

THESIS ABSTRACTS

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CLOVIS HAMEL, *New Results in Model Theory and Set Theory*. University of Toronto, Toronto, ON, 2023. Supervised by Franklin D. Tall. 2020 *Mathematics Subject Classification*. Primary 03C55, 54H99, 03E75, Secondary 03C85, 03C98. *Key words and phrases*. stability, definability, Tsirelson space, Grothendieck spaces, metastability, Morley's trichotomy, Turing determinacy.

Abstract

Traditionally, the role of general topology in model theory has been mainly limited to the study of compacta that arise in first-order logic. In this context, the topology tends to be so trivial that it turns into combinatorics, motivating a widespread approach that focuses on the combinatorial component while usually hiding the topological one. This popular combinatorial approach to model theory has proved to be so useful that it has become rare to see more advanced topology in model-theoretic articles. Prof. Franklin D. Tall has led the re-introduction of general topology as a valuable tool to push the boundaries of model theory. Most of this thesis is directly influenced by and builds on this idea.

The first part of the thesis will answer a problem of T. Gowers on the undefinability of pathological Banach spaces such as Tsirelson space. The topological content of this chapter is centred around Grothendieck spaces.

In a similar spirit, the second part will show a new connection between the notion of metastability introduced by T. Tao and the topological concept of pseudocompactness. We shall make use of this connection to show a result of X. Caicedo, E. Dueñez, J. Iovino in a much simplified manner.

The third part of the thesis will carry a higher set-theoretic content as we shall use forcing and descriptive set theory to show that the well-known theorem of M. Morley on the trichotomy concerning the number of models of a first-order countable theory is undecidable if one considers second-order countable theories instead.

The only part that did not originate from model-theoretic questions will be the fourth one. We show that $ZF + DC +$ "all Turing invariant sets of reals have the perfect set property" implies that all sets of reals have the perfect set property. We also show that this result generalizes to all countable analytic equivalence relations. This result provides evidence in



favour of a long-standing conjecture asking whether Turing determinacy implies the axiom of determinacy.

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ANDREAS LIETZ, *Forcing “NS $_{\omega_1}$ is ω_1 -Dense” from Large Cardinals*. Universität Münster, Münster, Germany, 2023. Supervised by Ralf Schindler. MSC: Primary 03E57, Secondary 03E35, 03E55, 03E60, 03E25. Keywords: nonstationary ideal, forcing axioms, large cardinals, axiom (*).

Abstract

We answer a question of Woodin [3] by showing that “NS $_{\omega_1}$ is ω_1 -dense” holds in a stationary set preserving extension of any universe with a cardinal κ which is a limit of $<\kappa$ -supercompact cardinals. We introduce a new forcing axiom Q-Maximum, prove it consistent from a supercompact limit of supercompact cardinals, and show that it implies the version of Woodin’s (*)-axiom for \mathbb{Q}_{\max} . It follows that Q-Maximum implies “NS $_{\omega_1}$ is ω_1 -dense.” Along the way we produce a number of other new instances of Asperó–Schindler’s $\text{MM}^{++} \Rightarrow$ (*) (see [1]).

To force Q-Maximum, we develop a method which allows for iterating ω_1 -preserving forcings which may destroy stationary sets, without collapsing ω_1 . We isolate a new regularity property for ω_1 -preserving forcings called respectfulness which lies at the heart of the resulting iteration theorem.

In the second part, we show that the κ -mantle, i.e., the intersection of all grounds which extend to V via forcing of size $<\kappa$, may fail to be a model of AC for various types of κ . Most importantly, it can be arranged that κ is a Mahlo cardinal. This answers a question of Usuba [2].

REFERENCES

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URL: <https://andreas-lietz.github.io/resources/PDFs/AJourneyGuidedByTheStars.pdf>.

ZHANSAYA TLEULIYEVA, *Algorithmic Properties of Rogers Semilattices* Nazarbayev University. Supervised by Manat Mustafa and Nikolay Bazhenov. MSC: 03D45. Keywords: theory of numberings, computable numbering, Rogers semilattice, limitwise monotonic, analytical hierarchy, projective determinacy, types of isomorphism.

Abstract

The thesis uses various approaches to explore the algorithmic complexity of families of subsets of natural numbers. One of these approaches involves investigating upper semilattices