In the preceding calculations, I have used the following well known process:—

If $p$ denote the number of dice, $n$ any particular number to be thrown, then $6^p$=the whole number of combinations; and the different ways in which $n$ can be thrown is the number of combinations in which $a+b+c+$, &c. $p$ terms can be made equal to $n$; the several numbers from 1 to 6 being successively substituted for $a$, $b$, $c$, &c. This will be the same as if we raise $(x+x^2+x^3+x^4+x^5+x^6)$ to the $p$th power, and determine the coefficient of $x^n$, which may readily be done as follows:—

$$(x+x^2+\ldots\ldots x^6)^p=x^p\left(\frac{1-x^6}{1-x}\right)^p=x^p(1-x^6)^p(1-x)^{-p}$$

giving to $p$ any of the values 2, 3, 4, &c., and expanding and performing the multiplication. Any coefficient in the resulting product, divided by $6^p$, will denote the probability of throwing the number which is the index of $x$, the term to which the coefficient belongs.

Your obedient Servant,

GEO. SCOTT, A.I.A.

*Fortescue House, Twickenham,*
*Dec.* 16*th*, 1853.

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## DETERMINATION OF SURPLUS.

*To the Editor of the Assurance Magazine.*

SIR,—To ascertain the sum which a Society may safely appropriate as a bonus being one of the most momentous problems that can fall within the scope of an actuary's duties, I may perhaps be permitted to offer a few observations on the subject.

I will suppose, then, that a mutual Society has been in existence five years, and that the amount of pure divisible surplus is sought, with a view to the declaration of a bonus. After payment of the preliminary expenses, or those attendant upon the formation of the Society, the cost of management, and the claims on account of deaths, the sum $s$ remains to credit of the Company. The present value of the future *gross* and *net* premiums on the existing policies $=$V and $v$ respectively, that of the policies themselves being $v'$. The working expenses hitherto average $e$ per annum; and $n$ policies on an equality have been issued yearly. Now $V-v'+s$ cannot be called actual surplus, since no allowance is made for future expenses, which must necessarily be incurred before the profits on the future premiums (of which $V-v$ is the present value) can be realized. To estimate this important deduction, we can but proceed upon the experience of the past; if therefore A be the *net* premiums receivable annually on the policies issued, $\frac{v}{A}$ will denote the average number of years these policies have to run, and the working expenses during such period $=\frac{ve}{A}$. Now we may reasonably assume, if nothing be known to the contrary, that in this time $\frac{nv}{A}$ new policies will be granted, so that the fair proportion of the sum $\frac{ve}{A}$ which should be borne by the $5n$ individuals already assured is

$$5n \times \frac{\frac{ve}{\overline{A}}}{n\left(5+\frac{v}{\overline{A}}\right)} = \frac{5ve}{5A+v};$$

thus reducing the surplus to $(V+s)-\left(v'+\frac{5ve}{5A+v}\right)$. This sum, however (which for brevity call $\beta$), being the present value of the entire profits which can ever be derived from the $5n$ policies, would, if it were all distributed forthwith, cut off the possibility of any further bonus being allotted to these members; but it is wished at present only to give such portion of this surplus as will leave a sufficiency for an *equal bonus at least*, to be divided every $m$ years in future. The number of prospective bonuses is $\frac{v}{mA}$; hence it follows, that the sum which may *now* be looked upon as divisible surplus is $\beta \div \left(1+\frac{v}{mA}\right)$. In deducing the expression $\frac{5ve}{5A+v}$, I have supposed the $5n$ persons to be all alive at the present time; if, however, the decrements should be sufficiently numerous to render such a step necessary, a correction might be applied.

Some of the preceding operations, it is true, are in a mathematical sense only approximative; but the discrepancies arising therefrom being of small moment, and all "on the safe side," a greater nicety of calculation would in actual practice be but labour thrown away. Any modifications which might be deemed prudent, such as assuming that the future expenses will increase progressively in a certain ratio, could easily be made, the foregoing being intended as a mere indication of the general method to be pursued.

I am, Sir,
Your obedient Servant,
SAMUEL YOUNGER.

*Engineers' Assurance Office*, 345, *Strand*,
18*th January*, 1854.

NOTE.—Our correspondent's object might perhaps be more easily obtained as follows:—Let the amount of premiums and interest received since the last division be denoted by S, the total expenses in the same interval by E, and the "loading" of the premiums per pound be $\phi$. Then the surplus fairly divisible in the case supposed will be nearly $\phi S - E$ (*see* vol. ii., page 334, of this *Journal*). Of course, the correctness of such a proceeding will depend on the accuracy with which the premiums for the risk have been assumed.—ED. *A. M.*

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## ASSURANCES ON ONE LIFE AGAINST ANOTHER, DURING THEIR JOINT DURATION, AND FOR $n$ YEARS LONGER.

*To the Editor of the Assurance Magazine.*

SIR,—In the second volume of the *Magazine*, page 95, Mr. Peter Hardy gives a formula for determining "the present value of a reversion