APPENDIX I

Counterfactuals and Spheres

The dualist argument for the counterfactual dependence of the *P**-instance on the *M*-instance from Section 2.5 goes as follows:

(16) If none of *M*'s bases had been instantiated, then *M* would not have been instantiated. ($\sim\cup\mathbf{P}_M \;\Box\!\!\rightarrow \sim$M)
(17) If *M* had not been instantiated, then none of *M*'s bases would have been instantiated. ($\sim$M $\Box\!\!\rightarrow \sim\cup\mathbf{P}_M$)
(18) If none of *M*'s bases had been instantiated, then *P** would not have been instantiated. ($\sim\cup\mathbf{P}_M \;\Box\!\!\rightarrow \sim$P*)

---

(19) If *M* had not been instantiated, then *P** would not have been instantiated. ($\sim$M $\Box\!\!\rightarrow \sim$P*)

The non-vacuous truth of the premises, (16)–(18), yields the existence of a sphere **S** in which *M* is instantiated just in case a base of *M* is instantiated and which contains a world *w* where neither *M* nor a base of *M* nor *P** is instantiated (see Figure A.1, which repeats the earlier Figure 2.5).[1] It follows from the existence of such a sphere that (19) is true.

To see why (19) follows, and to see how exactly we get to the situation represented in Figure A.1 in the first place, let us proceed in three steps.

First, given that (16) and (17) are non-vacuously true, there is a sphere in which *M* is instantiated just in case a base of *M* is instantiated and that contains a world where neither *M* nor a base of *M* is instantiated. Proof: By the non-vacuous truth of (16), there is a world *v* where neither a base of *M* nor *M* is instantiated that it closer to the actual world than all worlds where *M* is instantiated in the absence of a base. By the non-vacuous truth of (17), there is a world *u* where neither *M* nor a base of *M* is instantiated that is closer to the actual world than all worlds where a base of *M* is instantiated in the absence

[1] Throughout this appendix, phrases of the form 'a sphere in which . . . if and only if . . .' should be read such that 'a sphere in which' takes wide scope over the biconditional.

https://doi.org/10.1017/9781108762717.008 Published online by Cambridge University Press

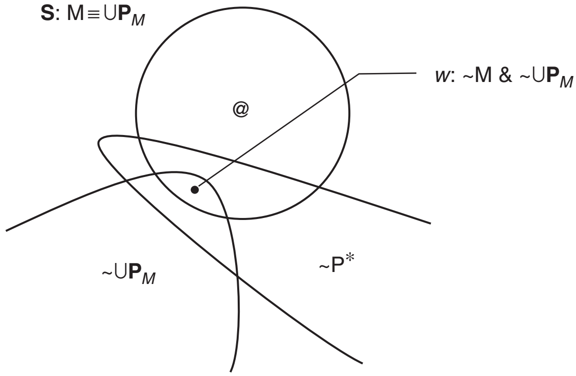

Figure A.1. (16) (~∪$\mathbf{P}_M$ □→ ~M), (17) (~M □→ ~∪$\mathbf{P}_M$), (18) (~∪$\mathbf{P}_M$ □→ ~P*) true

of *M*. Worlds *v* and *u* might be equally close to the actual world, or one might be closer to the actual world than the other.[2] If *v* and *u* are equally close, let **S** be a sphere that contains both worlds at the very edge; that is, let **S** be a sphere that contains no worlds that are less close to the actual world than *v* and *u* are. If one of *v* and *u* is closer to the actual world than the other, let **S** contain whichever world is closer at the very edge. Sphere **S** does not contain a world where *M* is instantiated in the absence of a base of *M*, for the worlds at its edge are closer to the actual world than any such worlds. Similarly, **S** does not contain a world where a base of *M* is instantiated in the absence of *M*. Thus, it is true in **S** that *M* is instantiated just in case a base of *M* is instantiated. By our choice of **S**, it contains a world where neither *M* nor a base of *M* is instantiated.

Second, given that (18) is non-vacuously true, any sphere that has the characteristics of **S**, that is, any sphere that contains a world where neither *M* nor a base of *M* is instantiated and in which it is true that *M* is instantiated just in case a base of *M* is instantiated, contains a world where neither a base of *M* nor *P** is instantiated that is closer to the actual world than any worlds where *P** is instantiated while no base of *M* is instantiated. Proof: Let **S** be a sphere in which it is true that *M* is instantiated just in case a base of *M* is instantiated. Let *w* be a world in **S** at which neither *M* nor a base of *M* is instantiated. Suppose that **S** does not contain a world where neither a base of *M* nor *P** is instantiated that is closer to the actual world than any worlds where *P** is instantiated while no base of *M* is instantiated. Suppose, in other

[2] It does *not* follow from the possibility that one of *u* and *v* is closer to the actual world than the other that the closest antecedent-worlds of (16) and (17) – if such there be – fail to coincide.

https://doi.org/10.1017/9781108762717.008 Published online by Cambridge University Press

words, that no world in **S** where neither a base of *M* nor P^* is instantiated is closer to the actual world than any worlds where P^* is instantiated while no base of *M* is instantiated. There are two cases to consider. Sphere **S** might not contain a world where neither a base of *M* nor P^* is instantiated at all. Then *w* must be a world where P^* is instantiated while no base of *M* is instantiated. Any worlds where neither a base of *M* nor P^* is instantiated are outside of **S** and hence less close to the actual world than *w*, which renders (18) false, in contradiction to its (non-vacuous) truth. In the other case, **S** does contain worlds where neither a base of *M* nor P^* is instantiated, but none of these worlds is closer to the actual world than any worlds where P^* is instantiated while no base of *M* is instantiated. In this case, too, (18) comes out false, contrary to our assumption of its truth. For there are no antecedent-cum-consequent-worlds of (18) that are closer to the actual world than the antecedent-cum-consequent-worlds of (18) in **S** are. Since all the antecedent-cum-consequent-worlds of (18) in **S** fail to be closer to the actual world than any worlds where the antecedent of (18) is true while its consequent is false, (18) comes out false.

Third, let *w* be a world where neither a base of *M* nor P^* is instantiated that is closer to the actual world than any worlds where P^* is instantiated while no base of *M* is instantiated. Let *w* be contained in a sphere in which it is true that *M* is instantiated just in case a base of *M* is instantiated. Then at *w* neither *M* nor P^* is instantiated, and *w* is closer to the actual world than any worlds where P^* is instantiated while *M* is not, such that (19) is non-vacuously true. Proof: Again, let **S** be a sphere in which it is true that *M* is instantiated just in case a base of *M* is instantiated. Let *w* be a world in **S** where neither a base of *M* nor P^* is instantiated that is closer to the actual world than any worlds where P^* is instantiated while no base of *M* is instantiated. By the equivalence of *M*'s instantiation with the instantiation of a base of *M* within **S**, *w* is a world where neither *M* nor P^* is instantiated. Further, by the equivalence, all worlds in **S** where P^* is instantiated while *M* is not instantiated are worlds where P^* is instantiated while no base of *M* is instantiated. By assumption, *w* is closer to the actual world than worlds of the latter kind. Therefore, *w* is also closer to the actual world than any worlds where P^* is instantiated while *M* is not instantiated. (If **S** does not contain any worlds where P^* is instantiated while *M* is not instantiated, then all such worlds are less close to actuality than *w* by virtue of being outside of **S**.)

In sum, and returning to Figure A.1, from the non-vacuous truth of (16), (17), and (18) we get the existence of a sphere **S** in which *M* is instantiated just in case a base of *M* is instantiated which contains a world *w* where neither *M* nor a base of *M* (nor P^*) is instantiated. It follows from the existence of such a sphere that (19) is non-vacuously true.

https://doi.org/10.1017/9781108762717.008 Published online by Cambridge University Press