BARTER, MONEY, AND COMMERCIAL ARITHMETIC

BY

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I. INTRODUCTION

The manuscripts dealing with commercial practices in Europe from the thirteenth to early seventeenth centuries have been the subject of numerous studies by those interested in the history of mathematics and sociology of knowledge. Indeed, these manuscripts shed a great deal of light on how modern mathematics, as well as mechanistic naturalism of the Enlightenment, can be traced to commercial arithmetic of the medieval and Renaissance periods. As such, the recent interest in these early manuscripts is well understood. But there is another aspect of these writings that has received far less attention: the economic contents of these manuscripts and their implications for economic theory. In what follows, I will attempt to remedy this shortcoming by looking at the monetary lessons of these manuscripts concerning barter. It will be shown that these lessons defy two fundamental assumptions of monetary theory: (1) the origin of money lies in the difficulties of barter, and (2) exchange is characterized by equality. With regard to the first issue, it will be argued that rules of barter in the period under consideration show that barter not only is not difficult, but it is in fact preferred to monetary transactions. With regard to the second issue, it is contended that the medieval relation of exchange is characterized not by equality, but by inequality, not by honesty, but by fraud and deceit.

II. THE COMMERCIAL REVOLUTION AND THE *ABACI* TRADITION

The thirteenth century, as most economic historians would have it, is the "culminating age of medieval achievement" (Bridbury 1992, p. 154). This is a period characterized by "drastic change in the methods of doing business or in the organization of business enterprise" (de Roover 1953, p. 23). Among these adaptations is the change in the lifestyle of the merchants, from traveling to a sedentary way of life. This change is made possible by a number of factors, such as increased security, the development of maritime insurance, and the use of bills of exchange or letters of credit.[1] This sedentary way of life in turn resulted in

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[1] For an extensive and remarkably lucid discussion of this transition, see Warren Van Egmond (1976).

the emergence of special educational facilities to teach the children of the merchants the way of doing business. Given the unique position of the Italians in adopting the new business methods, these educational facilities are sometimes referred to as the "schools of abacus" or, more accurately, *"scuola"* or *"bottegha d'abaco"* (Van Egmond 1976, p. 65; Swetz 1987, p. 21). Their reckoning masters were, in turn, called the *"masteri d'abaco"* (Van Egmond 1976, p. 66; Swetz 1987, p. 21). The manuscripts written by these masters are referred to by Van Egmond (1976, p. 4) as "books of the abacus" or *"abaci."*

The term *"abaco"* in this context should not, of course, be confused with the medieval reckoning board. Instead, the expression stands for a tradition in mathematical computation which was borrowed by the Italian merchants and scholars from the works of the Moslem mathematicians, such as Muhammad Ibn Musa Al-Khwarizmi's *Algebra* (1831).[2] At the apex of this tradition stands Leonardo of Pisa or Fibonacci and his *Liber abaci* (1202). By its particular format, *Liber abaci* established a format for teaching mathematics, a format which, not only in Italy but in many parts of Europe, lasted well into the sixteenth century.[3] In a broad outline, the format consists of introducing the reader to Arabic numerals, followed by teaching the four basic arithmetical operations: addition, subtraction, multiplication, and division. In this exposition, how to deal with fractions and, in particular, the "golden rule" or the "rule of three"—where a missing fourth number in two equal ratios is found—is often explained.[4] Having established the basis of arithmetical operation, the discussion turns to business problems, such as those dealing with barter, pricing, profit calculations, partnerships, and mixture of coins. Thereafter, more esoteric and academic concepts dealing with algebra and geometry may be analyzed. Between all of these, many problem situations are presented. Some of these problems are real, dealing with everyday living scenarios, while some are fictitious, simply designed for pedagogical purposes.[5] Problems dealing with business practices fall within the first group.

What distinguishes the manuscripts written by *"masteri d'abaco"* from those written by Fibonacci, and a host of academic mathematicians who followed him, is that the former are geared more toward the practical side of affairs, particularly the business side.[6] These types of manuscripts, as Warren Van Egmond

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[2] On the possible date of the publication of Al-Khwarizmi's *Algebra* and the structure of the work, see G. J. Toomer (1973). For the contribution of medieval Islamic civilization to the development of mathematics see J.L. Berggren (1986).

[3] See Kurt Vogel (1976) for an account of the structure of Fibonacci's *Liber abaci*. Swetz points out that *Liber abaci* should be translated as "Book of Computation" rather than "Book of Abacus" (Swetz 1987, pp. 325–26).

[4] For a recent discussion of the importance of the "rule of three" in Renaissance trade, see Judy L. Klein (1997, pp. 25–34). On the use of this rule in art, as well as commerce, see Michael Baxandall (1988, pp. 94–102).

[5] For an extensive discussion and classification of problem situations in the abacus manuscripts see Van Egmond (1976, pp. 164–228).

[6] "Sons of German businessmen," Swetz writes, "flocked to Venice to study *Welsche Praktik*, the foreign practices of business, commercial arithmetic and currency exchange. After acquiring these skills, they returned home with a new Italian vocabulary which included terms as: *disagio*, discount; *credito*, credit; *valuta*, value; *netto*, at net price, etc" (Swetz 1987, p. 10).

points out, may also deal with more esoteric and theoretical subjects, such as geometry and algebra, insofar as the immediate usefulness of these subjects for solving everyday problems is concerned (Van Egmond 1976, pp. 12–18).[7] However, the primary intention of these writings is to teach mathematics for business practices. For example, the text of the *Treviso Arithmetic* of 1478, which is the first printed book of its kind, starts by saying:

> Here beginneth a Practica, very helpful to all who have to do with that commercial art commonly known as the abacus.
>
> I have often been asked by certain youths in whom I have much interest, and who look forward to mercantile pursuits, to put into writing the fundamental principles of arithmetic, commonly called the abacus (Swetz 1987, p. 40).[8]

The tradition of writing and teaching books of abacus, however, is not confined to Italy. Elsewhere in Europe, but somewhat later, the same type of manuscripts appeared, usually having in their titles the word "arithmetic." For example, Natalie Zemon Davis (1960, p. 19) points out that the "French commercial arithmetics were the major and almost the only printed material for the technical instruction of French merchant and banker in the XVIth century." The same holds for other countries in Europe, particularly England, where books on commercial arithmetic appear well into the sixteenth century. For example, Humfrey Baker published his *Well-Spring of Science* in 1568. The subtitle of the work, *Teaching the Perfect Worke and Practise of Arithmetick, both in Numbers and Fractions*, clearly sums up the nature of the work. The table of contents of the book, centuries after the Fibonacci's *Liber abaci*, still shows the usual format of the books of abacus, starting with an introduction to Arabic numerals and ending with the issues immediately relevant to the practices of the merchants.

In the following section, I will look at some of the manuscripts in the *abaci* tradition, not from the perspective of history of mathematics or the sociology of knowledge but from the perspective of their economic contents. In particular, I will examine the content of these manuscripts to see what they have to offer on the subject of exchange.

III. THE "RULES OF BARTER"

The importance of barter in the medieval and Renaissance periods is well known. Even though the monetary economy became more and more pervasive as Europe passed through the Middle Ages, "[p]ure barter did not immediately disappear from the local economy even in the sixteenth and seventeenth century" (Van Der Wee 1977, p. 290). Indeed, the "use of credit facilities in the money sector even enhanced the opportunities for barter transactions in so far as it encouraged barter on an extended time basis" (Van Der Wee 1977, p. 290). This

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[7] For a detailed discussion of distinctive features of different kinds of mathematical manuscripts written between the years 1300–1500 and, in particular, the distinction between *"abaci"* and "algorism," see Van Egmond (1976, Chapters I and VI).

[8] Swetz, therefore, refers to these types of manuscripts as "practica" (Swetz 1987, p. 29).

persistent and pervasive use of the barter mechanism is clearly evident in the books of abacus.

As mentioned earlier, after some preliminary teachings of simple arithmetic, the authors of *abaci* manuscripts usually turn their attention to examples or problem situations that are real and related to business activities. In particular, most of these manuscripts have a section on the "Rules of Barter," a fact that indicates that for the medieval and Renaissance merchants barter is "one of the major forms of commerce" (Van Egmond 1976, p. 185).

The extent to which the problems dealing with barter appear in the abacus manuscripts varies. Some manuscripts provide numerous examples of barter and others, only a few. For example, the *Treviso Arithmetic* provides only three examples of barter situations. The *Well-Spring of Science*, on the other hand, devotes an entire chapter, "Of the Rules of Barter: that is to say, to change Ware for Ware," to the issue, and presents twelve problem situations (Baker 1568, p. 248). Let me point out that throughout this essay I use the term "barter" in the sense defined by Baker, namely, the exchange of ware for ware.[9]

The complexity of these problem situations in the abacus manuscripts also varies. Indeed, there are usually different types of problem situations with increasing complexity. The first type is a very simple case of barter, where one good is simply exchanged for another. Van Egmond (1976, p. 186) notes that in some of the abacus manuscripts these type of problems are called "*baratto semplice*" or "simple barter." An example of this kind can be found in the *Treviso Arithmetic*:

> Two merchants wish to barter. The one has cloth at 5 lire a yard, and the other has wool at 18 lire a hundredweight. How much cloth should the first have for 4564 hundredweights of wool? (Swetz 1987, p. 151)

This is, of course, an example of the "Rule of Three" or the "Golden Rule" (Baker 1568, p. 70). Note that even though goods are directly exchanged for one another in the above example, the unit of account is money. This, as we shall see below, holds for all other examples concerned with barter.

The problems dealing with simple barter themselves can become more extensive. For example, the third example given in *Treviso Arithmetic* consists of one merchant exchanging "1 pexo of balsam worth 150 ducats" against another merchant's "wax of 5 ducats per hundredweight, sugar at 6 ducats a hundredweight, and ginger at 8 ducats a hundredweight" (Swetz 187, pp. 152–53).

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[9] There is little agreement among economists with regard to the meaning of "barter." For example, under the entry "barter" in *The New Palgrave Dictionary of Economics*, Keith Hart defines barter as "a simultaneous exchange of commodities, whether goods or labor services, with bargaining and without using money" (Hart 1998, p. 196). He further adds that barter "is not abolished by money and indeed sometimes transforms money itself into an item of barter," a view which is supported in this essay (p. 198). However, the next entry, "barter and exchange," written by F.Y. Edgeworth for the original *Palgrave*, states: "Barter, as distinct from exchange, is defined by the absence of money both as a medium of exchange and measure of value" (Edgeworth 1998, p. 198). This definition not only is not identical to that of Hart, but it seems to deny the simultaneous existence of barter and money.

There are more complex cases of barter involving the exchange of two goods, where the barter value and price are different.[10] Consider for example the following:

> There are two merchants of whom the one has cloth worth 22 soldi a yard, but holds it in barter at 27 soldi. The other has wool which is worth in the country 19 lire per hundredweight. Required is to know how much he must ask per hundredweight in barter so that he may not be *cheated* (Swetz 1987, p. 152, emphasis added).

Similar examples can be found in the *Well-Spring of Science*:

> Two Merchants will change their merchandize, the one with the other. The one of them hath Cloth of 7 s. 1 d. the yard, to sell for ready money, but in barter he will sell it for 8 s. 4 d. The other hath Cinnamon of 4 s. 7 d. the pound, to sell for ready money; I demand how he shall sell for it in barter that he be no *loser*? (Baker 1568, p. 248, emphasis added).

Note that in the above examples, the price of cloth is less than its barter value. This difference is not accidental. As Van Egmond notes, the "majority of barter problems" in the abacus manuscripts are of this nature (Van Egmond 1976, p. 186).

But why is the barter value higher than the price? Two different explanations have been put forward. In reference to the first example cited above, Swetz argues that the

> reasons for preferring barter to direct monetary payment were twofold: there was a shortage of currency (not until the discovery of gold in the New World and its import into Europe did a large scale money economy function), and the existence of trade fairs ... [where] in the absence of trusted "international" currency, it was easier and safer to barter goods (Swetz 1987, p. 239).

But this is not a sound explanation. Even though, as we shall see below, it is true that barter is preferred to monetary payment, the higher barter value per se does not indicate this preference. Moreover, the money stock in Europe increased in the late thirteenth and early fourteenth centuries before it started to decrease.[11]

Van Egmond gives a more meaningful explanation. He argues that in the *abaci* tradition the "cash price" was "the true price of the good" and the "barter price" was the "price at which it was being offered" (Van Egmond 1976, p. 186). This argument is more in accord with the concepts of getting "cheated" or becoming a "loser," which appear in some of the above passages quoted from the *abaci* manuscripts. The argument is also supported by the second example of barter offered in the *Well-Spring of Science*:

> Two Merchants will barter their merchandize the one with the other: The one of them hath Chamblets, of 2ℓ. 18 s. 4 d. the piece, to sell for ready money, and in barter he will sell the piece for 4ℓ. 3 s. 4 d. The other hath fine Caps

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[10] According to Van Egmond, these still fall in the category of "simple barter" (Van Egmond 1976, p. 187).

[11] On the estimate of European money stock in the late Middle Ages, see Peter Spufford (1988, Table 2, p. 420).

of 35 s. 10 d. the dozen, to sell in barter. I demand what the dozen of Caps were worth in ready money? Ans. Say, if 4ℓ. 3 s. 4 d. which is the *over-price* of the piece of chamblet, become of 2ℓ. 8 s. 4 d. which was the *just price* of the same, of what shall come 35 s. 10 d. which is the overprice of the dozen caps? (Baker 1568, pp. 248–49, emphasis added).

The above distinction between the "just price" and the "over price" indicates the correctness of Van Egmond's distinction between the "true price" and the "barter price." Baker's "just price" appears to be the actual price of the commodity, that is, the actual money value. His "over price" or barter value, however, includes an increment above the actual price. Many of the subsequent examples of rules of barter provided by Baker involve this same distinction.

It should be noted that the actual price itself includes not only the cost per unit, but a desired profit per unit. This is evident from the fact that prior to the discussion of barter, in a chapter on "Of losses and gains in the trade of Merchandize," Baker already discusses how to calculate the price, which includes a profit margin (Baker 1568, p. 193). This is also evident from Davis's investigation of the sixteenth century French writings on commercial arithmetic. In reference to these writings Davis states:

> The authors of these arithmetics may have to some extent worried about the sin of avarice and whether it inevitably expressed itself in commerce and finance. When they came to write texts, however, whatever moral concerns they had were generally put aside, and they became first and foremost conveyors of technique. Thus the setting of prices was discussed solely in terms of costs and desired profits, with none of the familiar references of the theologian and moralist to just price[12] (Davis 1960, p. 21).

Davis then quotes the sixteenth century writer La Roche on how to calculate the cost per unit of a good and "sell it to one's profit at so much gain per livre or at the rate of so much per cent" (Davis 1960, p. 21). Thus, in the final analysis, the "over price" or the barter value represents a profit margin more than the usual markup.

According to Van Egmond, the difference between "cash price" and "barter price" was a "stratagem apparently designed to fix the merchant's profit margin" (Van Egmond 1976, p. 186). "The barter problems given in the *abaci* generally revolve around setting the barter price, either in order to achieve a desired profit or to offset the other merchant's barter price" (Van Egmond 1976, p. 186). The example given by Van Egmond in the second case is similar to the last two examples given above. But in the case of fixing the profit rate, Van Egmond provides the following example:

> Two men want to barter wool and cloth. A canna of cloth is worth 6 lire and in barter it is valued at 8 lire. The hundredweight of wool is worth 25 lire and in barter it is offered at such a price that the man with the cloth finds he has earned 10 per cent. At what price was the hundredweight of wool offered in barter? (Van Egmond 1976, p. 187).

It should be noted, once again, that the profit margin that Van Egmond is

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[12] As we saw, Baker does refer to "just price." But his phrase has no moral connotation.

referring to must not be the usual markup that is included in the price, but something over and above this markup, which can only be obtained in barter.

The *Well-Spring of Science* also contains problems that involve calculating profit rate in barter. For example, the third example offers the following problem and its solution:

> Two merchants will change their merchandize the one with the other: the one of them has Fustians of 18 s. 4 d. the piece to sell for ready money, and in barter he will sell the piece for 26 s. 8 d. The other hath Tapestry of 15 d. the ell to sell for ready money, and in barter he will sell it for 20 d. the ell. I demand which of them gaineth, and how much upon the 100 li. of money? Answ. Say if 18 s. 1/3 (which is the just price of Fustian) be sold in barter for 26 s. 2/3: for how much shall 1 shil. 1/4, (which is the just price of the ell of Tapestry) be sold in barter? multiply and divide, and you shall find 21 d. 9/11. And he doth over-sell it but for 20 d. So that of 21 9/11 he maketh but 20 d. And therefore say by the Rule of 3, if the second Merchant, of 21 9/11 do make but 20/1, how much shall he lose in the 100/1? multiply and divide, and you shall find 91 2/3 the which being abated from 100, there will remain 8 1/3. And after the rate of 8 1/3, doth the second Merchant lose in 100. And consequently, the first Merchant of 20 d. maketh 21 9/11, and therefore say againe by the Rule of Three, if the first Merchant make 21 9/11, how much shall he gain upon 100/1? multiply and divide, and you shall find 109ℓ.1/11. And thus the first Merchant gaineth after the rate of 9ℓ. 1/11 upon 100ℓ. of money (Baker 1568, pp. 249–50).

Baker's problem and its solution are somewhat mechanically stated and difficult to comprehend. The difficulty is also compounded by the absence of the concept of decimal. They can, however, be restated this way: The two merchants merely offset each other's "over price" if the second merchant sets the "over price" of tapestry at 21.8181 d. per ell. This would make the ratio of "just price" of fustian to that of tapestry equal to 14.$\overline{6}$—that is, 220 d./15 d.—a ratio which is equal to the ratio of "over-prices" of the two goods, respectively. Lacking full knowledge, however, the second merchant sets his "over price" at 20 d. Thus, according to Baker, the second merchant loses 1.8181 d. per his correct "over price" of 21.8181 d., a loss which is equivalent to 8.$\overline{3}$ per cent. As far as the first merchant is concerned, he can now sell the tapestry for an "over price" of 21.8181 d. This, according to Baker, is a gain of 1.8181 d. per 20 d., or 9.0909 per cent.

The view that barter was a way of making extra profit through deceit is also discussed in other sources. For example, in reference to French commercial practices, Davis writes: "Deception was a frequently alleged temptation of the business life" and the process of bartering "involved a certain amount of evident deception" (1960, p. 22). Davis's example is from an early sixteenth century manuscript: "Two merchants want to barter their merchandise and deceive (tromper) each other, one in pepper, the other in cloth. He who has the pepper wants to sell at 25 francs the hundred in barter what is worth only 20 francs in current cash (Davis 1960, p. 22). Davis then states: "Although a work of about the same time claimed the goal of the Rule of Barter was 'to keep oneself from being deceived or tricked and from deceiving or tricking others,' later in the

century we see a non-committal description: 'Bartering is nothing but giving one piece of merchandise for another in hope of coming out in a better position' " (Davis 1960, p. 22). Actually, this definition of barter appears in earlier manuscripts as well. Kurt Vogel (1978, p. 68), for example, quotes the following from Luca Pacioli's 1494 *Summa de Arithmetica*: "Barter is nothing but giving a good for another in order to get more." This concept of barter is then generalized by Vogel:" "If one of the two parties (A) sets the cash value of his good b_1 and the barter value s_1, and the other party (B) sets the cash value b_2 and the barter value s_2, then the exchange is only right and nobody is cheated, if the following is true: $b_1: s_1 = b_2: s_2$." (Vogel 1978, p. 68). Richard W. Hadden also supports the above view and, in reference to Vogel's analysis, states that in exchange "proportion becomes the language of gain" (Hadden 1994, p. 90).

It should be emphasized that profit making in this fashion depends, in the final analysis, on the level of arithmetical knowledge of the two parties. Complete knowledge on the part of one merchant and complete lack of it on the part of the other results in maximum gain. Full knowledge on the part of the two merchants, however, results in a fair exchange, where the barter values are proportional to prices. One can add to this the conjecture that complete lack of knowledge on the part of the two merchants may also result in a fair exchange. We will return to this issue in the next section.

There is, however, an even more complex case of barter, involving payments both in cash and in kind. According to Van Egmond, these types of problems are known as *"baratto composto"* or "compound barter" (1976, p. 187). Baker (1568) explains the problem this way:

> When a **merchant overselleth his** merchandize, and he will have also some **part** of his **over-price in ready money**: as the 1/2 the 1/3, or the 1/4 etc. He **must** subtract the same part of money from the just price, and also from the over-price of his merchandize: and the two numbers that remain after the subtraction is made, shall be the two numbers in the Rule of Three: the just price of the second merchant shall be the third number: to know how much he shall over-sell the part of his merchandize (pp. 255–56).

Baker's first example of compound barter is this:

> Two merchants will change their merchandize the one with the other, the one of them has Wooll at 5ℓ. the 100 pounds **weight** to sell for ready money, and in barter he will sell it for 6 li. And yet he **will** have the 1/3, in ready money. The other hath cloth of 13 s. 4 d. the yard to sell for ready money. I would know how he shall sell the same in barter? Answ. Take the 1/3 of 6 li which is the over-price of 100 of Wooll, and that is 2 li. the which you must abate from 5 li. Which is the just price of the C of Wooll, and also abate it from 6ℓ. which is the over-price, and there shall rest 3ℓ *l*. and 4 l. for the two first numbers in the Rule of Three; then take 13 s. 4 d. which is the just price of a yard of cloth, for the third number: Then multiply and divide, and you shall find 17 s. 9 d. 1/3 for so much shall the second sell his cloth in barter (p. 256).

The problem, as posed by Baker, is not clear. The solution offered is also unclear, since it is stated mechanically and it lacks explanation. This is

apparently not a unique case. After discussing the case of complex barter, Vogel (1978, p. 69) writes:

> In another, especially frequent group of problems, it is stated that A demands from B that certain part of his good, 1/n, be paid in cash. Yet without any explanation, the following formula is used:
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> $$(b_1 \quad s_1/n): (s_1 \quad s_1/n) = b_2: s_2.$$

Vogel himself does not fully explain how the formula is derived. In the next section, I will explain and formalize the solution, but for the time being this much can be said.

Consider the above problem posed by Baker. The first merchant sets the price of wool at 1200 d.—that is, 5 li. x 240 d./li.—per hundredweight and the second merchant sets the price of cloth at 160 d. This means effectively that 160/1200 hundredweights of wool can barter for 1 yard of cloth. When it comes to complex barter, however, the first merchant asks the second 1440 d.—that is, 6 li. x 240 d./li.—worth of cloth per hundredweight. This means that as far as the wool merchant is concerned): 160/1440 hundredweights of wool barters for 1 yard of cloth. In the case of compound barter, however, the wool merchant would like to barter 160/1440 hundredweights of wool for 2/3 of a yard of cloth and 1/3 of a yard worth of cloth in cash. But then how much cash is this? Since, from the perspective of the wool merchant, 160 d. purchases a yard of cloth, then 1/3 of a yard worth of cloth is equivalent to 160/3 d. Thus the wool merchant would deduct 160/3 d. worth of wool from his own payment of wool in exchange for 2/3 of a yard of cloth. But then again, how much wool is 160/3 d. worth of wool? Since, 1200 d. purchases 1 hundredweight of wool, then 160/3 d. worth of wool is equivalent to 160/3600 hundredweight of wool. Hence, according to the wool merchant, $0.0\bar{6}$ hundredweight of wool—that is, 160/1440 hundredweight of wool minus 160/3600 hundredweight of wool—barters for 2/3 of a yard of cloth. Or, alternatively, $0.0\bar{9}$ hundredweight of wool should barter for 1 yard of cloth. But since each yard of cloth sells for 160 d., from the perspective of the wool merchant, a hundredweight of wool should sell for 160 d./$0.0\bar{9}$ or $6.\bar{6}$ li. per hundredweight of wool. The cloth merchant would then have to match this price if he is to avoid being cheated. Using the rule of three, he should set his price at $213.\bar{3}$ d.—that is, (160 d. $6.\bar{6}$ li.)/5 li.—per yard or $17.\bar{7}$ s. per yard, which is the price arrived at by Baker. This mode of calculation, as we shall see soon, will lead to the same numerical solution proposed by Baker and the algebraic solution offered by Vogel.

Before leaving this section, it should be pointed out that more complex cases of barter, involving more deceit and profit-making opportunities, were also possible. For example, as Vogel points out, in some cases the exchange of goods involves future payments. In these problems, which Vogel, following Geronimo Cardano (1539), calls "*Transmutatio cum expectatione temporis*," the two merchants set their prices and barter values as before; however, one merchant wants to deliver his goods in the future and the second merchant has to decide when to deliver his own (Vogel 1978, p. 71). Such cases would obviously involve calculation of interest payments. The complexity of the problem is

increased when one side also asks for a fraction of goods to be delivered in cash (Vogel 1978, p. 72).

IV. A FORMAL PRESENTATION OF THE "RULES OF BARTER"

As we saw in the previous section, there are at least three modes of exchanging "ware for ware," each more complex than the other. A better understanding of these exchanges can be achieved by formalizing them in the following manner.

Consider two merchants wanting to exchange wool and cloth. Let us refer to wool and cloth by W and C, their respective units by w and c, and their respective merchants by M_W and M_C. Also, let D represent the commodity money, whose unit is d. Moreover, let p_W^D and p_C^D stand respectively for price per unit of wool and cloth expressed in units of the commodity money:

$$p_W^D = x \ d/w,$$

$$p_C^D = y \ d/c,$$

where x and y are numbers expressing the magnitudes of the prices of the two goods.

In the case of simple barter, the barter value of wool in terms of cloth and barter value of cloth in terms of wool can be expressed as:

$$b^1 \ _W^C = p_W^D/p_C^D = (x/y) \ c/w$$

$$b^1 \ _C^W = p_C^D/p_W^D = (y/x) \ w/c.$$

This is, of course, the simple case of barter. If we let **B** stand for the relation "barters for," then effectively:

$$y \ w \ \boldsymbol{B} \ x \ c.$$

Let us now proceed to the case of complex barter. In this case, as we saw earlier, the two merchants continue to set their prices as before. However, when it comes to barter, cheating can occur, and there are different possible scenarios: (1) M_W will cheat, while M_C will not cheat at all. In this case, M_W will set his barter value in terms of commodity money as:

$$b^2 \ _W^D = u \ d/w,$$

where $u > x$. This same barter value expressed in cloth is:

$$b^2 \ _W^C = (u/y) \ c/w.$$

M_C, however, who does not cheat, or is perfectly ignorant, continues to set his barter value equal to his price, which means effectively

$$b^2 \ _C^W = (y/x) \ w/c.$$

In the final analysis exchange takes place in such a way that

$$y \ w \ \boldsymbol{B} \ u \ c,$$

and M_C is cheated.

(2) M_C will cheat, while M_W will not cheat at all. In this case, M_C will set his barter value in terms of commodity money as:

$$b^2 \, _c{}^D = v \; d/c$$

where $v > y$. The barter value expressed in wool is:

$$b^2 \, _c{}^W = (v/x) \; w/c.$$

M_W, however, who does not cheat at all, or is perfectly ignorant, continues to set:

$$b^2 \, _w{}^C = (x/y) \; c/w.$$

In the final analysis

$$v \; w \; \boldsymbol{B} \; x \; c,$$

and M_W is cheated.

(3) Both M_W and M_C engage in deception, or both are perfectly informed in barter. Thus:

$$b^2 \, _w{}^C = (u/y) \; c/w,$$

$$b^2 \, _c{}^W = (v/x) \; w/c.$$

In the final analysis,

$$y \; w \; \boldsymbol{B} \; x \; c$$

and no one is cheated.

(4) Neither M_W nor M_C engages in cheating. This, of course, reduces to the case of "simple barter":

$$b^2 \, _w{}^C = (x/y) \; c/w,$$

$$b^2 \, _c{}^W = (y/x) \; w/c.$$

And, once again,

$$y \; w \; \boldsymbol{B} \; x \; c.$$

In sum, complex barter resembles a game theoretic matrix of payoffs with two contenders and two possible strategies.

The above analysis, of course, shows only four extreme cases. But, as we saw in one of Baker's examples, in the real world there are many more possibilities. How the trade is settled ultimately would depend on the magnitude of cheating, and this itself would depend on each side's knowledge of arithmetic.

In the case of compound barter, the magnitude of cheating intensifies. Once again, there are different scenarios, but let us just mention two: (1) M_W cheats more intensely, and M_C does not cheat at all. In this case, M_W, having first set a complex barter value, will ask M_C to pay a fraction of his good, $1/n$, in cash. The question now becomes, how much is that fraction worth to M_W? The answer involves the following set of arguments.

In the case of complex barter, M_W expected:

$$(y/u) \; w \; \boldsymbol{B} \; 1c.$$

With his new condition he expects:

$$(y/u) \; w \; \boldsymbol{B} \; [(n-1)c/n - (1/n) \; c \text{ in terms of D}]$$

But, from M_W's perspective, "$(1/n)$ c in terms of D" is the same as $(y/n) \, d$, since

$$y \; d \; \boldsymbol{P} \; c,$$

where P stands for the relation "purchases." Thus we have:

$$(y/u) \text{ w } B \text{ } [(n-1)c/n - (y/n) \text{ d}].$$

But since from the perspective of M_W,

$$(y/n) \text{ d } P \text{ } (y/nx) \text{ w,}$$

we can rewrite the above relation as:

$$[(y/u) - (y/nx)] \text{ w } B \text{ } (n-1)c/n$$

or simply

$$[n/(n-1)] \text{ } [(y/u) - (y/nx)] \text{ w } B \text{ c.}$$

Given the above barter relation, M_W sets:

$$b^3 {}_w{}^C = c/[n/(n-1)] \text{ } [(y/u) - (y/nx)]w.$$

Expressed in money commodity, this barter value would be:

$$b^3 {}_w{}^D = y \text{ d}/[n/(n-1)] \text{ } [(y/u) - y/xn)]w = d/[n/(n-1)] \text{ } [(1/u - (1/nx)]w.$$

If M_C is completely ignorant, he would set his barter value expressed in the commodity money as in the case of simple barter:

$$b^3 {}_c{}^D = y \text{ d }/c.$$

(2) M_C cheats more intensely, and M_W does not cheat at all. In this case M_C asks $1/n$ of the M_W's wool in cash. Following the logic of the previous case, M_C sets:

$$b^3 {}_c{}^D = x \text{ d}/[n/(n-1)][(x/v) - (x/ny)] \text{ c} = d/[n/(n-1)][(1/v) - (1/ny)] \text{ c.}[13]$$

If M_W is completely ignorant, he would set his barter value expressed in commodity money as in the case of simple barter:

$$b^3 {}_w{}^D = x \text{ d }/w.$$

Exchange is "fair" if, when both sides cheat, the ratio of barter values is the same as the ratio of prices, that is:

$$b^3 {}_c{}^D/b^3 {}_w{}^D = y \text{ w}/x \text{ c}$$

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[13] The logical steps are as follows:

$$(x/v) \text{ c } B \text{ } [(n-1)w/n - (1/n) \text{ w in terms of D}].$$

But, from M_C's perspective, "$(1/n)$ w in terms of D" is the same as (x/n) d, since

$$x \text{ d } P \text{ w,}$$

where P stands for the relation "purchases." Thus we have:

$$(x/v) \text{ c } B \text{ } [(n-1)w/n - (x/n) \text{ d}].$$

But from the perspective of M_C,

$$(x/n) \text{ d } P \text{ } (x/yn) \text{ w.}$$

Therefore, we get:

$$[(x/v) - (x/ny)] \text{ c } B \text{ } (n-1)w/n,$$

or

$$[n/(n-1)][(x/v) - (x/ny)] \text{ c } B \text{ w.}$$

Given the above, M_C sets:

$$b^3 {}_c{}^W = w/[n/(n-1)] \text{ } [(x/v) - (x/ny)] \text{ c.}$$

or

$$[(1/u) - (1/nx)] : [(1/v) - (1/ny)] = y/x.$$

Multiplying both sides by x, we have:

$$[(x/u) - (1/n)] : [(1/v) - (1/ny)] = y$$

Dividing both sides of the equation by $b^3 \, c^D$, we get:

$$(x - u/n) : (u - u/n) = y/b^3 \, c^D.$$

This is, of course, the same result as the unexplained solution proposed by Baker and Vogel's formulation of it.[14]

What are the implications of all of these for economic theory? Before answering this question let us briefly review the prevailing view on barter and monetary relation in economics, as well as some criticism of this view.

V. THE CONCEPT OF BARTER IN ECONOMIC THEORY

Modern monetary theories concerning properties of exchange relation and origin of money have not changed much since at least the time of Aristotle. "In the first community," which is the "family," writes Aristotle (1921, 1257a19–20), there is no exchange. Later, "when the family divided into parts," primitive exchange begins, "a kind of barter which is still practiced among barbarous nations who exchange with one another the necessities of life and nothing more" (1257a23–26). Eventually, a "more complex form of exchange grew" between nations (1257a23). But since "various necessities of life are not easily carried about ... men agreed to employ in their dealings with each other something which was intrinsically useful and easily applicable to the purpose of life, for example, iron, silver, and the like" (1257a35–39). In the "process of time they put stamp upon it" (1257a40). But beside facilitating exchange, money fulfills another function. It makes "different and unequal" goods, and even their owners, "somehow comparable":

> It is for this end that money has been introduced, and in a sense an intermediate; for it measures all things, and therefore the excess and defects— how many shoes are equal to a house or a given amount of food. The number of shoes exchanged for a house [or for a given amount of food] must therefore correspond to the ratio of builder to shoemaker. (Aristotle 1915, 1133a17–25)

Of course, money itself "is not always worth the same; yet it tends to be steadier," and this "is why all goods must have price set on them; for then there will always be exchange, and if so association of man with man" (1133b10–15). "Money, then, acting as a measure, makes goods commensurable and equates them; for neither would there have been association if there were not exchange, nor exchange if there were not equality, nor equality if there were not commensurability" (1133b15–20).

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[14] Vogel's formula "$(b_1 - s_1/n): (s_1 - s_1/n) = b_2: s_2$ " is notationally confusing, since "s_2," which here is used in the sense of compound barter value of merchant B, was used previously by him as the simple barter value.

Aristotle's monetary theories are obviously no more than storytelling. The "first community" and non-existence of exchange among them, a primitive exchange developing among "barbarous nations," the difficulty associated with barter, the agreement among men to solve this difficulty, and thus the development of money are mere fictions. But Aristotle's storytelling is also combined with a peculiar concept of the relation of exchange. Setting aside the notoriously ambiguous and controversial argument concerning the "ratio of builder to shoemaker," exchange is viewed as an equation.[15] The reason for this is Aristotle's assumption that in barter equal magnitudes of a uniform substance are exchanged and money merely measures this uniform substance. Exactly what this uniform substance is has itself been subject of numerous controversies. Aristotle himself states: "All goods must be therefore measured by some one thing, as we said before. Now this unit is in truth demand, which holds all things together (for if men did not need them equally, there would be either no exchange or not the same exchange); but money has become by convention a sort of representative of demand" (Aristotle 1915, 1133a26–30).[16]

Many readers of Aristotle, however, including a whole array of scholastic writers, classical political economists, and even some modern economists, such as Joseph A. Schumpeter (1954, pp. 60–61), interpreted the uniform substance to be labor. Whatever the nature of the substance, barter or monetary relation is viewed by Aristotle as a mathematical relation in which equal amounts of this substance are exchanged, and price becomes merely the monetary expression of this substance.

Aristotle's views concerning the difficulties of barter, the creation of money and the equation of exchange have left a long-lasting legacy in economic theory. In the same period when commercial revolution and the *abaci* tradition began, the scholastic school followed the monetary conjectures of Aristotle and put forward the concept of just price.[17] In the next few centuries, the Aristotelian views became ingrained in monetary theories, particularly in the writings of classical political economists. The most famous of these writings are, of course, those of Adam Smith. The story that Smith recites concerning the development of money is essentially that of Aristotle with a few twists of its own: Aristotle's evolutionary theory is replaced by the "four distinct stages which mankind pass thro: 1ˢᵗ, the Age of Hunters; 2ᵈˡʸ, the Age of the Shepherds; 3ᵈˡʸ, the Age of Agriculture; and 4ᵈˡʸ, the Age of Commerce" (Smith 1973, p. 14). In these stages of development, the "propensity to truck, barter and exchange one thing for another" gives rise to "division of labor" (Smith 1776, p. 25). Division of labor, in the last stage of history, would turn every man into a "merchant" (p. 37). In the earlier time, however, exchange of good for good must have been difficult,

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[15] On the incomprehensible nature of Aristotle's statement, see M.I. Finely (1977, particularly p. 148). For a different view of Aristotle's concept of exchange and the debt of modern economic theory to the ancients, see S. Todd Lowry (1987, pp. 182–212, 250–51). A more recent discussion of Aristotle's "geometry of exchange" can be found in Joel Kaye (1998, Chapter 2).

[16] Kaye (1998, p. 48) points out that the term "demand" in this passage should have been translated as "human need."

[17] For a more recent and thorough analysis of the relation between Aristotle and the scholastic writers see Kaye (1998).

it "must frequently have been very much clogged and embarrassed in its operation" (p. 37). These difficulties must have led every "prudent man" to carry "certain quantities of some commodity or other" for the purpose of exchange (p. 38). Over time, however, metals, having certain unique characteristics, replaced all other commodities for this purpose, and were subsequently coined. It is in "this manner that money becomes in all civilized nations the universal instrument of commerce" (p. 44). But besides facilitating exchange, money does something else: "exchangeable value of every commodity is more frequently estimated by the quantity of money, than by the quantity of labor or any other commodity which can be had in exchange for it" (p. 48). That is, as a measure of value, money—similar to labor—makes commodities equal to one another.

Some aspects of this theory do not escape even the critics of the classical political economy. Karl Marx, for example, who avoids the stages of development theory of money, views the relation of exchange as an equality.[18] "Money," he writes in *Capital*, "necessarily crystallizes out of the process of exchange, in which different products of labor are in fact equated with each other" (Marx 1977, p. 181). He, himself, explicitly attributes this view "to the great investigator," Aristotle, whose equation "5 beds = 1 house" is virtually "indistinguishable from 5 beds = a certain amount of money" (p. 151). It is worth noting here that by using the equality sign, " = " Marx turns the rhetorical mathematics of Aristotle into a symbolic one, making the relation of exchange even more ambiguous.[19] It is this concept of equality which leads Marx to state, at the beginning of *Capital*, that "clearly, the exchange relation of commodities is characterized precisely by abstraction from their use-values" (p. 127). That is, the equation of exchange implies for Marx that commodities are equal to one another. But since they cannot possibly be equal as heterogenous objects, we must abstract from their physical being altogether to arrive at what they have in common, namely, abstract labor.[20]

The early proponents of marginal analysis also viewed exchange as a relation of equality, as is evident from the "equation of exchange" in William Stanley Jevons (1957, p. 105) and Leon Walras (1954, p. 87).[21] They also followed the classical school and, by extension, Aristotle, when it came to the origin and functions of money. In his *Money and the Mechanism of Exchange*, Jevons presents a stage of development theory of money which is very much Smithian. The primordial stage is, once again, the "hunting state" in which "subsistence is gained by hunting wild animals" (Jevons 1902, pp. 19–20). In this stage, "skin," rather than "meat," becomes the "currency" because it is less perishable than meat. In the second stage, the "pastoral state," cattle "performs some of the functions of

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[18] According to Karl Marx, the "exchange of commodities begins where communities have their boundaries, at their point of contact with other commodities, or members of the latter" (Marx 1977, p. 182). But the exact account of this exchange is left out by Marx.

[19] The symbol " = " first appears in the writings of the Renaissance mathematician Robert Recorde (1557). However, it does not seem to be widely used in economic theory until the end of the classical era.

[20] For a detailed discussion of this issue, see Sasan Fayazmanesh (1994).

[21] Indeed, even the more modern concepts of money and monetary relation in the marginal analysis tradition are based on the assumption of "delivery of goods of equal value, *quid pro quo*" (see Ostroy and Starr 1974, p. 1093, original emphasis).

money" (p. 21). In yet another stage, the "agricultural state," even some "vegetable production," such as "corn," becomes money. However, because of certain qualities of metals, such as "utility," "portability," "stability in value," metals replace all other goods as money. But when does barter begin? Jevons's narrative is somewhat ambiguous here. He writes, without being specific, that the "earliest form of exchange *must have* consisted in giving what was not wanted directly for that which was wanted," that is, "barter or truck" (Jevons 1902, p. 3, emphasis added). Barter, however—and here Jevons tries to refine the Aristotelian-Smithian view—involves three difficulties or inconveniences: "want of coincidence in barter," "want of measure of value," and "want of means of subdivision" (pp. 3–7). The first difficulty is that of finding "two persons whose disposable possessions mutually suit each other's wants" (p. 3). It is in this discussion that Jevons's concept of the origin of barter becomes somewhat less ambiguous: "A hunter having returned from a successful chase has plenty of game, and may want arms and ammunition to renew the chase. But those who have arms may happen to be well supplied with game, so that no direct exchange is possible" (p. 4). The second difficulty has to do with the question of "[h]ow much beef for how much flax"? (p. 5). That is, in a "state of barter the price current-list would be a most complicated document, for each commodity would have to be quoted in terms of every other commodity" (p. 5). The third difficulty "is a minor inconvenience of barter," according to Jevons; the "tailor, as we are reminded in several treatises on political economy, may have a coat ready to exchange, but it much exceeds in value the bread which he wishes to get from the baker, or the meat from the butcher. He cannot cut the coat up without destroying the value of his handiwork" (p. 6).

Jevons's refinement of Aristotelian-Smithian conjectures ultimately became the most authoritative account of the difficulties of barter in the twentieth century. For example, the entry "barter" in the early twentieth century Palgrave *Dictionary of Political Economy* begins with the concept of the direct exchange of goods in the "primitive state of society" and immediately proceeds to Jevons's three "inconveniences" of barter, necessitating the development of money (see J.E.C. Munro 1926, pp. 121–22). Similarly, in the late twentieth century *New Palgrave Dictionary of Money and Finance*, where the entry on barter is merely subsumed under money, it is stated that barter is bilateral and thus "in Jevons's famous phrase it requires 'double coincidence [of wants], which will rarely happen' " (see James Tobin 1992, p. 770). Between these two citations, numerous monetary theorists would further the authority of Jevons on the subject by mentioning his work and refining it.[22] Moreover, almost all modern textbooks on money refer to Jevons's "double coincidence of wants" when mentioning the difficulties of barter and the development of money.

VI. EARLIER CRITICISMS OF ECONOMIC THEORIES OF BARTER

Anthropologists studying modern pre-capitalist economies have often disputed the soundness of economic theories of barter and development of money.

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[22] See, for example, Robert Clower (1970).

Caroline Humphrey (1985, 1992), for example, correctly points out that economists' notion of a pure barter economy and the emergence of money is no more than a myth. She writes: "No example of barter economy, pure and simple, has ever been described, let alone the emergence from it of money; all available ethnography suggests that there never has been such a thing" (Humphrey 1985, p. 48). According to Humphrey, had economists actually studied modern cases of communities in which barter does take place, they would have been led to different conclusions concerning barter and the origin of money. By using the Lhomi of northeast Nepal as a case study, Humphrey demonstrates that, contrary to the prevalent economic theories, the "search for coincidence of wants is not necessary, since the time and place of barter for common items was established long in the past" (Humphrey 1985, p. 56). Or with regard to the complication arising from keeping track of numerous exchange ratios, she contends that in "practice this is not the case … many items are never traded," and Lhomi's barter "in practice, follows limited and well known 'tracks' " (p. 57). In general, Humphrey argues that barter "has been misconstrued largely because of the persistence of the creation-myth in classical and neoclassical economics that in barter lie the origins of money and hence of modern capitalism. In this perspective money originates as a solution to the problem of barter" (1992, p. 2).

Many historians of money have also expressed similar views, but mostly without the usual case studies of the anthropologists. Paul Einzig, for example, has been a staunch critic of the orthodox theories of barter. In reference to the usual citations of Jevons's anecdotal examples of difficulties of barter—such as the one involving a Parisian singer and the inhabitants of Society Islands—Einzig writes: "What the economists quoting these and other similar instances do not appear to realize is that the difficulties complained of are not inherent in the system of barter. They are largely anomalies arising from sudden contact between two civilizations" (Einzig 1966, p. 342). According to Einzig, the " 'double coincidence' (to quote the word of Jevons) that must arise is by no means difficult to achieve in a small community where everybody knows a great deal about everybody else's products and requirements" (Einzig 1966, p. 342). "Nor is the difference between value of the objects to be exchanged, or time lag between their delivery," Einzig goes on to say, "an unsurmountable obstacle" (p. 344).

Einzig concludes his criticism of Jevons by saying that "it seems to be a mistake to assume that barter became reduced to absurdity at an early phase of economic progress. The chances are that in the majority of communities it survived long after its replacement by some form of monetary system" (p. 344). Similar views are expressed more recently by Glyn Davies (1994, pp. 9–10), who argues that most "modern textbooks on money" base their account of barter not on "the mainstream of human progress" but on existing pre-capitalist communities. Such books, according to Davies, present the "rise of money on the misleadingly narrow and mistaken view of the alleged disadvantages of barter to the exclusion of other factors" (p. 10). "Barter," Davies goes on to say, "has, undeservedly, been given a bad name in conventional economic writing, and its alleged crudities have been exaggerated" (p. 10).

The received view of barter has also been challenged occasionally by those

economists who are aware of anthropological studies. For example, in their macroeconomics textbook, James K. Galbraith and William J. Darity (1994, pp. 182–83) call the usual account of the development of money the "*rationalist* explanation," according to which "a need for money came to exist, at some long distance date in the past, as trade developed between individuals and communities and as societies wrestled with the difficulties of systems of barter." They then dismiss the validity of this "rationalist idea" and argue that anthropological "evidence on primitive societies suggests that where money is absent, so too is exchange itself" (p. 183). "That is," they contend, "without money no alternative mechanisms, such as barter, exist for establishing the equivalence of value of one good for another" (p. 183). They conclude by arguing that if "the anthropological evidence is a guide to history, it would be more accurate to say that the invention of money coincides with the innovation of trade rather than that money merely provides a convenient and efficient way to conduct a human activity conceived and initiated without it" (p. 183).

But the most well-known economic criticism of the received view of barter appears to belong to Karl Polanyi. In reference to those who equate the principle of the division of labor with barter, Polanyi states that a "host of writers on political economy, social history, political philosophy, and general sociology had followed in Smith's wake and established his paradigm of the bartering savage as an axiom of their respective science" (Polanyi 1957, p. 44). According to Polanyi, this paradigm has shaped the orthodox economic theory of the development of market, a theory in which propensity to exchange eventually would give rise to long-distance trade. However, Polanyi (1957) goes on to argue:

> The logic of the case is, indeed, almost the opposite of that underlying the classical doctrine. The orthodox teaching started from the individuals's propensity to barter; deduced from it the necessity of local markets, as well as division of labor; and inferred, finally, the necessity of trade, eventually foreign trade, including long-distance trade. In light of our present knowledge we should almost reverse the sequence of the argument: the true starting point is long-distance trade, a result of geographical location of goods, and of the "division of labor" given by location. Long-distance trade often engenders markets, an institution which involves acts of barter, and if money is used, of buying and selling, thus eventually, but by no means necessarily, offering to some individuals an occasion to indulge in their alleged propensity for bargaining and haggling (p. 58).

In supporting this argument, besides referring to anthropological studies, Polanyi alludes to medieval history of Europe, when the "short flowering of the famous fairs of Europe was another instance where long-distance trade produced a definite type of market; England's staples were another example" (1957, p. 60).

VII. THE "RULES OF BARTER" AND BARTER IN ECONOMIC THEORY

As noted in the previous section, the common criticisms of the orthodox theories of exchange are mostly based on anthropological studies. To the extent that these criticisms point out the mythical nature of primordial exchange and the rational-

ist explanation of the development of money, the criticisms are helpful in recognizing the need for reconstructing the theories of exchange and money. However, insofar as these critical views seem to suggest that such theories must be based on studies of exchange among modern pre-capitalist societies, they are less helpful. After all, it is difficult today to find societies that are untouched by capitalist relations of exchange. Also, what holds true of modern pre-capitalist economies may not necessarily be true of early stages of the development of exchange.

There is, however, a more direct way of challenging the conventional view of exchange and reconstructing a new theory: an examination of centuries of actual practices of the merchants in the medieval and Renaissance periods as they are evident in the abacus manuscript. From what has been said in the previous sections, we can conclude the following concerning the so-called difficulties of barter and the relation of exchange.

The first difficulty of barter, namely, what Jevons called the "double coincidence of wants," does not seem to arise at all in the age of merchant capitalism, when barter is one of the major forms of commerce. Indeed, there is no mention of such a difficulty in any of the manuscripts examined. This, similar to what Humphrey points out concerning Lhomi's practices, may be due to the fact that barter appears to follow limited and well-known tracks. As Van Egmond notes, there are certain "major products" that are dealt with in the *abaci*, such as "wool, finished cloth, pepper, grain, and wine" (Van Egmond 1976, p. 188). There are also some other, minor products, which appear in these manuscripts. For example, in the fifteenth century *Treviso Arithmetic*, beside wool and cloth, one example of barter involves balsam, wax and ginger. Similarly, in the sixteenth century *Well-Spring of Science*, which was written when markets had become more extensive, besides the usual commodities, we encounter the exchange of such things as cinnamon, tapestry, caps, tin, lead, iron, and steel. But in general, the number of commodities exchanged appears to be very limited and usually involves the major products of wool and cloth. This limited extent of the commodities exchanged, and the fact that the trade routes and the locations of markets are well established, prevents the emergence of the problem of "double coincidence of wants."

The limited numbers of goods exchanged also preclude the existence of Jevons's second difficulty of barter. If the numbers of commodities bartered are relatively small, as is evident from the examples repeatedly used in the *abaci* manuscripts, the complication of keeping track of numerous exchange ratios will not arise. Indeed, as we have seen, the *abaci* examples of the exchange of "ware for ware" do not seem to convey any difficulty whatsoever with keeping track of complicated numbers, since all barter values are expressed in terms of money of account. This brings up an important issue often mentioned by the historians of money. Contrary to the usual assumption of economic theory, according to which money of account must also fulfill the function of medium of exchange, in the medieval and Renaissance periods the medium of exchange often does not play the role of money of account. Peter Spufford, for example, points out that in "most parts of late medieval Europe, and in many places up to the eighteenth or even the nineteenth century, a dichotomy existed in the functions of money.

On the one hand money of account was the *measure of value*, whilst on the other, the actual coin was the *medium of exchange* and the *store of wealth*" (Spufford 1988, p. 411, original emphasis). Similarly, John F. Chown distinguishes between money of account or "ghost money" and the "real coins" (Chown 1994, pp. 17–20).[23] This dichotomy between different functions of money points out that, in reality, barter can be conducted merely on the basis of a ghost money, a unit of account based on non-existing real coins. In other words, the mere existence of a fictitious unit of account can prevent the emergence of the complication of keeping track of a large number of exchange ratios.

As far as Jevons's third and "minor inconvenience of barter" is concerned— that is, his so-called "want of means of subdivision"—the problem is also non-existent, or at least not important when it comes to the reality of the medieval barter. As we have seen, most of the major goods bartered in this period are for practical purposes divisible. There are, occasionally, some non-divisible items bartered as well. For example, Baker's second problem involves the exchange of "Chamblets" that in barter sell for "4ℓ.3 s. 4 d." per piece and "Caps" that sell, also in barter, for "35 s. 10 d." per "dozen" (Baker 1568, p. 248). This means that one piece of "Chamblet" should exchange for 2.33 dozen "Caps," which is, of course, not a round number. But Baker does not seem to be bothered at all by this problem. It appears, therefore, that for practical purposes, the non-divisibility of some commodities does not inhibit barter. This is particularly true when money is available and when part of the payment, as we have seen, may involve cash.

In sum, the difficulties of barter, assumed by monetary theory, appear to be non-existent when it comes to actual medieval and Renaissance exchange. But the *abaci* manuscripts have greater implications for economic theory. As it was pointed out earlier, the most common assumption in economic theory since at least the time of Aristotle, is that exchange is characterized by equality. This assumption eventually leads to conceptualizing the relation of exchange by the mathematical symbol of equality. This characterization of exchange is extremely problematic since the relation "barters for," which we encountered in the *abaci* manuscripts and designated by B, has little in common with the relation " $=$ ".[24] Yet in the eyes of economic theory they appear identical because it is assumed that in barter equal amounts of some substance, be it need, labor, utility, or some other substance, are exchanged. Once this substance is manifested in the commodity money, the relation B is transformed into " $=$ " by virtue of equality of prices.

The "rules of barter," however, deny the validity of the above argument by undermining its basic premise. As noted earlier, the *abaci* manuscripts point out that the merchants set certain prices for their goods. But when it comes to actual

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[23] As Peter Spufford points out, the term "imaginary money," which is often used in this context, is a misnomer, since "on closer inspection an historical explanation may be found for the existence of each money of account, and that such an historical explanation will indicate to which real coin the system continued to be attached" (Spufford 1988, pp. 413–14).

[24] A full discussion of the properties of exchange relation, both in theory and in reality, is beyond the scope of this essay and should be dealt with elsewhere.

exchange they try not to trade their goods at these prices. Rather, they attempt to set higher barter values for their goods. Indeed, as it was observed earlier, the first rule of barter is "giving a good for another in order to get more." This "getting more" involves taking advantage of the other side's ignorance of the basic rules of arithmetic. The greater this ignorance, the more the possibility of deceit and the greater the opportunity for profit making. Thus, in the final analysis, the relation of barter is characterized by deceit and inequality rather than honesty and equality of exchange.[25] If this is the case, we should rethink one of the most fundamental assumptions in monetary theory.

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[25] James Tobin (1992, p. 774) has argued that fiat money is "an embarrassment to value theory." To this, one can add that the practices of medieval merchants are also embarrassing when it comes to such a theory.

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