Global Patterns

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Filaments and the Solar Dynamo

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Abstract.
Filaments are a global phenomenon and their formation, structure and dynamics are determined by magnetic fields. So they are an important signature of the solar magnetism. The central mechanism in traditional mean-field dynamo theory is the alpha effect and it is a major result of this theory that the presence of kinetic or magnetic helicities is at least favourable for the effect. Recent studies of the magnetohydrodynamic equations by means of numerical bifurcation-analysis techniques have confirmed the decisive role of helicity for a dynamo effect. The alpha effect corresponds to the simultaneous generation of magnetic helicities in the mean field and in the fluctuations, the generation rates being equal in magnitude and opposite in sign. In the case of statistically stationary and homogeneous fluctuations, in particular, the alpha effect can increase the energy in the mean magnetic field only under the condition that also magnetic helicity is accumulated there. Generally, the two helicities generated by the alpha effect, that in the mean field and that in the fluctuations, have either to be dissipated in the generation region or to be transported out of this region. The latter may lead to the appearance of helicity in the atmosphere, in particular in filaments, and thus provide valuable information on dynamo processes inaccessible to in situ measurements.

1. Introduction

The dynamo for the global solar magnetic field is assumed to operate in the convection zone and to consist of the cyclic generation of a toroidal (azimuthal) field from a poloidal one (whose field lines lie in planes containing the rotational axis of the Sun) and the regeneration of a poloidal field from a toroidal one. If there exists a poloidal field, then a toroidal field is generated very effectively by differential rotation. But the regeneration of the poloidal field represents a problem. For this reason the theory of the turbulent dynamo has been developed (Krause and Rädler 1980). The central mechanism in this theory is the generation of a mean, or large-scale, electromotive force $\mathcal{E}$ by turbulently fluctuating, or small-scale, parts of velocity and magnetic field, and it is a major result of the theory that the presence of kinetic and magnetic helicities is favourable for a so-called alpha effect, i.e., a non-vanishing component $\mathcal{E}_{||} = \alpha\langle \mathbf{B}\rangle$ of $\mathcal{E}$ along the mean magnetic field $\langle \mathbf{B}\rangle$. The densities per unit volume of kinetic, magnetic

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and current helicity are defined by

$$H_{\mathrm{K}} = \mathbf{v} \cdot (\nabla \times \mathbf{v}),\ H_{\mathrm{M}} = \mathbf{A} \cdot \mathbf{B},\ H_{\mathrm{C}} = \mathbf{B} \cdot (\nabla \times \mathbf{B}), \tag{1}$$

where $\mathbf{v}$, $\mathbf{B}$ and $\mathbf{A}$ denote fluid velocity, magnetic field and a magnetic vector potential. H_{M} and H_{C} are closely related (cf., e.g., Seehafer 1990).

The usually quoted estimate for the alpha-effect parameter α is (Krause and Rädler 1980, Eq. (3.31))

$$\alpha \approx -\frac{\tau}{3}\langle \mathbf{v}' \cdot \nabla \times \mathbf{v}' \rangle, \tag{2}$$

where τ is the correlation time of the velocity fluctuations $\mathbf{v}'$ (angular brackets denote averages and primes the corresponding residuals). This estimate, which relates α to the kinetic helicity of the fluctuations, is derived under the following approximations and assumptions:
1) The first order smoothing approximation (FOSA), which consists of neglecting the unpleasant term $\nabla \times (\mathbf{v}' \times \mathbf{B}' - \langle \mathbf{v}' \times \mathbf{B}' \rangle)$ in the equation for the time evolution of the magnetic fluctuations. This approximation is valid for, e.g., wave turbulence, where a disturbance does not lead to the onset of convection but only to a wave. It is, however, rather doubtful in the case of conventional, convective turbulence, i.e., in the solar convection zone.
2) $\langle \mathbf{v} \rangle = \mathbf{0}$.
3) $\langle \mathbf{B} \rangle = constant$ (in space and time).
4) Statistically stationary and homogeneous fluctuations.
5) η (magnetic diffusivity) $\rightarrow 0$.

The alpha effect is more directly related to *current* helicity than to kinetic helicity, namely (see Sec. 3 and Keinigs 1983, Matthaeus et al. 1986, Rädler & Seehafer 1990, Seehafer 1994a, b, Seehafer 1996),

$$\alpha \stackrel{\mathrm{def}}{=} \frac{\mathcal{E} \cdot \langle \mathbf{B} \rangle}{\langle \mathbf{B} \rangle^2} = -\frac{\eta}{\langle \mathbf{B} \rangle^2}\langle \mathbf{B}' \cdot (\nabla \times \mathbf{B}') \rangle. \tag{3}$$

For deriving this relation, of the above five conditions only the fourth one is needed. On the other hand, the traditional estimate, Eq. (2), gives an information on which type of *fluid motion* can produce an alpha effect.

The majority of dynamo studies, in particular those in the frame of mean-field theory, have been kinematic. Kinematic dynamo theory studies the conditions under which a prescribed velocity field can amplify, or at least prevent from decaying, some seed magnetic field, completely disregarding the equations governing the motion of the fluid. A step towards a self-consistent, nonlinear theory is taken by models containing, mainly on the base of physically plausible assumptions, a back reaction of a generated mean magnetic field on the generating turbulent fluid motions. Here, in particular, models with the so-called α-quenching are studied, in which the alpha-effect parameter α is a function of the mean magnetic field (e.g., Rädler et al. 1990).

In principle totally self-consistent are numerical simulations of the complete system of the nonlinear magnetohydrodynamic (MHD) equations (e.g., Meneguzzi et al. 1981, Meneguzzi and Pouquet 1989, Glatzmaier 1984, 1985). In some sense still a step further goes a bifurcation, or qualitative, analysis, by which one tries to get an overview of the attractor structure of the system, i.e., of the set of the possible time-asymptotic states. In Sec. 2 an example of a numerical bifurcation analysis is presented. The Reynolds numbers reachable here presently are by many orders of magnitude smaller than those at the Sun. Therefore, the statistical mean-field approach remains indispensible. In Sec. 3 it is demonstrated that the alpha effect generates simultaneously and at equal rates fluctuating (turbulent) and mean-field magnetic helicities of opposite signs. Section 4 then gives a final discussion.

2. Bifurcation Analysis of a Magnetofluid with Helical Forcing

Simple examples of strongly helical flows are provided by the so-called ABC flows (see, e.g., Dombre et al. 1986), given by

$$\mathbf{v} = \mathbf{v}_{ABC} = (A\sin z + C\cos y,\ B\sin x + A\cos z,\ C\sin y + B\cos x), \tag{4}$$

where A, B and C denote constant coefficients. The ABC flows are steady solutions of the incompressible Navier–Stokes equation [Eq. (6) below with the magnetic field dropped] if an external body force

$$\mathbf{f} = -\Delta\mathbf{v}_{ABC} = \mathbf{v}_{ABC} \tag{5}$$

– in the following called ABC forcing – just compensating for viscous losses is applied. Here we report results of numerical studies of the complete system of the incompressible MHD equations with this kind of forcing as well as with a generalized ABC forcing with a variable degree of helicity. Comprehensive accounts of the corresponding studies may be found in Seehafer et al. (1996), Feudel et al. (1995, 1996), and Schmidtmann et al. (1998).

We use the incompressible MHD equations in the nondimensional form

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \Delta\mathbf{v} - \nabla p - \frac{1}{2}\nabla\mathbf{B}^2 + (\mathbf{B}\cdot\nabla)\mathbf{B} + \mathbf{f}, \tag{6}$$

$$\frac{\partial\mathbf{B}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{B} = P_m^{-1}\Delta\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{v}, \tag{7}$$

$$\nabla\cdot\mathbf{v} = 0,\ \ \nabla\cdot\mathbf{B} = 0, \tag{8}$$

where p is the thermal pressure and P_m the magnetic Prandtl number (the ratio between magnetic diffusivity and kinematic viscosity). Periodic boundary conditions are applied and the spatial means of $\mathbf{v}$ and $\mathbf{B}$, and consequently also of $\mathbf{f}$ are assumed to vanish. The ABC forcing, given by Eqs. (4) and (5), is used with with

$$A = B = C = R \tag{9}$$

where R is referred to as Reynolds number. For this forcing the MHD equations are equivariant with respect to a discrete symmetry group which is isomorphic to the octahedral group O (the rotation group of the cube).

Besides the pure ABC forcing also a generalized ABC forcing is applied, given by

$$\mathbf{f} = (1-\lambda)\mathbf{v}_{ABC} + \lambda\mathbf{v}^{-}_{ABC}, \tag{10}$$

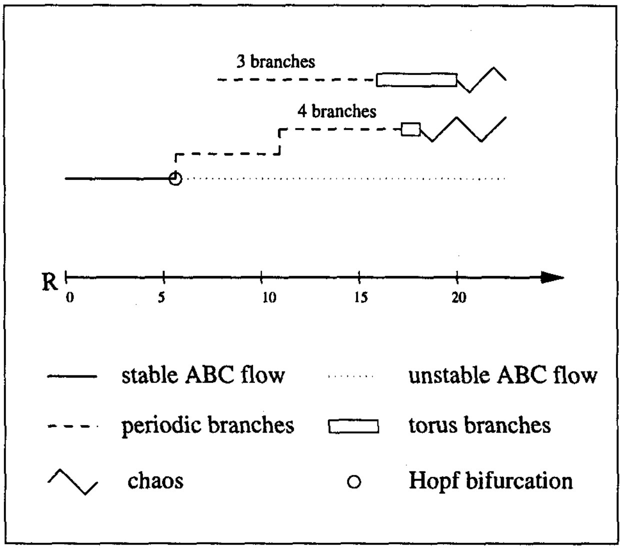

Figure 1. Schematic bifurcation diagram for pure ABC forcing.

where

$$\mathbf{v}^{-}_{ABC} = (A\cos z + C\sin y,\ B\cos x + A\sin z,\ C\cos y + B\sin x) \qquad (11)$$

and λ is a parameter varying between 0 and 0.5. $\mathbf{v}^{-}_{ABC}$ satisfies $\nabla \times \mathbf{v}^{-}_{ABC} = -\mathbf{v}^{-}_{ABC}$, and for $\lambda = 0.5$ its addition in the forcing term "kills" the helicity on average in the volume, while $\lambda = 0$ corresponds to the original ABC forcing.

We restrict ourselves to the case of $P_m = 1$ and R and λ are our bifurcation parameters.

An overview of the bifurcation structure for pure ABC forcing is depicted in Figure 1. For weak forcing (small R), there exists a stable stationary solution, namely the ABC flow [given by Eq. (4)] with vanishing magnetic field, and all system trajectories are attracted by this solution. If R is raised, the steady state loses stability in a Hopf bifurcation, leading to a periodic solution with a magnetic field as the only time-asymptotic state. The periodic magnetic solution is at first symmetric to the full group O, but for further raised R it bifurcates into four new periodic solutions, which can be be transformed into each other by certain elements of O. Besides that another periodic magnetic branch appears, consisting of three solutions which can be transformed into each other. Both branches undergo secondary Hopf bifurcations leading to quasiperiodic or torus solutions, which in turn eventually decay to chaotic states.

The volume-averaged magnetic helicity [cf. Eq. (1)] is negative, thus opposite in sign to the kinetic helicity, as also found by Galanti et al. (1992).

For the case of the generalized ABC forcing given by Eq. (10), the locations of primary and secondary bifurcations in the parameter plane are shown in Figure 2. For weak forcing (small R), there always exists a stable stationary,

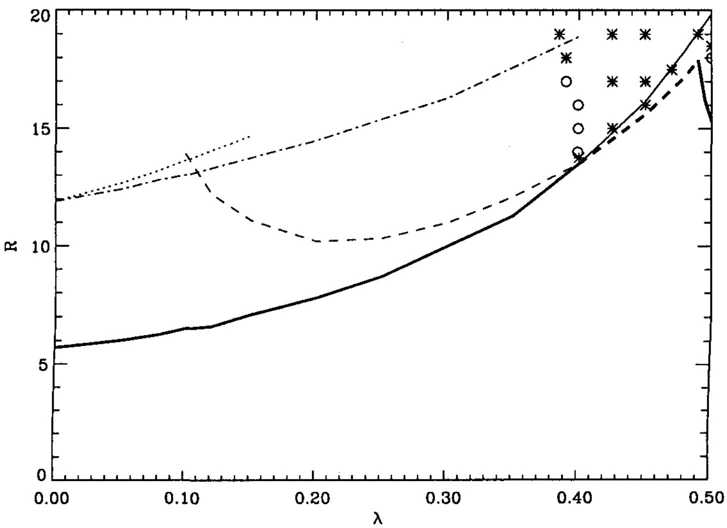

Figure 2. Locations of primary and secondary bifurcations of the original stationary solution in the λ-R plane. Solid line and dashed-dotted line: a single pair of complex conjugate eigenvalues crosses the imaginary axis; dashed line: two real eigenvalues pass through zero; dotted line: two pairs of complex conjugate eigenvalues cross the imaginary axis. Asterisks indicate points at which, by means of simulations, non-magnetic chaotic time-asymptotic states have been found, while circles correspond to magnetic periodic attractors.

nonmagnetic, globally attracting solution (which coincides with the the original ABC flow only in the special case of $\lambda = 0$). Keeping fixed λ and raising R, this steady-solution branch has been traced. Thick solid and dashed lines, respectively, indicate the primary bifurcation of the original steady state. For $\lambda < 0.4$ the steady state loses stability in a Hopf bifurcation, but at $\lambda = 0.4$ the type of the first bifurcation, as well as the character of the time-asymptotic states after this bifurcation, change. While for $\lambda < 0.4$ a magnetic periodic state is the (only) new attractor, for λ between 0.4 and 0.5 new non-magnetic states emerge. Only if the helicity exceeds a certain threshold value, a Hopf bifurcation leads to a magnetic periodic state (i.e., to a dynamo effect). For helicities below the threshold value the transition is more complex, but always the ensuing time-dependent states, including chaotic ones, are non-magnetic (which does not exclude, of course, a dynamo effect for higher Reynolds numbers).

3. Alpha Effect and the Generation of Magnetic Helicity

The mean value of the magnetic helicity can be written as the sum of two contributions resulting from the mean and fluctuating magnetic fields, respectively, namely

$$\langle H_{\rm M}\rangle = H_{\rm M}^{\rm MEAN} + H_{\rm M}^{\rm FLUC}, \tag{12}$$

with

$$H_{\rm M}^{\rm MEAN} = \langle \mathbf{A}\rangle \cdot \langle \mathbf{B}\rangle, \quad H_{\rm M}^{\rm FLUC} = \langle \mathbf{A}' \cdot \mathbf{B}'\rangle. \tag{13}$$

For the time evolutions of $H_{\rm M}^{\rm MEAN}$ and $H_{\rm M}^{\rm FLUC}$ one finds (Seehafer 1996)

$$\begin{aligned}\frac{\partial H_{\rm M}^{\rm MEAN}}{\partial t} = \quad & -2\eta\nabla\times\langle\mathbf{B}\rangle\cdot\langle\mathbf{B}\rangle + 2\mathcal{E}\cdot\langle\mathbf{B}\rangle \\ & + \left(\frac{\partial H_{\rm M}^{\rm MEAN}}{\partial t}\right)_{\rm transport}\end{aligned} \tag{14}$$

and

$$\begin{aligned}\frac{\partial H_{\rm M}^{\rm FLUC}}{\partial t} = \quad & -2\eta\langle\nabla\times\mathbf{B}'\cdot\mathbf{B}'\rangle - 2\mathcal{E}\cdot\langle\mathbf{B}\rangle \\ & + \left(\frac{\partial H_{\rm M}^{\rm FLUC}}{\partial t}\right)_{\rm transport}\end{aligned} \tag{15}$$

These equations show that the alpha effect (the terms $\mp 2\mathcal{E}\cdot\langle\mathbf{B}\rangle$ on the right-hand sides) corresponds to the simultaneous generation of magnetic helicities in the mean field and in the fluctuations, the generation rates being equal in magnitude and opposite in sign. The mean total magnetic helicity, which is an invariant of ideal magnetohydrodynamics, is not influenced by the alpha effect. This may equally be considered as a transfer of magnetic helicity between the fluctuating (or small-scale) and the mean (or large-scale) fields, mediated by the alpha effect, or as a helicity cascade (cf., Frisch et al. 1975, Pouquet et al. 1976, Stribling and Matthaeus 1990, 1991).

Consider now a situation in which the magnetic fluctuations are statistically stationary. Actually it is assumed throughout traditional turbulent-dynamo theory that the magnetic fluctuations have settled down to a statistically stationary state. If then, furthermore, the fluctuations are spatially homogeneous, Eq. (15) implies that the alpha-effect parameter α is connected to the mean current helicity of the fluctuations by Eq. (3).

Let us, next, examine under which conditions there is a turbulent dynamo effect, i.e., under which conditions the turbulent emf increases the energy in the mean magnetic field. For that purpose we assume $\langle\mathbf{v}\rangle = \mathbf{0}$, since we are not interested in the dynamo action of the mean flow. For the change of the mean-field magnetic energy density one then finds

$$\frac{\partial}{\partial t}\frac{\langle\mathbf{B}\rangle^2}{2} = -\eta(\nabla\times\langle\mathbf{B}\rangle)^2 + \mathcal{E}\cdot(\nabla\times\langle\mathbf{B}\rangle) + \nabla\cdot(\text{Poynting flux}), \tag{16}$$

which shows that the alpha effect contributes to the growth of the mean magnetic field if $\alpha(\nabla\times\langle\mathbf{B}\rangle\cdot\langle\mathbf{B}\rangle) > 0$ or, equivalently (see the definition of α in Eq. (3), $\mathcal{E}\cdot\langle\mathbf{B}\rangle(\nabla\times\langle\mathbf{B}\rangle\cdot\langle\mathbf{B}\rangle) > 0$. For $\mathcal{E}\cdot\langle\mathbf{B}\rangle(\nabla\times\langle\mathbf{B}\rangle\cdot\langle\mathbf{B}\rangle) < 0$ the alpha effect lowers the mean-field energy.

Consider again the case of statistically stationary and homogeneous fluctuations. The condition for a dynamo action of the alpha effect, $\mathcal{E}\cdot\langle\mathbf{B}\rangle(\nabla\times\langle\mathbf{B}\rangle\cdot$

$\langle\mathbf{B}\rangle) > 0$, then becomes $\eta\langle\nabla \times \mathbf{B}' \cdot \mathbf{B}'\rangle(\nabla \times \langle\mathbf{B}\rangle \cdot \langle\mathbf{B}\rangle) < 0$, i.e., as first noted by Keinigs and Gerwin (1986), the current helicities in the fluctuating and the mean magnetic fields must have opposite signs.

Assume now that the alpha effect really overcomes the dissipative term in Eq. (16), i.e., $\mathcal{E} \cdot (\nabla \times \langle\mathbf{B}\rangle) > \eta(\nabla \times \langle\mathbf{B}\rangle)^2$. By using Eq. (3) and the Schwarz inequality $(\nabla \times \langle\mathbf{B}\rangle)^2\langle\mathbf{B}\rangle^2 \geq (\nabla \times \langle\mathbf{B}\rangle \cdot \langle\mathbf{B}\rangle)^2$ one then finds as a necessary condition for the growth of $\langle\mathbf{B}\rangle^2$

$$-\langle\nabla \times \mathbf{B}' \cdot \mathbf{B}'\rangle(\nabla \times \langle\mathbf{B}\rangle \cdot \langle\mathbf{B}\rangle) > (\nabla \times \langle\mathbf{B}\rangle \cdot \langle\mathbf{B}\rangle)^2. \tag{17}$$

That is, the current helicity of the fluctuations must exceed that of the mean field by modulus.

Condition (17) has an implication for the evolution of the mean-field magnetic helicity: Since $|\eta\langle\nabla \times \mathbf{B}' \cdot \mathbf{B}'\rangle| = |\mathcal{E} \cdot \langle\mathbf{B}\rangle|$ due to the assumed stationarity and homogeneity of the fluctuations, $|\eta\nabla \times \langle\mathbf{B}\rangle \cdot \langle\mathbf{B}\rangle| < |\mathcal{E} \cdot \langle\mathbf{B}\rangle|$. Then, according to Eq. (14) helicity is accumulated in the mean magnetic field, with sign given by the sign of $\mathcal{E} \cdot \langle\mathbf{B}\rangle$, i.e., by the sign of α.

4. Discussion: The Helicity-Sign Puzzle

The two helicities generated by the alpha effect, that in the mean field and that in the fluctuations, have either to be dissipated in the generation region or to be transported out of this region. The latter may lead to the appearance of helicity in the atmosphere, e.g., in filaments (Martin 1998, these proceedings), and through solar eruptions even in interplanetary space. There has been accumulated strong evidence that the atmospheric and interplanetary magnetic helicity is predominantly negative in the northern and positive in the southern hemisphere (Seehafer 1990, Rust 1994, Rust and Kumar 1994, Pevtsov et al. 1995, Abramenko et al. 1996). It is not clear yet, however, whether the fields observed in the atmosphere, e.g., in active regions, can be interpreted as mean fields or fluctuations in the sense of mean-field theory.

Assume that the observed fields are either mainly mean fields or mainly fluctuations. The magnetic helicity accumulated in the mean field has the same *sign as the alpha-effect parameter α. So α should be negative in the northern* hemisphere if the observed fields are mean fields. Vice versa, α should be positive in the northern hemisphere if the atmospheric fields have to be interpreted as fluctuations. For a proper propagation of the dynamo waves (from the poles to the equator), a negative (positive) α requires a decrease (an increase) of the angular velocity of the solar rotation with depth in the convection zone. Helioseismological measurements (Christensen-Dalsgaard and Schou 1988) indicate near the equator a decrease with depth (the decrease occurs rather low in the convection zone). Consequently, $\alpha < 0$ in the northern hemisphere and the fields observed in active regions are mean fields. It will be interesting to carry out improved helicity measurements in the solar atmosphere as well as in the solar wind and to analyze them with respect to signatures of the two helicities.

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