OPENING OF THE MAGNETIC FIELD LINES IN A FAST ROTATING MAGNETOSPHERE, WITH AN APPLICATION TO JUPITER

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In this Communication, we consider a simple model of magnetosphere around a fast rotating Jupiter-like object possessing a spin-aligned dipolar moment μ . In this model, low-energy plasma released by inner sources located beyond the corotation radius diffuses outward through closed lines, forming a thin equatorial disk in which there is a quasi-static balance between the centrifugal force and the magnetic tension. At some critical radius r_o, however, the magnetic field is no longer strong enough to hold the matter, and blows open, the matter escaping freely (Fig. 1) (this model has been introduced by Hill and Carbary, 1978).

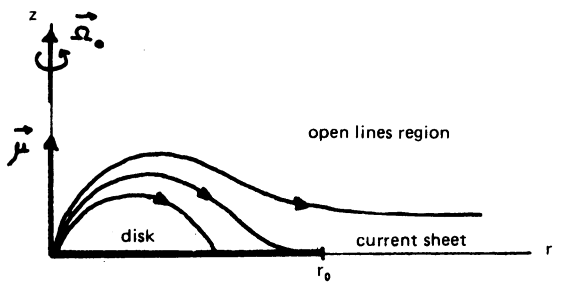

Figure 1: Geometry of the model

The complete solution of the whole set of equations of this model is well beyond the scope of this Paper, in which we just would like to present the solution of the following restricted sub-problem, which can be solved analytically: for given functional forms for the quantities characterizing the plasma in the closed region (angular frequency $\Omega(r)$ and flux-tube mass-contents $\eta(r)$ for $r < r_o$), what is the "self-consistent" value of r_o, and what is the structure of the magnetic field (B_r, B_ϕ, B_z) in the region outside the disk and the central object (we will assume this region to be force-free, and the currents flowing

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through it to have a negligible effect on the poloidal component $\vec{B}_p$ of $\vec{B}$).

We first consider the following problem for the (quasi-vacuum) potential $A = \int_0^\infty c(k)e^{-k|z|}J_1(kr)dk$ of that part of $\vec{B}_p$ created by the disk currents: to find the function c(k) such that: i) for $z = 0^+$, $r < r_o$, the B_r component of the field is given by $B_r = + 2\pi r\Omega^2(r)\eta(r) = \chi(r)$ (from the radial component of the momentum balance equation) and then $\int_0^\infty kc(k)J_1(kr)dk = \chi(r)$; ii) for z=0, $r>r_o$, the z-component of the field is zero (open field) and then $\int_0^\infty c(k)J_1(kr)dk = -\mu/r^2 + \phi/2\pi r$, where the constant of integration ϕ is equal to the flux on the open lines; iii) at $(z=0,\ r=r_o)$, the field is continuous (this condition will determine the value of ϕ).

Solving this problem by standard techniques (Sneddon, 1966), one gets a formal solution for the poloidal structure of the field. For this solution to be physical, it is necessary that the components B_r and B_z of the field keep the same sign on $\{z=0^+,\ r<r_o\}$ ($B_r > 0$, $B_z < 0$ for $\mu > 0$) and that the surface mass density $\sigma = -\eta B_z > 0$ (this is obvious from the momentum balance equation above, and the fact that the plasma diffuses). Then we choose $\chi(r) > 0$ and $\eta(r) > 0$, and look for a condition on these functions for $B_z(r) < 0$. A necessary condition writes

$$\int_0^1 \frac{x}{(1-x^2)^{1/2}} \frac{d}{dx}\left[x^2\chi(r_o x)\right] \leq \frac{2\mu}{r_o^3} \qquad (1)$$

(1) is also sufficient if we restrict our attention to the large class of function χ for which $\frac{d}{dr}\left[r\frac{d}{dr}(r^2\chi)\right] \geqslant 0$. The maximum value of r_o for which (1) holds has clearly to be interpreted as the opening radius. If we take for instance Hill and Carbary's assumptions ($\Omega=\Omega_o=$ angular frequency of the central object, $\eta=\eta_o$), one gets $r_o = (\mu/2\pi\eta_o\Omega_o^2)^{1/4}$.

The toroidal component of the field is easily computed with the assumptions above. At the surface of the disk, one has, because of angular momentum conservation,

$$B_\phi = \frac{2\pi\dot{M}}{B_z}\frac{d}{dr}(r^2\Omega) \qquad (2)$$

($\dot{M}$ is the rate at which plasma is released by the inner sources), and this value can then be transported along the lines of the zero-order poloidal structure by using the well-known force-free relation rB_ϕ= constant along a line.

To apply the results presented in this Paper to Jupiter's magnetosphere, it is necessary to do some specific hypotheses on Ω and η, as do Hill and Carbary. Using their scaling and their value for $\dot{M}$, one gets $r_o \simeq 35\ R_J$. Choosing, instead of $\Omega = \Omega_o$, $\Omega = \Omega_o a^2/(r^2+a^2)$ to take into account the observed lack of strict corotation, one gets a larger value for r_o (one can take e.g. a equal to the "corotation breaking" radius introduced by Hill (1979)).

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DISCUSSION

Sturrock: Your assumption that the system has linear translational symmetry has the drawback that an open field system would have infinite energy per unit length. Similarly, the assumption that $j = \mu B$, where μ is a constant, involves an infinite total current in the system. Hence it would be better to relax these two assumptions although this would, of course, make the problem much more difficult.

Aly: That's right, considering a translational invariant configuration makes any energetic considerations impossible to apply to actual situations, as the energy per unit of length of the open field is indeed infinite (the energy of the sequence of force-free field $\{A_\mu\}$ increases as Log μ when $\mu \to \infty$). However, it is possible to prove an asymptotic result, similar to the one I have presented here, for other 2-D configurations for which the energy of the open field, corresponding to the given fixed value of the normal component B_n of the field on the boundary is finite. In that case, you find that the energy of the force-free field increases monotonically with the parameter μ which measures the degree of shearing of the structure, and approaches asymptotically the energy of the open field when $\mu \to \infty$. This proves indirectly that the energy of the open field is the least upper bound for the energy of the force-free configurations having the same given B_n on the boundary and satisfying the conditions I have quoted in my paper.

Birn: In your theorem, did you have to assume that a solution A_μ exists for all values of μ?

Aly: Yes, the asymptotic result I have presented rests on the assumption that the sequence $\{A_\mu\}$ does exist for all values of μ (actually, it can be also applied if $\{A_\mu\}$ exists only for large values of μ, $\mu > \mu_1$ say). At the present time, the existence of $\{A_\mu\}$ is still a conjective, there is not yet a complete proof of this point. However, it is worth noticing that we are sure that $\{A_\mu\}$ exists in some cases, as we know explicitely some particular sequences, as the one you have computed yourself with Goldstein and Schindler.