colliding particles do no work, so that the total kinetic energy is conserved (*i.e.* the coefficient of restitution is unity if there are two particles). It may be noticed that this terminology is customary in the theory of atomic collisions. A collision that involves loss of kinetic energy is *inelastic*; when the loss of energy is as great as possible (coefficient of restitution zero) the collision may be called *completely inelastic*.

I illustrate the use of these terms by a few examples. Steel can often be treated as rigid and elastic, lead as rigid and completely inelastic. Putty is deformable and completely inelastic. A spring obeying Hooke's law is extensible and elastic. Mr. Lightfoot requires of his string that it should be inextensible and completely inelastic, and his comment might read " although real strings are often practically inextensible, no real string fulfils the condition that it is *completely* inelastic ". I am, Yours, etc., F. C. Powell.

COORDINATE NOTATION.

To the Editor of the *Mathematical Gazette*.

Sir,—A problem that crops up in many practical connections is that of the labelling, schedulling, and classifying of points, areas, and lines. I refer to such cases as the numbering and listing of the buildings of a camp or village, the preparation of a county valuation roll in such a way that the properties can readily be identified on the Ordnance Survey plan of the area, or the numbering of a system of roads and their identification on a road map.

A method of labelling that has been gaining in favour in recent years is that based on coordinates; and it is difficult to find a better one. But why is it not universally adopted? I am convinced that the answer is that the accepted method of representing the coordinates is at fault:. each is shown separately; this allows of classification according to one or other of the coordinates: not according to both.

Consider, for instance, the point P with coordinates x 284, y 407. On Army grid maps this would be written 284407. Now this number gives no basis for classifying a system of points, but, if rewritten as

$$24, \quad 80, \quad 47,$$

a simple basis of classification is at once available: write down the points in the numerical order of their coordinates in the new forms. The point P falls in the square to the north-east of the point 24, 00, 00. And all points in that square will be scheduled together.

The next obvious reform will be to introduce negative digits as well as positive. P now takes the form

$$34, \quad \bar{2}1, \quad 4\bar{3}$$

(x $3\bar{2}4$, y $41\bar{3}$). All points commencing with the number 34 will be scheduled together in the square whose *centre* is the point 34, 00, 00, a more symmetrical arrangement. If P represents a building it might be labelled and identified locally by the last two digits only, $4\bar{3}$.

This notation could be extended to vectors and possibly to tensors of even higher order. But an objection would probably be raised because of the labour involved in separating the components when required for calculations involving them. Would it be possible to carry out these calculations in the new symbols as they stand? I have investigated this possibility and find that not only would it be possible, but in most cases of distinct advantage to do so. The following three examples will illustrate this; the notation is duodecimal—

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why put new wine in old bottles? The multiplication table can be memorised in half-an-hour.

Consider two vectors A^r, B^r, referred to rectangular Cartesian coordinates and having as covariant components :

$$\begin{array}{llll} A_1 & \bar{4}\ \ 42, & B_1 & 0\,.\,\bar{3}\bar{4}, \\ A_2 & 6\ \ \bar{5}4, & B_2 & \bar{5}\,.\,1\bar{6}, \\ A_3 & 0\,.\,1\bar{1}, & B_3 & 4\ \ 06 \end{array}$$

(the contravariant components will of course have the same values). The digits have purposely been selected at random : note how seldom it is necessary to carry forward.

Addition $\qquad C^r = A^r + B^r$

$$\begin{array}{lll} A^r & \bar{4}60\,.\,4\bar{5}1, & 24\bar{1} \\ B^r & 0\bar{5}4\,.\,\bar{3}10, & \bar{4}\bar{6}6 \\ \hline C^r & \bar{4}14\,.\,1\bar{4}1, & \bar{2}\bar{2}5 \end{array}$$

The digits are added as in ordinary arithmetic but any " carry forward " is to be carried forward *three* places.

Scalar Product $\qquad N = A^rB_r = A^1B_1 + A^2B_2 + A^3B_3$

$$\begin{array}{rrrr} & \bar{4}60\ . & 4\bar{5}1, & 24\bar{1} \\ & 0\bar{5}4\ . & \bar{3}10, & \bar{4}\bar{6}6 \\ \hline 0\bar{2}0 & 0\bar{4}0 & 0\bar{1}4 & 04\bar{4} \\ & 100 & \bar{1}60 & 0\bar{5}0 \\ & & 1\bar{3}0 & 400 \\ \hline \bar{2} & \bar{3}\ \ . & 6 & \bar{1} \end{array} \qquad N = \bar{2}\bar{3}\,.\,6\bar{1}$$

The first digit of each trio of B^r is multiplied by all the first digits of A^r; the second by the second, and the third by the third. Products not affecting the second duodecimal place are omitted. The digits of N are got by summing all those in each three-square compartment and carrying forward if necessary.

The square of the magnitude of a vector, A^rA_r, is found in the same way.

Vector Product $\qquad C^r = e^{rst}\,A_sB_t$

$$\begin{array}{rrrrl} & \bar{4}60 & 4\bar{5}1, & 24\bar{1} & \\ & 0\bar{5}4\ . & \bar{3}10, & \bar{4}\bar{6}6 & \\ \hline ,20 & ,\bar{2}0 & ,5\bar{5} & ,45 & \\ , & ,00 & ,00 & ,01 & A_2B_3 - A_3B_2 \\ , & , & ,30 & ,00 & \\ \hline \bar{1},0 & \bar{3},0 & 5,0 & \bar{4},0 & \\ , & , & 0,0 & 0,\bar{3} & A_3B_1 - A_1B_3 \\ , & , & \bar{2},0 & 0,0 & \\ \hline 20, & \bar{6}0, & 30, & 20, & \\ , & 0\bar{1}, & \bar{4}\bar{5}, & 43, & A_1B_2 - A_2B_1 \\ , & , & 2\bar{2}, & 00, & \\ \hline C^r = 212, & \bar{1}3\bar{4}\ . & 1\bar{3}\bar{4}, & \bar{2}13 & \end{array}$$

The first three lines of the product represent A_2B_3 (second columns of each trio) and A_3B_2 (third columns). The differences give the first digits of each trio of C^r in the bottom line. The second and third digits of C^r are found in the same way. The same method could be extended to find the contracted product of a vector and tensor of the second order if three lines are allowed for the latter.

Yours, etc., J. HALCRO JOHNSTON.

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