supporting F_0, let $A_{F_0} = \{kL^2 \mid L \in F_0^*\}$ and $\overline{A}_{F_0}$ be the convex hull of A_{F_0}. Applying Section 4 in [1] to the vector space $Z \simeq \mathbb{R}^k$, we obtain that the maximal nonnegative quadratic form Q on $\mathbb{R}^k \simeq Z$ satisfies $Q = k\sum a_l B_{p_l'}^2$ where $\sum a_l = 1$, $a_l \geqslant 0$ and $p_l' \in Z$. Then

$$k\int_F \sum a_l B_{p_l'}(q)^2 dm(q) = \int_F Q(q)\ dm(q) \leqslant 1.$$

From $Q(F) \geqslant 1$, with the above inequality, we obtain that $m\{q \in F \mid Q(q) > 1\} = 0$. Since Q is continuous and m is dense, $F = \{x \in Z \mid Q(x) = 1\}$. So Q is the Euclidean norm if we restrict to Z. This completes the proof of Lemma 4.1. □

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